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# Educational Inequality

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## *Abstract*

*This paper develops a theoretical model of the inequality in wages and salaries associated with differences in years of schooling (educational inequality, for short). Our model assumes that in the long run individual decisions to become more educated equalize the lifetime earnings of more educated workers and comparable less educated workers. Given this assumption our model implies that innovations that increase the relative demand for more educated labor, and which cause short-run increases in educational inequality, in the long run induce offsetting increases in the relative supply of more educated labor. But, our model also has the novel implication that innovations that increase differences between the wages and salaries received by workers with the same years of education who are more or less able (ability premiums, for short) cause a smaller fraction of workers to choose to become more educated. Consequently, innovations that increase ability premiums in the long run also cause educational inequality to be larger than otherwise. In applying our theory to recent changes in educational inequality in the United States, we suggest that, to the extent that innovations that increase ability premiums are contributing to educational inequality, the increases in educational inequality during the 1980s and 1990s are unlikely to be soon reversed.*

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In recent decades American labor markets have seen increases both in the inequality in wages and salaries associated with differences in years of schooling (educational inequality, for short) and in the inequality in wages and salaries associated with differences in ability among workers with the same years of schooling (ability premiums, for short). This paper develops a theoretical model that explores the relation between educational inequality and ability premiums.

In the short run, because the relative supply of more educated labor is given, innovations that increase the relative demand for more educated labor cause an increase in educational inequality. In contrast, in the long run educational inequality depends both on innovations that affect the relative demand for more educated labor and on induced changes in the relative supply of more educated labor. To model the relative supply of more educated labor, we assume that in the long run individual decisions to become more educated equalize the lifetime earnings of more educated workers and comparable less educated workers.

Given this assumption, our model implies that innovations that increase the relative demand for more educated labor in the long run cause a larger fraction of workers to choose to become more educated and, hence, induce offsetting increases in the relative supply of more educated labor. More interestingly, our model also implies that innovations that increase ability premiums cause a smaller fraction of workers to choose to become more educated and, hence, cause the relative supply of more educated labor to be smaller than otherwise. As a result, we have the novel implication that innovations that increase ability premiums cause educational inequality to be larger than otherwise. To the extent that innovations that increase ability premiums are contributing to educational inequality, our analysis suggests that the increases in educational inequality during the 1980s and 1990s are unlikely to be soon reversed.

## 1. Definitions and Some Facts

We define educational inequality to be the ratio of the average wage or salary of workers

with more years of schooling to the average wage or salary of workers with fewer years of schooling.<sup>1</sup> Educational inequality comprises the relative earnings of efficiency units of more and less educated labor and the average abilities of more and less educated workers, measured as the average number of efficiency units of more or less educated labor that a worker supplies.

Looking at workers who have and have not attended college, Claudia Goldin and Lawrence Katz (2001) found that in the United States educational inequality, on our definition, exhibited a generally U-shaped pattern over the twentieth century. Educational inequality apparently decreased from 1900 until 1950, but then increased from 1950 to the end of the century. Strikingly, Goldin and Katz estimate that “the relative earnings of the more-educated [in 1999] are similar to that which prevailed in the early twentieth century.”

This U-shaped pattern, however, was not smooth. Looking at the first half of the century, Goldin and Katz found that educational inequality decreased in “two giant steps”, one in the years before and after 1920 and the other during the 1940s. Turning to the second half of the century, David Autor, Katz, and Alan Krueger (1998) found that educational inequality increased from 1950 to 1970, but decreased during the 1970s. Autor, Katz, and Krueger also quantified the large and widely discussed increase in educational inequality during the 1980s. Lawrence Mishel, Jared Bernstein, and John Schmitt (2001) find that educational inequality increased further over the 1990s, although at a slower rate than in the 1980s.

As indicated, the novel aspect of our analysis is to address how educational inequality is related in the long run to ability premiums, which we define to be differences between wages and salaries received by workers with the same years of education who are more or less able.

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<sup>1</sup>We should not confuse educational inequality with the return to schooling, which is a measure of the effect of years of schooling on a worker’s wage or salary. Educational inequality would provide an unbiased estimate of the return to schooling if and only if a worker’s years of schooling was uncorrelated with other characteristics that affect a worker’s wage or salary. See James Heckman and Edward Vytlacil (2000) on the problem of estimating the return to schooling.

Ability premiums result from the advantage in production associated with being more able, combined with factors that determine differences in ability, such as the innate characteristics of individual workers and the quality of the educations that workers receive.

Richard Murnane, John Willett, and Frank Levy (1995) report that the dependence of wages or salaries on a measure of mathematical skill, which we can take to be a proxy for ability, increased between the 1970s and the 1980s especially for college-educated workers and also for less educated workers. In addition, analyzing American data from 1963 to 1989, Chinhui Juhn, Kevin Murphy, and Brooks Pierce (1993) found that the residual inequality in wages or salaries that remains after accounting for years of education and experience increased steadily from the late 1960s through the 1980s. According to Mishel, Bernstein and Schmitt (2001) more recent data indicate that residual inequality peaked about 1995 and since then has declined. We can presume that these changes in residual inequality reflect, either wholly or in part, changes in ability premiums.<sup>2,3</sup>

## 2. Analytical Framework and Related Literature

Our framework for analyzing changes in educational inequality allows both for innovations that increase the relative demand for more educated labor and for innovations that increase ability premiums. In modeling the relative demand for more educated labor we focus on technological innovations that increase the relative importance of more educated labor in

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<sup>2</sup>Some authors refer to residual inequality as within-group inequality. Eric Gould, Omer Moav, and Bruce Weinberg (2001) point out that since the late 1960s inequality among less educated workers has increased as much as inequality among more educated workers. This observation, together with the results of Murnane, Willett, and Levy, suggests that an increased ability premium does not account completely for increased inequality among less educated workers. Appealing to the distinction introduced by Steven Davis and John Haltiwanger (1991) between inequality within plants and inequality among plants, Gould, Moav, and Weinberg associate increased inequality among less educated workers with “increasing variance of technological implementation across industries”.

