Unemployment and Indeterminacy

Tomoyuki Nakajima*
Brown University and Kyoto University

November, 2003

Abstract

Using an efficiency-wage model, we examine the relationship between indeterminacy and unemployment insurance. It is shown that the less unemployment insurance is, the more likely equilibrium is to be indeterminate. Equilibrium can be indeterminate even without externalities or increasing returns, which makes a sharp contrast to the recent literature on indeterminacy. Our result is based on the fact that the no-shirking condition with marginal utility of wealth kept constant is downward sloping when income insurance is not perfect.

Key words: involuntary unemployment; efficiency wage; indeterminacy; sunspots.
JEL classification numbers: E10; E24; E32; J41.

*Address: Department of Economics, Brown University, Box B, Providence, RI 02912, USA. Tel: 401-863-3807. Email: tomoyuki_nakajima@brown.edu.

1 Introduction

In the last decade or so, there has been a growing body of research on indeterminacy in dynamic general equilibrium models.[1] In the context of macroeconomics, the existence of a continuum of equilibria raises the possibility that the economy fluctuates purely driven by self-fulfilling expectations and may justify the "animal spirit hypothesis" of business cycles. In their seminal work, Benhabib and Farmer (1994) have obtained indeterminacy in a one-sector neoclassical growth model with externalities. For this, they require that externalities be so large that the labor demand curve is upward sloping and steeper than the labor supply curve. The major criticism against their work has been that the amount of externalities required for indeterminacy is too large. Since then, a lot of research has made a progress to modify the model of Benhabib and Farmer (1994) to obtain indeterminacy with a lower amount of externalities. Examples include, among others, the model with sector-specific externalities by Benhabib and Farmer (1996); the model with home production by Perli (1996); the two-sector model by Benhabib and Nishimura (1998); the model with variable capital utilization by Wen (1998); the monetary model with borrowing constraint by Barinci and Chéron (2001).

Here, we take a different approach by looking at a source of indeterminacy different from externalities, that is, limited risk sharing between the employed and the unemployed. Specifically, we consider a version of the efficiency wage model of Alexopoulos (2002). The model is similar to the standard one-sector neoclassical growth model except that a worker's effort is not perfectly observable by firms. The wage rate should, therefore, be set to prevent workers from shirking. Given the wage rate set in such a way, firms hire workers according to their labor demand, which generates unemployment in equilibrium. With full income insurance, the model reduces to that of the standard growth model with utility linear in leisure, and the result of Behnabib and Farmer (1994) applies. With partial income insurance, however, the model can generate indeterminacy even without externalities.

To see why indeterminacy is possible in our model without externalities, it is useful to look at the labor market equilibrium, as illustrated by Benhabib and Farmer (1994) and others. There, the relevant labor supply curve is the one with constant marginal utility of wealth (the "Frisch labor supply curve"). In the standard model where the steady state is a saddle point, the labor demand curve slopes down and the Frisch labor supply curve slopes up; an increase in marginal utility of wealth shifts down the Frisch labor supply curve, and increases labor input and output. In the model of Benhabib and Farmer (1994), if the labor demand curve is upward sloping and

[1] A useful survey on this area of research is given by Benhabib and Farmer (1999).

steeper than the labor supply curve, higher marginal utility of wealth reduces labor and output by shifting down the labor supply curve. This non-standard feature of the labor market is the key for indeterminacy in their model. Also, in the two-sector model of Benhabib and Farmer (1996), if sector-specific externalities are large enough, higher marginal utility of wealth shifts up the Frisch labor supply curve, resulting in a decrease in labor input and output, even when the labor demand curve is downward sloping.

The mechanism that generates indeterminacy in our model is also understood by looking at the labor market equilibrium. As is well known (Shapiro and Stiglitz, 1984), in an efficiency wage model the labor market equilibrium is described by the incentive-compatibility condition (or "no-shirking condition") in place of the labor supply curve. The relevant no-shirking condition in our dynamic model is the one with constant marginal utility of wealth, which is referred to as the "Frisch NSC." With full income insurance, the Frisch NSC is horizontal, which makes the model equivalent to the standard model with utility linear in leisure, leading to determinacy with constant-returns technology. When income insurance is only partial, however, the Frisch NSC becomes downward sloping. This is because when the level of consumption differs between the employed and the unemployed, an increase in employment tends to reduce the average level of marginal utility of consumption, because marginal utility of consumption of the employed is lower. If marginal utility of wealth is kept constant, this tends to reduce the average amount of consumption, which, in turn, tends to reduce the incentive-compatible wage rate. It is also shown that the Frisch NSC gets steeper with less unemployment benefits, thus with higher inequality in consumption between the employed and the unemployed. This is because the effect of a change in employment on the average marginal utility of consumption is greater with less unemployment benefit. Thus, for a sufficiently low level of unemployment benefits, the Frisch NSC is steeper than the labor demand curve. In such a case, higher marginal utility of wealth is associated with lower labor input and output, even when the labor demand curve slopes down. This is the mechanism that generates indeterminacy in our model.[2]