<sup>3</sup>For simplicity, our theoretical analysis abstracts from inequality associated with experience. Juhn, Murphy, and Pierce (1993) report that their data also show an increase in the relative return to experience.

production. We could, however, readily generalize the analysis to take into account other factors that can increase the relative demand for more educated labor, such as innovations that increase the relative demand for education-intensive products, such innovations being a possible result of the removal of barriers to international trade.<sup>4</sup> Examples of innovations that increase ability premiums include increases in the advantage in production associated with higher ability and increases in differences in the quality of education that can underlie differences in ability.

The innovations that drive our analysis can be correlated. Also, as many authors have suggested, these innovations can be a concomitant of technological progress and economic growth. For example, Oded Galor and Moav (2000) analyze a model in which both the relative importance of more educated labor and the advantage associated with high ability depend on the rate of technological progress.<sup>5</sup> In their analysis the innovations on which we focus are perfectly correlated.

But, in both principle and in practice these innovations can occur independently of technological progress and economic growth and independently of each other. For example, an increase in the relative importance of more educated labor in production could be associated either with an increase or with a decrease in total factor productivity. As another example, increases in differences in the quality of education could result either from improvements in higher quality education or, as some people think has been happening, deterioration in

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<sup>4</sup>Autor, Levy, and Murnane (2001) claim that computer technology, which substitutes for routine tasks and complements problem-solving tasks, alone “explains thirty to forty percent of the observed relative demand shift favoring college versus non-college labor during 1970 to 1998, with the largest impact felt after 1980.” Similarly, Robert Baldwin and Glen Cain(1997) claim that technological innovations have been the main force in increasing educational equality, but James Harrigan and Rita Balaban (1999) report that both capital accumulation and a decrease in the price of traded goods also have contributed.

<sup>5</sup>Papers that make similar assumptions include Galor and Daniel Tsiddon (1997), Francesco Caselli (1999), Huw Lloyd-Ellis (1999), Philippe Aghion, Peter Howitt, and Giovanni Violante (1999), and Gould, Moav, and Weinberg (2001).

lower quality education. Also, from a theoretical perspective we think that distinguishing the effects of these innovations from each other and from the effects of technological progress, as we do in the present paper, provides a useful clarification.

In other related papers Daron Acemoglu (1998, 2000) assumes that the unusually rapidly increases in the relative number of more educated workers in the 1910s and 1920s and again in the 1970s induced the development of more education-intensive production technologies. He suggests that in subsequent decades the resulting endogenous response of demand for more educated labor was more than sufficient to reverse the initial decrease in educational inequality. The analysis in the present paper is complementary in that it treats the relative demand for more educated labor as exogenous and the relative supply of more educated labor as endogenous. Together Acemoglu's analysis and the present analysis imply that either an exogenous increase in the relative number of more educated workers or an exogenous increase in the relative demand for more educated labor could lead to an endogenous cycle in educational inequality. But, our analysis also shows that innovations that increase ability premiums cause the relative supply of more educated labor to be smaller than otherwise and, as a result, cause educational inequality to be larger than otherwise.

### 3. Educational Inequality in the Short Run

Assume that more educated labor and less educated labor perform complementary functions in the production process. Specifically, assume that the output per period of a representative firm in a representative industry, denoted by  $Y$ , is a Cobb-Douglas function of inputs of more educated labor and less educated labor, as in

$$(1) \quad Y = L_m^\sigma L_\ell^{1-\sigma}, \quad \sigma \in (0, 1),$$

where  $L_m$  and  $L_\ell$  denote the numbers of efficiency units of more educated labor and less educated labor that the firm employs per period. This formulation implies that more educated labor and less educated labor differ qualitatively. The parameter  $\sigma$  measures the

relative importance of more educated labor in production.<sup>6</sup>

Let  $\tilde{L}_m$  and  $\tilde{L}_\ell$  denote the quantities supplied per firm per period of efficiency units of more educated labor and less educated labor, and let  $w_m$  and  $w_\ell$  represent the earnings per period of an efficiency unit of more educated labor and less educated labor. Assume that the firm takes the earnings of an efficiency unit of labor as given and demands quantities of each type of labor such that the marginal product of an efficiency unit of labor equals the earnings of an efficiency unit of labor. Calculating marginal products from equation (1), and using the market-clearing conditions that  $\tilde{L}_m$  equals  $L_m$  and  $\tilde{L}_\ell$  equals  $L_\ell$ , we find that

$$(2) \quad \frac{w_m}{w_\ell} = \frac{\sigma}{1 - \sigma} \frac{\tilde{L}_\ell}{\tilde{L}_m}.$$

Equation (2) shows that the relative earnings per period of efficiency units of more and less educated labor depend negatively on the relative supply of more educated labor and positively on the relative importance of more educated labor in production.