Our analysis shows that indeterminacy is more likely to occur if there is more inequality in consumption between the employed and the unemployed. In our calibration exercise, indeterminacy occurs with constant returns technology if the consumption of the unemployed is less than

[2]Our work is closely related to Bennett and Farmer (2000), who have emphasized the potential importance of a downward sloping Frisch labor supply curve to generate indeterminacy. As shown by Hintermaier (2003) and Nakajima (2001), however, as long as utility is concave between consumption and leisure, the Frisch labor supply curve cannot slope down. Here, the Frisch NSC is downward sloping in spite of the concavity of the utility function.

75 percent of that of the employed. This seems to be a quite plausible number. For example, based on the evidence of Gruber (1997), Alexopoulos (2002) and Burnside, Eichenbaum and Fisher (2000) set this value to 78 percent in their quantitative analysis of the related model.

The rest of the paper is organized as follows. Section 2 describes the economy. The case of full income insurance is analyzed in Section 3. The case of partial income insurance is in Section 4. Section 5 provides concluding remarks.

2 Description of the Economy

The economy considered in this paper is a version of the efficiency wage model of Alexopoulos (2002). It is similar to the one-sector neoclassical growth model except that a worker's effort is imperfectly observable by firms. Firms set the wage rate so that prevents workers from shirking on the job. Given the wage rate set in such a way, the number of employed workers is determined according to the demand for labor, which generates unemployment in equilibrium.

2.1 Households

The representative household consists of a unit-measure continuum of individuals. As shown below, household members differ in the level of consumption, which depends on their employment status. To maintain the representative-agent framework, we follow Alexopoulos (2002) to assume that the household owns the stock of capital and makes all capital (saving) related decisions. Individuals are not allowed to borrow or save: they simply consume their income at each point in time.

2.1.1 Capital accumulation

Let K_t be the stock of capital owned by the household at time t. It evolves over time as

$$\dot{K}_t = I_t - \delta K_t, \tag{1}$$

where I_t is investment at time t, and δ is the depreciation rate.

At each point in time, the household rents capital to firms at the rate R_t. The rental income net of investment, $R_t K_t - I_t$, is distributed equally to each household member, which is the minimum level of income guaranteed regardless of their employment status or unemployment benefits. Let C_t^h denote this amount:

$$C_t^h \equiv R_t K_t - I_t. \tag{2}$$

2.1.2 Unemployment insurance

At each point in time, household members are different in the following respects. First, they are either employed or unemployed. Second, employed individuals may or may not shirk. Third, shirkers may or may not be caught. At each point in time, randomly picked N_t members of the household receive job offers (as we shall see below, no one will turn down the job offer).

To share the risk of unemployment, the household organizes a fully funded unemployment insurance program for its members.[3] Let F_t be the transfer that employed individuals make for the unemployed. The total amount of the fund, N_tF_t, is distributed equally among (eligible) unemployed members.

Depending on the size of unemployment benefits, unemployment is either voluntary or involuntary. If unemployment benefits are large, an unemployed individual's utility can be higher than that of an employed (non-shirking) individual, and unemployment is voluntary. This is the case, for example, when the income insurance is full, that is, when unemployed individuals receive the same income as employed (non-shirking) individuals. In such a case, we need a mechanism that prevents individuals from turning down a job offer. For that purpose, we assume that the household observes which members receive job offers and that individuals rejecting offers are not eligible for unemployment benefits. With this mechanism, we shall ignore the possibility of job offers' being turned down in what follows.

2.1.3 Individuals

Each employed individual works for a fixed number of hours, h. Firms offer a contract that specifies the required effort level, e_t and a wage rate W_t. It also stipulates that if a worker is caught shirking his wage rate will go down to sW_t, where $s \in (0, 1)$ is an exogenous parameter. A shirker is caught with probability $d \in (0, 1)$.

Let C_t be the consumption of an employed individual who does not shirk:

$$C_t = C_t^h + hW_t - F_t. \tag{3}$$

The consumption of an employed individual who shirks but does not get caught is also given by C_t. Since a detected shirker receives only the fraction s of the wage rate, his consumption, C_t^s, is

$$C_t^s = C_t^h + shW_t - F_t. \tag{4}$$

[3] It is straightforward to reformulate the model so that the unemployment insurance program is provided by the government rather than the household.