A worker's wage or salary equals the product of the number of efficiency units of labor that he (or she) is discerned to supply per period and the earnings per period of an efficiency unit of his type of labor. We assume that each worker coarsely discerns his ability to supply efficiency units of either more educated labor or less educated labor. Specifically, each worker discerns whether he has high ability, or ordinary ability, or low ability.<sup>7</sup>

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<sup>6</sup>The Cobb-Douglas function in equation (1) is a special case of an aggregate production function that exhibits constant elasticity of substitution between more educated labor and less educated labor, as in  $Y = [\sigma L_m^\rho + (1 - \sigma) L_\ell^\rho]^{1/\rho}$ ,  $\rho < 1$ ,  $\rho \neq 0$ . Equation (1) obtains in the limit as the parameter  $\rho$  goes to zero. If we were to replace equation (1) with the general CES function, then we would have to replace subsequent equations that are derived using equation (1) with more complicated equations involving  $\rho$ . These equations are given in the mathematical appendix. As we can see, as long as  $\rho$  is not a large negative number, none of our qualitative conclusions, which we draw mainly from equations (11) and (12), are changed. Autor, Katz, and Krueger (1998) cite various estimates of the elasticity of substitution between workers who are or are not college educated, all of which imply that  $\rho$  is in the neighborhood of 1/3.

<sup>7</sup>As long as the innovations that we consider are not too small, the main qualitative results of the analysis

Low ability can be either innate or a result of the quality of the basic education that a worker receives. Ordinary ability or high ability can be either innate or a result of a combination of the quality of the basic education that a worker receives and the quality of the advanced education that is available to a worker. Whatever the cause of differences in ability, we assume that each worker discerns his ability before he decides either to become more educated or to remain less educated.<sup>8</sup>

We also assume that a worker's ability affects his possibilities for educational achievement in the following ways: (1) Workers with either high ability or ordinary ability are capable of becoming more educated. (2) Workers with high ability can realize their advantage over workers with ordinary ability only by becoming more educated. (3) Workers with low ability are not educable beyond a basic education.

In addition, we assume that employers can discern only coarsely the number of efficiency units of labor that a worker supplies per period. Specifically, employers discern that more educated workers with high ability on average supply  $\alpha$  efficiency units of labor per period and that more educated workers with ordinary ability on average supply  $\beta$  efficiency units of labor per period, where  $\beta < \alpha$ . Accordingly, the wage or salary of a more educated worker with high ability is  $\alpha w_m$ , and the wage or salary of a more educated worker with ordinary ability is  $\beta w_m$ . The ratio  $\alpha/\beta$  measures the ability premium for more educated workers.

Similarly, employers discern that less educated workers with ordinary ability on average supply  $\gamma$  efficiency units of labor per period and that less educated workers with low ability would not change if workers and employers were able to discern workers' abilities somewhat less coarsely into a larger, but finite, number of ability levels. The more elegant alternative of assuming that workers' abilities are finely discernable along a continuum seems unrealistic.

<sup>8</sup>This assumption implies that our analysis abstracts from stochastic elements in the effect of schooling either in making a worker more educated worker or in determining the number of efficiency units of labor that a worker supplies.



on average supply  $\delta$  efficiency units of labor per period, where  $\delta < \gamma$ . Accordingly, the wage or salary of a less educated worker with ordinary ability is  $\gamma w_\ell$ , and the wage or salary of a less educated worker with low ability is  $\delta w_\ell$ . The ratio  $\gamma/\delta$  measures the ability premium for less educated workers.

Let  $N_{mh}$  denote the number of more educated workers who have high ability, and let  $N_{mo}$  denote the number of more educated workers who have ordinary ability. Also, let  $N_{\ell o}$  denote the number of less educated workers who have ordinary ability, and let  $N_{\ell \ell}$  denote the number of less educated workers who have low ability. We define the short run to be an interval over which all of these numbers are predetermined. Given the quadruple  $\{\alpha, \beta, \gamma, \delta\}$ , the relative supply of efficiency units of more educated labor is related to the numbers of workers of each type according to

$$(3) \quad \frac{\tilde{L}_\ell}{\tilde{L}_m} = \frac{\gamma N_{\ell o} + \delta N_{\ell \ell}}{\alpha N_{mh} + \beta N_{mo}}.$$

Let  $W_m$  denote the average wage or salary per period of more educated workers, and let  $W_\ell$  denote the average wage or salary per period of less educated workers. We can express  $W_m$  in terms of the variables that we have already introduced as the aggregate earnings per period of more educated workers,  $(\alpha N_{mh} + \beta N_{mo})w_m$ , divided by the number of more educated workers,  $N_{mh} + N_{mo}$ , and we can express  $W_\ell$  as the aggregate earnings per period of less educated workers,  $(\gamma N_{\ell o} + \delta N_{\ell \ell})w_\ell$ , divided by the number of less educated workers,  $N_{\ell o} + N_{\ell \ell}$ . Equivalently, we can express  $W_m$  as the product of the earnings of an efficiency unit of more educated labor,  $w_m$ , and the average ability of more educated workers,  $(\alpha N_{mh} + \beta N_{mo})/(N_{mh} + N_{mo})$ , and similarly for  $W_\ell$ .

Using these expressions for  $W_m$  and  $W_\ell$  we can express the ratio  $W_m/W_\ell$ , which measures educational inequality, as

$$(4) \quad \frac{W_m}{W_\ell} = \frac{\alpha N_{mh} + \beta N_{mo}}{N_{mh} + N_{mo}} \frac{N_{\ell o} + N_{\ell \ell}}{\gamma N_{\ell o} + \delta N_{\ell \ell}} \frac{w_m}{w_\ell}.$$

Substituting equations (2) and (3), which determine the ratio  $w_m/w_\ell$ , into equation (4),

we obtain

$$(5) \quad \frac{W_m}{W_\ell} = \frac{\sigma}{1 - \sigma} \frac{N_{\ell o} + N_{\ell \ell}}{N_{mh} + N_{mo}}.$$

Equation (5) implies that educational inequality depends positively on the number of less educated workers relative to the number of more educated workers. Robert Topel (1997) reports that the data for many countries are consistent with this implication. In addition, equation (5) implies that, given the relative number of more educated workers, a technological innovation that increases the parameter  $\sigma$ , the relative importance of more educated labor, increases educational inequality.

Equation (5) also implies that educational inequality does not depend on the numbers of efficiency units of labor that workers on average supply per period, as given by quadruple  $\{\alpha, \beta, \gamma, \delta\}$ . An innovation that increases either  $\alpha$  or  $\beta$  would increase the average wage or salary of a more educated worker for given earnings per efficiency unit of more educated labor. But, an increased supply of efficiency units of more educated labor also would decrease the relative earnings per efficiency unit of more educated labor. With a Cobb-Douglas production function these two effects on educational inequality are exactly offsetting. A similar analysis applies to  $\gamma$  and  $\delta$ .<sup>9</sup>

#### 4. The Long Run

The relative number of more educated workers is predetermined only in the short run. In the long run the fraction of workers that chooses to become more educated responds both to innovations that increase the relative demand for more educated labor and to innovations that increase ability premiums. To facilitate the analysis we define the long run to be a steady state in which the fraction of workers with ordinary ability that chooses to become

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<sup>9</sup>With the general CES production function educational inequality in the short run would be either positively or negatively related to both  $\alpha$  and  $\beta$  and either negatively or positively related to both  $\gamma$  and  $\delta$  as  $\rho$  is positive or negative. But, as noted above, as long as  $\rho$  is not a large negative number, our qualitative conclusions are robust with respect to this complication. See the mathematical appendix.

more educated is the same in every age cohort and in which this fraction is such that the lifetime earnings of more educated workers and comparable less educated workers are equal. This definition implies that a worker chooses to become more educated or to remain less educated according to which choice yields him higher lifetime earnings.<sup>10</sup>

Realization of a new long run in which a larger fraction of workers in every age cohort chooses to become more educated would require that the structural parameters remain unchanged for as long as it takes for the educational sector to expand to accommodate the required larger fraction of each age cohort and, once the educational sector has expanded appropriately, for long enough for an age cohort to pass completely through a life cycle.<sup>11</sup> Thus, in practice, the long run only specifies a value to which at any point in time the fractions of more educated workers and less educated workers are tending. Nevertheless, by focusing on the properties of the long run we can see most simply and clearly how endogeneity of the relative supply of more educated labor affects educational inequality.

To analyze educational inequality in the long run, consider a constant population of workers per firm, normalized to one, with constant fractions of workers that have high ability, ordinary ability, and low ability. Let  $A_h$ ,  $A_o$ , and  $A_\ell$ , respectively, denote these fractions, where  $A_h + A_o + A_\ell = 1$ . Recall that workers with high ability can realize their advantage over workers with ordinary ability only by becoming more educated and that workers with low ability are not educable beyond a basic education.

Let  $M$  denote the fraction of workers with ordinary ability that chooses to become more

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<sup>10</sup>Our analysis abstracts from liquidity constraints on the ability to finance education. Galor and Moav (2000) point out that, if technological progress relaxes liquidity constraints on the ability to finance education, then technological progress can be associated with a decrease in educational inequality. Gould, Moav, and Weinberg (2001) introduce an additional precautionary motive for becoming more educated.

<sup>11</sup>An increasing fraction of more educated workers and an expanding educational sector is historically relevant. The hypothetical realization of a new long run with a smaller fraction of more educated workers and a smaller educational sector presumably would involve a different dynamic process.

educated. The fraction  $1 - M$  remains less educated. For ability premiums to be observed both for more educated workers and for less educated workers, both  $M$  and  $1 - M$  must be positive. Hence, in the long run workers with ordinary ability must be indifferent between becoming more educated and remaining less educated.<sup>12</sup> Moreover, if some workers with ordinary ability choose to become more educated, then all workers with high ability choose to become more educated.

Assume that each worker is active for  $T$  periods and that to become more educated a worker must spend  $\tau$  periods in school rather than in the work force, where  $\tau < T$ . Thus, in the long run  $A_h\tau/T$  young workers who have high ability and  $A_oM^*\tau/T$  young workers who have ordinary ability are in school, where the  $*$  denotes a value that obtains in the long run. Accordingly, in the long run the number of workers of each type who are in the work force is

$$\begin{aligned}
 N_{mh}^* &= A_h(T - \tau)/T, \\
 N_{mo}^* &= A_oM^*(T - \tau)/T, \\
 N_{\ell o}^* &= A_o(1 - M^*), \text{ and} \\
 N_{\ell \ell}^* &= A_\ell.
 \end{aligned}
 \tag{6}$$

Substituting equations (6) into equation (5) we obtain

$$\frac{W_m^*}{W_\ell^*} = \frac{\sigma}{1 - \sigma} \frac{A_o(1 - M^*) + A_\ell}{A_h + A_oM^*} \frac{T}{T - \tau}.
 \tag{7}$$

Equation (7) relates educational inequality in the long run to the fractions of the workers who have high, ordinary, or low ability, the fraction of the workers with ordinary ability

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<sup>12</sup>If workers' abilities were discernable, somewhat less coarsely, into a larger, but finite, number of ability levels, then the existence of ability premiums for both more educated and less educated workers would be consistent with all workers with ability higher than some critical level choosing to become more educated and all workers with ability lower than that critical level choosing to remain less educated. In this case, as noted above, the main qualitative results that we derive would obtain as long as the innovations that we consider are not too small.

that chooses to become more educated, the fraction of his active life that a more educated worker spends in the work force, and the relative importance of more educated labor in production. Again, we see that with a Cobb-Douglas production function, given the relative number of more educated workers, educational inequality does not depend on the quadruple  $\{\alpha, \beta, \gamma, \delta\}$ , which denotes the numbers of efficiency units of labor per period that workers supply.