The consumption of an unemployed individual, C_t^u, is

$$C_t^u = C_t^h + \frac{N_t}{1 - N_t} F_t. \tag{5}$$

We assume that the intra-household transfer, F_t, is determined by

$$F_t = \sigma(1 - N_t) h W_t, \tag{6}$$

where $\sigma \in [0, 1]$ is the exogenous parameter that measures the degree of income insurance. For example, when $\sigma = 1$, income insurance is full, and the consumption of each type of individuals becomes

$$\begin{aligned} C_t &= C_t^h + N_t h W_t, \\ C_t^s &= C_t^h + (N_t + s - 1) h W_t, \\ C_t^u &= C_t^h + N_t h W_t. \end{aligned}$$

Thus, $C_t = C_t^u$ if $\sigma = 1$. When income insurance is partial, $\sigma < 1$,

$$C_t = C_t^h + \big[1 - \sigma(1 - N_t)\big] h W_t, \tag{7}$$

$$C_t^s = C_t^h + \big[s - \sigma(1 - N_t)\big] h W_t, \tag{8}$$

$$C_t^u = C_t^h + \sigma N_t h W_t. \tag{9}$$

In this case, $C_t^u < C_t$, and, for a sufficiently small σ, unemployment becomes involuntary. Alexopoulos (2002) and Burnside, Eichenbaum and Fisher (2000) consider the particular case of partial income insurance, in which $\sigma = s$. When $\sigma = s$, $C_t^u = C_t^s$, and the unemployed and the detected shirkers get the same utility.

The instantaneous utility of an individual with a consumption level C and an effort level e is given by

$$U(C, e) = \begin{cases} \ln(C) + \theta \ln(T - \xi - he), & \text{if } e > 0, \\ \ln(C) + \theta \ln(T), & \text{if } e = 0, \end{cases} \tag{10}$$

where $\theta > 0$, T is the time endowment, and ξ is the fixed cost of exerting nonzero effort.[4] An employed non-shirker obtains instantaneous utility of $U(C_t, e_t)$. It is clear that any shirker chooses $e = 0$, so that a shirker's expected utility is $(1-d)U(C_t, 0) + dU(C_t^s, 0)$. An unemployed worker's utility flow is $U(C_t^u, 0)$.

[4] As explained in Appendix, considering a more general class of utility of the form,

$$U(C, e) = \ln(C) + \frac{\theta}{1 - \gamma}(T - \xi - he)^{1-\gamma},$$

does not affect our result on indeterminacy at all.

2.1.4 The household's problem

Let N_t^s be the number of employed individuals who shirk at time t. The household takes $\{R_t, W_t, N_t, N_t^s\}$ as given, and chooses $\{C_t^h, I_t\}$ to maximize the lifetime utility of its members:

$$\int_0^\infty e^{-\rho t}\Big\{(N_t - N_t^s)U(C_t, e_t) + N_t^s\big[(1-d)U(C_t, 0) + dU(C_t^s, 0)\big] \\ + (1 - N_t)U(C_t^u, 0)\Big\}\, dt, \tag{11}$$

subject to (1), (2), and (7)-(9).

2.2 Firms

At each time t, output is produced by perfectly competitive firms using the technology:

$$Y_t = A_t K_t^\alpha \big[h e_t (N_t - N_t^s)\big]^{1-\alpha},$$

where A_t is the externality factor:

$$A_t = \overline{K}_t^{\alpha\eta}\big[h \overline{e}_t(\overline{N}_t - \overline{N}_t^s)\big]^{(1-\alpha)\eta}.$$

When $\eta = 0$ there are no externalities. As we shall see below, when $\eta = 0$, equilibrium is determinate under full insurance, but it can be indeterminate if insurance is only partial.

2.2.1 Profit maximization

It is not profitable for firms to allow worker to shirk, so that they offer a contract to make $N_t^s = 0$. The problem of a firm is thus given by

$$\max_{e_t, W_t, N_t, K_t} \Big\{A_t K_t^\alpha (h e_t N_t)^{1-\alpha} - hW_t N_t - R_t K_t\Big\},$$

subject to the incentive compatibility constraint:[5]

$$U(C_t, e_t) \geq (1-d)U(C_t, 0) + dU(C_t^s, 0), \tag{12}$$

where $U(C, e)$ is as defined in (10), and C_t and C_t^s are given in (3)-(4). In (3)-(4), firms take C_t^h and F_t as given.

[5] Remember that no individual will turn down the job offer. In other words, the individual rationality constraint is satisfied. In addition, it is non-binding.

The incentive compatibility constraint (12) will bind in the firms' problem, so that we can solve (12) for e_t as a function of C_t/C_t^s:[6]

$$e_t = \frac{T-\xi}{h} - \frac{T}{h}\left(\frac{C_t}{C_t^s}\right)^{-\frac{d}{\theta}}. \tag{13}$$

Note that since the firm takes F_t and C_t^h as given, it views C_t/C_t^s as a function of W_t:

$$\frac{C_t}{C_t^s} = \frac{hW_t + C_t^h - F_t}{shW_t + C_t^h - F_t},$$

Hence, we write the right-hand side of (13) as a function of W_t: $e(W_t; C_t^h, F_t)$.

Given the incentive compatibility constraint $e_t = e(W_t; C_t^h, F_t)$, the first-order conditions for profit maximization are

$$\frac{e'(W_t)W_t}{e_t} = 1,$$

$$(1-\alpha)\frac{Y_t}{N_t} = hW_t, \tag{14}$$

$$\alpha\frac{Y_t}{K_t} = R_t. \tag{15}$$

Here, the first equation is the Solow condition that implies the firm chooses the wage rate to minimize the cost per unit effort (Solow, 1979). It implies that the firm sets the wage rate W_t so that the consumption ratio, C_t/C_t^s, remains constant over time:

$$\frac{C_t}{C_t^s} = \chi, \qquad \text{all } t,$$

where the constant $\chi \geq 1$ is implicitly defined by

$$Td(1-s\chi)(\chi-1) = \theta(1-s)\Big[(T-\xi)\chi^{1+\frac{d}{\theta}} - T\chi\Big]. \tag{16}$$

We assume that

$$\frac{d}{\theta}T - \frac{\theta+d}{\theta}\xi > 0, \quad \text{and} \quad (T-\xi)s^{-1-\frac{d}{\theta}} - Ts^{-1} > 0,$$

which guarantees a unique solution $\chi \in (1, s^{-1})$ in (16).

Since C_t/C_t^s is constant over time, the level of effort is also constant, $e_t = e$, all t, where

$$e = \frac{T-\xi}{h} - \frac{T}{h}\chi^{-\frac{d}{\theta}}.$$

[6]This is true for a larger class of utility functions than considered here. For example, any utility function of the form $U(C,e) = (1/(1-\sigma))[C(T-\xi-he)^{\theta}]^{1-\sigma}$, or $U(C,e) = \ln(C) + V(T-\xi-he)$ has this property.

2.2.2 No shirking condition

As is well known, in an efficiency wage model the labor market equilibrium is described by using the "no shirking condition" in place of the labor supply curve (Shapiro and Stiglitz, 1984). A corresponding condition in our model is derived from the incentive compatibility condition, $C_t/C_t^s = \chi$.

Using (7)-(8), the incentive-compatibility condition, $C_t/C_t^s = \chi$, is rewritten as

$$hW_t = \frac{\chi - 1}{1 - s\chi + \sigma(\chi - 1)(1 - N_t)} C_t^h, \tag{17}$$

which is referred to as the no-shirking condition. It implies an upward relationship between the wage rate W_t and the employment N_t given C_t^h. This is an intuitive relationship: keeping the common income, C_t^h, fixed, a higher wage rate is needed to induce more individuals to work without shirking. The no shirking condition (17) plays the same role as the labor supply function does in the standard growth model.

Using equations (7), (9) and (17), the ratio of the consumption of the (non-shirking) employed to that of the unemployed is constant and given by

$$\frac{C_t}{C_t^u} = \mu \equiv \frac{(1 - s)\chi}{1 - s\chi + \sigma(\chi - 1)}. \tag{18}$$

In the calibration exercise below, μ plays an important role. Note that when $\sigma = 1$, $\mu = 1$, and when $\sigma = s$, $\mu = \chi$.

2.2.3 Resource Constraint

Since no individuals shirk in equilibrium, the economy-wide resource constraint at time t is given by

$$N_t C_t + (1 - N_t) C_t^u + I_t = Y_t, \tag{19}$$

where output, Y_t, is

$$Y_t = K_t^a (ehN_t)^b, \tag{20}$$

with $a = \alpha(1 + \eta)$ and $b = (1 - \alpha)(1 + \eta)$.

3 Equilibrium

Let us now discribe an equilibrium of the economy. We restrict our attention to an interior solution: $N_t \in (0, 1)$, all t.