## 5. The Choice to become More Educated

To determine the fraction of workers with ordinary ability that chooses to become more educated, let  $E_{mo}^*$  denote the lifetime earnings of a more educated worker with ordinary ability in the long run, and let  $E_{lo}^*$  denote the lifetime earnings of a less educated worker with ordinary ability in the long run. Given  $T$  and  $\tau$ , we have

$$(8) \quad E_{mo}^* = (T - \tau) \beta w_m^* \quad \text{and} \quad E_{lo}^* = T \gamma w_\ell^*.$$

In this formulation the cost of becoming more educated equals the earnings foregone while becoming more educated.

For workers with ordinary ability to be indifferent between becoming more educated and remaining less educated,  $E_{mo}^*$  must equal  $E_{lo}^*$ . Equating  $E_{mo}^*$  and  $E_{lo}^*$  we obtain

$$(9) \quad \frac{\beta w_m^*}{\gamma w_\ell^*} = \frac{T}{T - \tau}.$$

Given that we observe ability premiums both for more educated workers and for less educated workers, in the long run the fraction of the workers with ordinary ability that chooses to become more educated must be such that  $\beta w_m^*/\gamma w_\ell^*$  satisfies equation (9).

Equation (9) has the following implication:

*In the long run the average wage or salary of a more educated worker with ordinary ability relative to the average wage or salary of a less educated worker with ordinary ability depends only on the fraction of his active life that a more educated worker spends in the work force.*

Thus, equation (9) implies that in the long run, in response to an innovation that increases  $\sigma$ , the fraction of workers with ordinary ability that chooses to become more educated increases just enough to reverse the short-run increase in  $w_m/w_\ell$ .

Earnings per efficiency unit of more educated labor and less educated labor also must satisfy the market-clearing conditions. Substituting equation (3) into equation (2), and using equation (6), we find that market clearing implies that

$$(10) \quad \frac{\beta w_m^*}{\gamma w_\ell^*} = \frac{\sigma}{1-\sigma} \frac{N_{\ell o}^* + (\delta/\gamma)N_{\ell \ell}^*}{(\alpha/\beta)N_{mh}^* + N_{mo}^*} = \frac{\sigma}{1-\sigma} \frac{A_o(1-M^*) + (\delta/\gamma)A_\ell}{(\alpha/\beta)A_h + A_o M^*} \frac{T}{T-\tau}.$$

Solving equations (9) and (10) for  $M^*$ , we obtain

$$(11) \quad M^* = \sigma \left(1 + \frac{\delta A_\ell}{\gamma A_o}\right) - (1-\sigma) \frac{\alpha}{\beta} \frac{A_h}{A_o}.$$

We assume that the values of the parameters are such that  $M^*$ , as given by equation (11), is a positive fraction.

Equation (11) implies that  $M^*$  is positively related to  $\sigma$ , an effect that already was implicit in equation (9). Thus, equation (11), together with equation (7), implies that innovations that increase the relative demand for more educated labor in the long run induce an increase in the relative supply of more educated labor that counteracts the short-run increase in educational inequality.

More interestingly, equation (11) also implies that  $M^*$  is negatively related to the ratios  $\alpha/\beta$  and  $\gamma/\delta$ . We can understand this result as follows: For any given value of the ratio  $\beta/\gamma$ , the larger is either  $\alpha/\beta$  or  $\gamma/\delta$  the larger is the ratio  $\alpha/\delta$  and, hence, for any given value of  $M^*$ , the larger would be the supply of more educated labor relative to the supply of less educated labor. Accordingly, equation (10) implies that, given  $M^*$ , the larger is either  $\alpha/\beta$  or  $\gamma/\delta$  the smaller would be the value of the ratio  $\beta w_m^*/\gamma w_\ell^*$  that would satisfy the market-clearing conditions. But, the ratio  $\beta w_m^*/\gamma w_\ell^*$  also must satisfy the equality  $E_{\ell o}^* = E_{mo}^*$ , and the implied equation (9), according to which  $\beta w_m^*/\gamma w_\ell^*$  depends only on the fraction of his active life that a more educated workers spends in the work force. Thus,

given  $\beta/\gamma$ , to satisfy the equality  $E_{\ell o}^* = E_{m o}^*$ , the larger is either  $\alpha/\beta$  or  $\gamma/\delta$  the smaller must be  $M^*$  in order to offset the effect of either  $\alpha/\beta$  or  $\gamma/\delta$  on the supply of more educated labor relative to the supply of less educated labor.<sup>13</sup>

In sum, equation (11) has the following implications:

*Innovations that increase the relative importance of more educated labor cause a larger fraction of the workers with ordinary ability to choose to become more educated, whereas innovations that increase ability premiums cause a smaller fraction of workers with ordinary ability to choose to become more educated.*