3.1 Full Insurance

We start with the full insurance case, $\sigma = 1$, and show that the model is equivalent to the standard growth model with utility linear in leisure. Thus, equilibrium is (locally) determinate as long as the labor demand curve is downward sloping: $b < 1$.[7]

Letting Λ_t be the multiplier on (1), the household's utility maximization (11) yields the first-order conditions:

$$\frac{N_t}{C_t} + \frac{1 - N_t}{C_t^u} = \Lambda_t, \tag{21}$$

$$\dot{K}_t = I_t - \delta K_t, \tag{22}$$

$$\dot{\Lambda}_t = (\rho + \delta)\Lambda_t - R_t \Lambda_t. \tag{23}$$

Given the full income insurance, the first equation is simply

$$\frac{1}{C_t} = \Lambda_t. \tag{24}$$

The transversality condition is $\lim_{t \to \infty} e^{-\rho t} \Lambda_t K_t = 0$.

Substituting for C_t^h from (7) into (17) with $\sigma = 1$, the no-shirking condition is rewritten as

$$hW_t = \frac{\chi - 1}{(1 - s)\chi} \frac{1}{\Lambda_t}. \tag{25}$$

This is the no-shirking condition with constant marginal utility of wealth, Λ_t. It corresponds to the Frisch labor supply curve in the standard growth model, and is referred to as the Frisch NSC. The resource constraint (19) becomes

$$C_t + I_t = Y_t, \tag{26}$$

where Y_t is given by (20).

Given the initial condition, $K(0) = K_0 > 0$, an equilibrium is given by a set of time paths, $\{C_t, N_t, I_t, Y_t, W_t, R_t, \Lambda_t, K_t\}$, that satisfy the household's first-order conditions (22)-(24), the factor-demand equations (14)-(15), the Frisch NSC (25), the resource constraints (20), (26), and the transversality condition.

From those equilibrium conditions it immediately follows that our model with full income insurance is isomorphic to the one-sector growth model in Benhabib and Farmer (1994) with the instantaneous utility function which is linear in leisure:

$$\ln(C_t) + B(T - ehN_t)$$

[7] This equivalence result is shown by Alexopoulos (2002), but we repeat it here for completeness.

where $B = \frac{\chi-1}{(1-s)\chi}$. Thus, using the result of Benhabib and Farmer (1994), unless the labor demand curve is upward sloping, the steady state is a saddle point and (locally) determinate. That is, the necessary and sufficient condition for the model with full income insurance to exhibit the saddle path stability is that

$$(1+\eta)(1-\alpha) < 1. \tag{27}$$

3.2 Partial Insurance

Let us turn to the case of partial income insurance, $\sigma < 1$. As we shall see below, the crucial property of this case is that the Frisch NSC is downward sloping. The possibility that indeterminacy be caused by a downward sloping Frisch labor supply curve is emphasized by Bennett and Farmer (2000). As shown by Hintermaier (2003) and Nakajima (2001), however, when utility is concave in consumption and leisure, the Frisch labor supply curve cannot be downward sloping, which excludes the mechanism proposed by Bennett and Farmer (2002) in the standard growth model. Here, the utility function is strictly concave in consumption and leisure, but indeterminacy is generated by a downward sloping Frisch NSC, which is possible as long as unemployment insurance is imperfect.

3.2.1 Equilibrium conditions

The first-order conditions for the household's problem are, again, given by (21)-(23). Since $C_t = \mu C_t^u$, however, (21) is now rewritten as

$$\left[\mu - (\mu - 1)N_t\right]\frac{1}{C_t} = \Lambda_t. \tag{28}$$

The left-hand side of this equation is the average marginal utility of consumption in the household. Note that it is decreasing in the employment, N_t. Because under the partial income insurance, unemployed individuals consume less than employed individuals, $C_t^u = C_t/\mu < C_t$, the marginal utility of consumption of an unemployed individual is greater than that of an employed individual, $U_c(C_t, e_t) < U_c(C_t^u, 0)$. Hence, given C_t, when N_t goes up, the average marginal utility of the household goes down.

The Frisch NSC is obtained from (2), (17) and (28) as

$$hW_t = \frac{\chi - 1}{\chi(1-s)}\left[\mu - (\mu - 1)N_t\right]\frac{1}{\Lambda_t}. \tag{29}$$

Thus, the Frisch NSC is downward sloping: keeping marginal utility of wealth, Λ_t, constant, this implies an inverse relationship between the wage rate W_t and the employment N_t. To see this, suppose the employment, N_t, goes up with Λ_t kept constant. As we have seen, this tends to decrease the average marginal utility of consumption in the household. With Λ_t kept constant, this tends to reduce C_t and thus C_t^h. By the no-shirking condition (17), this, in turn, leads to a decrease in the wage rate, W_t.

The resource constraint is now written as

$$\frac{1}{\mu}\big[1+(\mu-1)N_t\big]C_t+I_t=Y_t, \tag{30}$$

where Y_t is given by (20).

A dynamic equilibrium in the partial insurance case is given by $\{C_t, N_t, I_t, Y_t, W_t, R_t, \Lambda_t, K_t\}$ that satisfy the household's first-order conditions (22)-(23) and (28), the factor-demand equations (14)-(15), the Frisch NSC (29), the resource constraints (20) and (30), and the transversality condition.

3.2.2 Linearization

For a variable X_t, let $\overline{X}$ denote its steady-state level, $x_t \equiv \ln(X_t)$, and $\hat{x}_t \equiv \ln(X_t) - \ln(\overline{X})$. Then the equilibrium conditions above are log-linearized as

$$-\frac{(\mu-1)\overline{N}}{\mu-(\mu-1)\overline{N}}\hat{n}_t-\hat{c}_t=\hat{\lambda}_t,$$

$$\hat{w}_t=-\frac{(\mu-1)\overline{N}}{\mu-(\mu-1)\overline{N}}\hat{n}_t-\hat{\lambda}_t, \tag{31}$$

$$S_C\frac{(\mu-1)\overline{N}}{1+(\mu-1)\overline{N}}\hat{n}_t+S_C\hat{c}_t+S_I\hat{i}_t=\hat{y}_t,$$

$$\hat{w}_t=\hat{y}_t-\hat{n}_t, \tag{32}$$

$$\hat{r}_t=\hat{y}_t-\hat{k}_t,$$

$$\hat{y}_t=a\hat{k}_t+b\hat{n}_t, \tag{33}$$

$$\dot{\lambda}_t=-(\rho+\delta)\hat{r}_t, \tag{34}$$

$$\dot{k}_t=\delta\hat{i}_t-\delta\hat{k}_t, \tag{35}$$

where S_C and S_I are the steady-state shares of consumption and investment, respectively.

Using (31)-(33), the equilibrium employment, $\hat{n}_t$, is written as

$$\hat{n}_t=an_\lambda\hat{k}_t+n_\lambda\hat{\lambda}_t, \tag{36}$$

where

$$n_\lambda \equiv -\frac{1}{b-1+\dfrac{(\mu-1)\overline{N}}{\mu-(\mu-1)\overline{N}}}.$$

As we shall see, the sign of n_λ is crucial for the determinacy of equilibria. We have $n_\lambda < 0$ if and only if

$$-\frac{(\mu-1)\overline{N}}{\mu-(\mu-1)\overline{N}} < b-1. \tag{37}$$

The left-hand side of this equation is the slope of the Frisch NSC (31), and is non-positive. It is zero when $\mu = 1$, and, as $\mu \to \infty$, declines monotonically to $-\overline{N}/(1-\overline{N}) < 0$. Thus, the greater μ is the steeper the Frisch NSC is. In other words, the Frisch NSC becomes steeper with less income insurance (remember that $\mu = C_t/C_t^u$). This is because the effect of N_t on the average marginal utility of consumption is larger with a greater μ. The right-hand of (37) is the slope of the labor demand curve, which is negative when externalities are small, i.e., when (27) is satisfied. Condition (37) says that employment N_t responds to Λ_t inversely if and only if the Frisch NSC is steeper than the labor demand curve. It is more likely to occur when income insurance is less.

From (33) and (36), it follows that $\hat{y}_t$ satisfies

$$\hat{y}_t = y_k \hat{k}_t + y_\lambda \hat{\lambda}_t, \tag{38}$$

where

$$y_k \equiv a(1+bn_\lambda), \quad \text{and} \quad y_\lambda \equiv bn_\lambda.$$

We can then show that $\dot{k}_t$ and $\dot{\lambda}_t$ satisfy the system of ODE's given by

$$\dot{k}_t = \delta\left(y_k + \frac{S_C}{S_I}\frac{an_\lambda}{1+(\mu-1)\overline{N}} - 1\right)\hat{k}_t + \delta\left(y_\lambda + \frac{S_C}{S_I}\frac{n_\lambda}{1+(\mu-1)\overline{N}}\right)\hat{\lambda}_t, \tag{39}$$

$$\dot{\lambda}_t = -(\rho+\delta)(y_k-1)\hat{k}_t - (\rho+\delta)y_\lambda\hat{\lambda}_t. \tag{40}$$

Since k is a predetermined variable and λ is a jump variable, local indeterminacy requires that both eigenvalues of the matrix associated with the system (39)-(40) be negative. Equivalently, for local indeterminacy, the trace should be negative and the determinant must be positive. It is straightforward calculation to show that the determinant is given by

$$\text{DET} = \frac{\delta(\rho+\delta)}{1+(\mu-1)\overline{N}}\frac{S_C}{S_I}n_\lambda(a-1). \tag{41}$$