The net change in the fraction of workers that chooses to become more educated depends on the net effect of innovations that increase the relative importance of more educated labor and innovations that increase ability premiums.<sup>14</sup>

## 6. Educational Inequality in the Long Run

Substituting equation (11) for  $M^*$ , the fraction of workers with ordinary ability who

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<sup>13</sup>Given a Cobb-Douglas production function only the ratios  $\alpha/\beta$  and  $\gamma/\delta$  appear in equation (11). With the general CES production function  $M^*$  also would depend on the ratio  $\gamma/\beta$ , either positively or negatively as  $\rho$  is negative or positive. But, as long as  $\rho$  is not a large negative number,  $M^*$  still would be negatively related to both  $\alpha$  and  $\gamma$  and positively related to both  $\beta$  and  $\delta$ . The same effect would carry over to the relations between  $W_m^*/W_\ell^*$  and the quadruple  $\{\alpha, \beta, \gamma, \delta\}$ . See equations (11') and (12') in the mathematical appendix.

<sup>14</sup>Equation (11) also tells us that  $M^*$  does not depend on either  $T$  or  $\tau$ . This result obtains because the larger is  $T/(T - \tau)$  the larger is  $E_{m o}^*$  relative to  $E_{\ell o}^*$  for a given ratio  $\beta w_m^*/\gamma w_\ell^*$ , but also the smaller is the ratio  $\beta w_m^*/\gamma w_\ell^*$  that satisfies market-clearing conditions. Given the assumed Cobb-Douglas production function, these two effects are offsetting. Hence, the value of  $M^*$  that satisfies the equality  $E_{\ell o}^* = E_{m o}^*$  is independent of  $T/(T - \tau)$ . With the general CES production function  $M^*$  would be either positively or negatively related to  $T$  as  $\rho$  is positive or negative. If  $\rho$  is positive, then the conclusions about the effect of  $T/(T - \tau)$  on  $W_m^*/W_\ell^*$  that we draw from equation (12) would be reinforced. See Sebnem Kalemli-Ozcan, Harl Ryder, and David Weil (2000) for a model in which the longer that a worker expects to live the larger is the number of years of schooling that he chooses.

choose in the long run to become more educated, into equation (7), which relates  $W_m^*/W_\ell^*$ , educational inequality in the long run, to  $M^*$ , we obtain a solution for  $W_m^*/W_\ell^*$  as a function of the exogenous variables,

$$(12) \quad \frac{W_m^*}{W_\ell^*} = \frac{1 + \frac{\sigma}{1-\sigma} A_\ell \left(1 - \frac{\delta}{\gamma}\right) + A_h \left(\frac{\alpha}{\beta} - 1\right)}{1 - A_\ell \left(1 - \frac{\delta}{\gamma}\right) - \frac{1-\sigma}{\sigma} A_h \left(\frac{\alpha}{\beta} - 1\right)} \frac{T}{T - \tau}.$$

Equation (12) tells us how educational inequality depends in the long run on the interplay between the relative importance of more educated labor in production and the ability premiums for both more educated workers and less educated workers.

From equation (12) we see that, depending on the ratios  $\alpha/\beta$  and  $\gamma/\delta$ ,  $W_m^*/W_\ell^*$  can be either positively or negatively related to  $\sigma$ . Specifically, equation (12) has the following implications about the long-run effects of increases in  $\sigma$ :

- For any value of  $\gamma/\delta$ , if  $\alpha/\beta$  is sufficiently close to one, then educational inequality in the long run is positively related to  $\sigma$ , whereas, if  $\alpha/\beta$  is sufficiently large, then educational inequality in the long run is negatively related to  $\sigma$ .
- For any value of  $\alpha/\beta$ , if  $\gamma/\delta$  is sufficiently close to one, then educational inequality in the long run is negatively related to  $\sigma$ .
- In the limit as both of the ratios  $\alpha/\beta$  and  $\gamma/\delta$  approach one,  $W_m^*/W_\ell^*$  becomes independent of  $\sigma$ . In this case, in the absence of ability premiums, induced long-run increases in the relative supply of more educated labor exactly offset the effect on educational inequality of innovations that increase the relative demand for more educated labor.

To understand these implications, recall from equation (9) that an increase in  $\sigma$  in the long run causes sufficiently more workers with ordinary ability to become more educated to offset the short-run effect of  $\sigma$  on the relative earnings per efficiency unit of more educated



labor and to leave  $w_m^*/w_\ell^*$  unchanged. But, an increase in the fraction of workers with ordinary ability who become more educated, and the associated decrease in the fraction of workers with ordinary ability who remain less educated, also cause a decrease both in the average number of efficiency units of more educated labor that more educated workers supply and in the average number of efficiency units of less educated labor that less educated workers supply. The net effect on  $W_m^*/W_\ell^*$  depends on which of these effects is more important.

Briefly stated we can summarize the relation between  $W_m^*/W_\ell^*$  and  $\sigma$  as follows:

*By causing a larger fraction of the workers with ordinary ability to choose to become more educated, innovations that increase the relative importance of more educated labor in the long run cause increases in the relative supply of more educated labor that counteract short-run increases in educational inequality and that, depending on ability premiums, can result in either smaller net increases or even decreases in educational inequality.*

Equation (12) also implies that, as we already saw in equation (7),  $W_m^*/W_\ell^*$  is larger the larger is  $T/(T-\tau)$ . The effect of  $T/(T-\tau)$  on educational inequality obtains because the smaller is the fraction of his active life that a more educated worker spends in the work force the smaller is the relative supply of more educated labor.