As long as we are interested in the case of moderate externalities so that $a-1=\alpha(1+\eta)-1<0$, the sign of the determinant is positive if and only if $n_\lambda < 0$, that is, if and only if the Frisch NSC is steepr than the labor demand curve. The trace is shown to equal

$$\mathrm{TR} = n_\lambda(1-N)\big[(1-N)\mu + N\big](N\mu + 1 - N)\Gamma, \tag{42}$$

where Γ is the quadratic equation in μ given by

$$\begin{aligned}\Gamma = {} & \delta(1-a)(2N-1)\Big\{N(1-N)\mu^2 + (2N^2-2N+1)\mu + N(1-N)\Big\} \\ & - \delta(1-a)\Big\{N^2\mu + N(1-N)\Big\} + \frac{\delta a S_C}{S_I}\Big\{(1-N)^2\mu + N(1-N)\Big\} \\ & - \rho b N\Big\{(1-N)^2\mu^2 + (1-N)(2N^2-2N+1)\mu + (1-N)^2\Big\}\end{aligned}$$

With our parameter values in the calibration exercise below, the trace has the same sign as n_λ.

3.2.3 Interpretation

As shown in the previous subsection, the determinacy of equilibrium depends on the sign of n_λ. For simplicity, suppose that there are no externalities ($a+b=1$). Then the determinant of the ODE system (39)-(40) is positive if and only if $n_\lambda < 0$, that is, if and only if higher marginal utility of wealth reduces employment and output. This corresponds to the earlier work on indeterminacy. For example, in Benhabib and Farmer (1994), when externalities are strong enough so that the labor demand curve is upward sloping and steeper than the labor supply curve, higher marginal utility of wealth reduces labor and output by shifting down the Frisch labor supply curve. Also, in Benhabib and Farmer (1996), when sector specific externalities are strong enough, higher marginal utility of wealth shifts up the Frisch labor supply curve; therefore, even when the labor demand curve is downward sloping, higher marginal utility of wealth results in lower employment and output.

In our model, $n_\lambda < 0$ if and only if the Frisch NSC is steeper than the labor demand curve. In such a case, higher marginal utility of wealth shifts down the Frisch NSC, but employment, and hence output, goes down. This is why indeterminacy is possible in our model without externalities. Also, note that the Frisch NSC gets steeper with a high μ, that is, with higher inequality in consumption between the employed and the unemployed. Thus, higher inequality (less risk sharing) makes indeterminacy more likely to occur.

3.3 Calibration

The parameters of the model to be calibrated are:

$$\{\rho, \delta, \alpha, \eta, S_C, S_I, \mu, \overline{N}\}.$$

Here we directly calibrate μ and $\overline{N}$ instead of assigning values to parameters such as θ, ξ, d, s, T, and h. Equations (16) and (18) show how μ depends on those underlying parameters. The (ranges of) values of the above parameters used in this paper are listed in Table 1. There, the values of ρ, δ, α, S_C and S_I are taken from Benhabib and Farmer (1996), and the value of $\overline{N}$ is from Burnside, Eichenbaum and Fisher (2000).

With those parameter values, we can use (42) to show that the trace of the matrix associated with (39)-(40) has the same sign as n_λ. Hence, when $n_\lambda < 0$, the trace is negative and the determinant is positive—the condition for local indeterminacy is satisfied.

Since $n_\lambda < 0$ is equivalent to the condition that the Frisch NSC is steeper than the labor demand curve, it is more likely to occur with less income insurance (large μ) or larger externalities (high η). Define $\mu^*(\eta)$ be the critical value of μ above which equilibrium is indeterminate:

$$\mu^*(\eta) \equiv \inf\{\mu : n_\lambda < 0\}.$$

Assuming $\overline{N} > (1-(1-\alpha)(1+\eta))/(2-(1-\alpha)(1+\eta)))$, which is satisfied for the values listed in Table 1, we can write $\mu^*(\eta)$ as

$$\mu^*(\eta) = \frac{(2-(1-\alpha)(1+\eta))\overline{N}}{(2-(1-\alpha)(1+\eta))\overline{N} - 1 + (1-\alpha)(1+\eta)}.$$

This is an increasing function of η. Figure 1 shows the area in which indeterminacy obtains in the (η, μ)-plane.