Most interestingly, equation (12) implies that  $W_m^*/W_\ell^*$  is larger the larger are the ratios  $\alpha/\beta$  and  $\gamma/\delta$ . This result obtains because, as we have seen from equation (11), an increase in either  $\alpha/\beta$  or  $\gamma/\delta$  causes a decrease in  $M^*$ . With a smaller fraction of workers with ordinary ability choosing to become more educated, a larger fraction of more educated workers has high ability, and a larger fraction of less educated workers has low ability. Consequently, the average wage or salary of more educated workers is larger and the average wage or salary of less educated workers is smaller.

Thus, our model yields the following novel result:

*By causing a smaller fraction of workers with ordinary ability to choose to be-*

*come more educated, innovations that increase ability premiums cause educational inequality in the long run to be larger than it otherwise would be.*

## 7. The Observed Pattern of Educational Inequality

Over the past few centuries life expectancy at all ages has steadily increased. A concomitant of this steady increase in life expectancy has been a steady increase in the fraction of his active life that a more educated worker can spend in the work force. In terms of our model, there has been a secular increase in  $(T - \tau)/T$ . Equation (12) implies that the secular increase in the relative supply of more educated labor resulting from the secular increase in  $(T - \tau)/T$  tends to decrease educational inequality. Goldin and Katz (2001) suggest that over the first half of the twentieth century the increased relative supply of more educated labor seems to have been the dominant influence on educational inequality, despite technological innovations that increased the relative importance of more educated labor in production.<sup>15</sup>

Now, suppose that during 1950s and 1960s and again during the 1980s and 1990s innovations that increased the relative demand for more educated labor were large enough to outweigh the effect of continuing increases in the relative supply of more educated labor. In our analysis we have modeled innovations that increased the relative demand for more educated labor by increases in  $\sigma$ . These innovations could account for the increases in educational inequality during these decades.<sup>16</sup>

Our analysis implies, however, that, to the extent that increases in educational inequality

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<sup>15</sup>Although not formally included in our model, expansion of publicly-financed higher education, which steadily decreased the out-of-pocket cost of becoming more educated, was another factor that caused an increase in the supply of more educated labor from the middle of the nineteenth century until at least the middle of the twentieth century.

<sup>16</sup>Autor, Katz, and Krueger (1998) speculate that a temporary slowing in the rate of increase in the relative supply of more educated labor, resulting from exogenous demographic factors, also contributed to the increases in educational inequality during the 1980s and 1990s.

have reflected the short-run effect of increases in  $\sigma$ , these increases in educational inequality will be reversed in the long run. We might speculate that the slowing of the rate of increase in educational inequality in the 1990s compared to the 1980s indicates the beginning of this process. As we have seen, in the long run increases in  $\sigma$  even can cause decreases in educational inequality.

But, our analysis also suggests that there is more to the story than increases in the relative demand for more educated labor. The data tell us that ability premiums increased steadily from the late 1960s until the mid 1990s. Moreover, as we have seen, our model has the critical implication that increases in ability premiums cause a smaller fraction of workers with ordinary ability to choose to become more educated. Hence, in the long run increases in ability premiums cause educational inequality to be larger than otherwise.

This implication suggests an additional reason for the increase in educational inequality during the 1980s and 1990s. More importantly, to the extent that increases in ability premiums are contributing in the long run to educational inequality, the theory implies that, although induced increases in the relative supply of more educated labor may moderate educational inequality, the increases in educational inequality during the 1980s and 1990s are unlikely to be soon reversed.

## 8. Summary

This paper has developed a theoretical model that relates changes in educational inequality to the combined effects of innovations that have increased the relative demand for more educated labor and innovations that have increased ability premiums. In the short run, in which the number of more educated workers is given, an increase in the relative demand for more educated labor causes an increase in educational inequality. In the long run, however, innovations that increase the relative demand for more educated labor also have the counteracting effect of inducing a larger fraction of workers to choose to become more educated.

But, our analysis also implies that educational inequality is positively related to ability premiums. Most interestingly, the larger are ability premiums the smaller is the fraction of workers that must choose to become more educated in order to equalize the lifetime earnings of more educated workers and comparable less educated workers. Consequently, in the long run innovations that cause increases in ability premiums also cause the relative supply of more educated labor to be smaller than otherwise and cause educational inequality to be larger than otherwise. In applying our theory to recent changes in educational inequality in the United States, we suggest that innovations that have increased ability premiums are preventing a reversal of recent increases in educational inequality.

## References

- Acemoglu, Daron. "Technical Change, Inequality, and the Labor Market." National Bureau of Economic Research (Cambridge MA) Working Paper No. 7800, July 2000.
- Acemoglu, Daron. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Quarterly Journal of Economics*, November 1998, *113*(3), pp. 1055-1089.
- Aghion, Philippe, Howitt, Peter, and Violante, Giovanni L. "General Purpose Technology and Within-Group Inequality." unpublished, December 1999.
- Autor, David H., Katz, Lawrence F., and Krueger, Alan B. "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics*, November 1998, *113*(3), pp. 1169-1213.
- Autor, David H., Levy, Frank, and Murnane, Richard J. "The Skill Content of Recent Technological Change: An Empirical Exploration." National Bureau of Economic Research (Cambridge MA) Working Paper No. 8337, June 2001.
- Baldwin, Robert E. and Cain, Glen G. "Shifts in U.S. Relative Wages: The Role of Trade, Technology and Factor Endowments." National Bureau of Economic Research (Cambridge MA) Working Paper No. 5934, February 1997.
- Caselli, Francesco, "Technological Revolutions." *American Economic Review*, March 1999, *89*(1), pp. 78-102.
- Davis, Steven J. and Haltiwanger, John. "Wage Dispersion Between and Within U.S. Manufacturing Plants, 1963-86." *Brookings Papers on Economic Activity: Microeconomics*, 1991, pp. 115-180.
- Galor, Oded and Moav, Omer. "Ability-Biased Technological Transition, Wage Inequality, and Economic Growth." *Quarterly Journal of Economics*, May 2000, *115*(2), pp. 469-497.
- Galor, Oded and Tsiddon, Daniel. "Technological Progress, Mobility, and Economic Growth." *American Economic Review*, June 1997, *87*(3), pp. 363-381.
- Goldin, Claudia and Katz, Lawrence F. "Decreasing and then Increasing Inequality in Amer-

- ica: A Tale of Two Half Centuries,” in Finis Welch, ed., *The Causes and Consequences of Increasing Inequality*, Chicago: University of Chicago Press, 2001.
- Gould, Eric D., Moav, Omer, and Weinberg, Bruce A. “Precautionary Demand for Education, Inequality, and Technological Progress.” *Journal of Economic Growth*, December 2001, 6(4), pp. 285-315.
- Harrigan, James and Balaban, Rita. “U.S. Wages in General Equilibrium: The Effects of Prices, Technology, and Factor Supplies, 1963-1991.” National Bureau of Economic Research (Cambridge MA) Working Paper No. 6981, February 1999.
- Heckman, James and Vytlačil, Edward. “Identifying the Role of Cognitive Ability in Explaining the Level of and Change in the Return to Schooling.” National Bureau of Economic Research (Cambridge MA) Working Paper No. 7820, August 2000.
- Juhn, Chinhui, Murphy, Kevin M., and Pierce, Brooks. “Wage Inequality and the Rise in Returns to Skill.” *Journal of Political Economy*, June 1993, 101(3), pp. 410-442.
- Kalemli-Ozcan, Sebnem, Ryder, Harl E., and Weil, David N. “Mortality Decline, Human Capital Investment, and Economic Growth.” *Journal of Development Economics*, June 2000, 62(1), pp.1-23.
- Lloyd-Ellis, Huw. “Endogenous Technological Change and Wage Inequality.” *American Economic Review*, March 1999, 89(1), pp. 47-77.
- Mishel, Lawrence, Bernstein, Jared, and Schmitt, John. *The State of Working America 2000-2001*, Ithaca: Cornell University Press, 2001.
- Murnane, Richard, Willett, John, and Levy, Frank. “The Growing Importance of Cognitive Skills in Wage Determination.” *The Review of Economics and Statistics*, May 1995, 77(2), pp. 251-266.
- Topel, Robert H. “Factor Proportions and Relative Wages: The Supply-Side Determinants of Wage Inequality.” *Journal of Economic Perspectives*, Spring 1997, 11(2), pp. 55-74.

## Mathematical Appendix: General CES Production Function

With a general CES production function, equations (1), (5), (11), and (12) would become the following:

$$(1') \quad Y = [\sigma L_m^\rho + (1 - \sigma) L_\ell^\rho]^{1/\rho}, \quad \rho < 1, \quad \rho \neq 0.$$

$$(5') \quad \frac{W_m}{W_\ell} = \frac{\sigma}{1 - \sigma} \frac{N_{\ell o} + N_{\ell \ell}}{N_{mh} + N_{mo}} \left( \frac{\alpha N_{mh} + \beta N_{mo}}{\gamma N_{\ell o} + \delta N_{\ell \ell}} \right)^\rho.$$

$$(11') \quad M^* = \frac{\left( \frac{\sigma}{1 - \sigma} \right)^{\frac{1}{1-\rho}} \left( 1 + \frac{\delta}{\gamma} \frac{A_\ell}{A_o} \right) - \frac{\alpha}{\beta} \frac{A_h}{A_o} \left( \frac{\gamma}{\beta} \frac{T}{T - \tau} \right)^{\frac{\rho}{1-\rho}}}{\left( \frac{\sigma}{1 - \sigma} \right)^{\frac{1}{1-\rho}} + \left( \frac{\gamma}{\beta} \frac{T}{T - \tau} \right)^{\frac{\rho}{1-\rho}}}.$$

$$(12') \quad \frac{W_m^*}{W_\ell^*} = \frac{\left( \frac{\sigma}{1 - \sigma} \right)^{\frac{1}{1-\rho}} \frac{T}{T - \tau}}{\left( \frac{\gamma}{\beta} \frac{T}{T - \tau} \right)^{\frac{\rho}{1-\rho}} T - \tau} \bullet$$

$$\frac{\left( \frac{\sigma}{1 - \sigma} \right)^{\frac{1}{1-\rho}} A_\ell \left( 1 - \frac{\delta}{\gamma} \right) + \left[ A_h \left( \frac{\alpha}{\beta} - 1 \right) + 1 \right] \left( \frac{\gamma}{\beta} \frac{T}{T - \tau} \right)^{\frac{\rho}{1-\rho}}}{\left( \frac{\sigma}{1 - \sigma} \right)^{\frac{1}{1-\rho}} \left[ 1 - A_\ell \left( 1 - \frac{\delta}{\gamma} \right) \right] - A_h \left( \frac{\alpha}{\beta} - 1 \right) \left( \frac{\gamma}{\beta} \frac{T}{T - \tau} \right)^{\frac{\rho}{1-\rho}}}.$$