Note that

$$\mu^*(0) = \frac{(1+\alpha)\overline{N}}{(1+\alpha)\overline{N} - \alpha} \approx 1.33.$$

Thus, as long as $\mu > 1.33$, equilibrium is indeterminate without externalities. In other words, indeterminacy obtains if the consumption of an unemployed is less than 75 percent of that of an employed. Based on Gruber (1997), Burnside, Eichenbaum and Fisher (2000) set this value to 1.285 (the consumption of an unemployed is about 78 percent of that of an employed). Given such evidence, the requirement that $\mu > 1.33$ does not seem unrealistic.

4 Concluding Remarks

Using an efficiency-wage model of Alexopoulos (2002), we have considered the relationship between the likelihood of indeterminacy and the degree of unemployment insurance coverage. It is shown that the less unemployment insurance is, the more likely indeterminacy is to obtain. Equilibrium can be indeterminate even without externalities or increasing returns, which makes a sharp contrast to the models of sunspots such as Benhabib and Farmer (1994, 1996), Benhabib and Nishimura (1998), Farmer and Guo (1994), Perli (1998), Wen (1998), and Barinci and Chéron (2001). Our result is based on the fact that the Frisch no-shirking condition is downward sloping when income insurance is not perfect. In this sense, it is related to Bennett and Farmer (2000).

In the simple setup in this paper, sunspot shocks generate countercyclical consumption without strong externalities, which is inconsistent with the stylized facts of the business cycle. This may be overcome in different ways. In Nakajima (2003), we develop a business cycle model with variable capacity utilization to pursue such a direction.

5 Appendix

In this appendix, we consider a class of instantaneous utility functions which is slightly more general than the one in the main text:

$$U(C,e) = \ln(C) + \frac{\theta}{1-\gamma}(T-\xi-he)^{1-\gamma}. \tag{A1}$$

This will only change the expressions for the no-shirking levels of $\chi = C_t/C_t^s$ and e. Let $\chi(W_t)$ be

$$\chi(W_t) = \frac{hW_t - F_t + C_t^h}{shW_t - F_t + C_t^h}.$$

With the instantaneous utility function (A1), the incentive compatibility constraint (12) leads to

$$e(W_t) = T - \xi - \left[T^{1-\gamma} - \frac{d(1-\gamma)}{\theta}\ln(\chi(W_t))\right]^{\frac{1}{1-\gamma}}.$$

The Solow condition $e'(W_t)W_t = e(W_t)$ implies that χ is a constant defined implicitly by

$$\frac{d(1-s\chi)(\chi-1)}{(1-s)\theta\chi}\left(T^{1-\gamma} - \frac{d(1-\gamma)}{\theta}\ln(\chi)\right)^{\frac{\gamma}{1-\gamma}} = T - \xi - \left(T^{1-\gamma} - \frac{d(1-\gamma)}{\theta}\ln(\chi)\right)^{\frac{1}{1-\gamma}}.$$

Given χ, the incentive-compatible level of effort, e, is given by

$$e = T - \xi - \left(T^{1-\gamma} - \frac{d(1-\gamma)}{\theta} \ln(\chi) \right)^{\frac{1}{1-\gamma}} .$$

The rest of analysis goes exactly like the one in the main text.

References

[1] Alexopoulos, M., 2002, "Unemployment and the business cycle," *Journal of Monetary Economics*, forthcoming.

[2] Barinci, J. P., and A. Chéron, 2001, "Sunspots and the business cycle in a finance constrained economy," *Journal of Economic Theory* 97, 30-49.

[3] Benhabib, J., and R. E. A. Farmer, 1994, "Indeterminacy and increasing returns," *Journal of Economic Theory* 63, 19-41.

[4] Benhabib, J., and R. E. A. Farmer, 1996, "Indeterminacy and sector specific externalities," *Journal of Monetary Economics* 37, 421-443.

[5] Benhabib, J., and R. E. A. Farmer, 1999, "Indeterminacy and sunspots in macroeconomics," in J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Volume 1A, Elsevier, 387-448.

[6] Benhabib, J., and K. Nishimura, 1998, "Indeterminacy and sunspots with constant returns," *Journal of Economic Theory* 81, 58-96.

[7] Bennett, R. L., and R. E. A. Farmer, 2000, "Indeterminacy with non-separable utility," *Journal of Economic Theory* 93, 118-143.

[8] Burnside, C., M. Eichenbaum, and J. D. M. Fisher, 2000, "Fiscal shocks in an efficiency wage model," NBER Working Paper 7515.

[9] Farmer, R. E. A., and J. T. Guo, 1994, "Real business cycles and the animal spirit hypothesis," *Journal of Economic Theory* 63, 42-72.

[10] Gruber, J., 1997, "The consumption smoothing benefits of unemployment insurance," *American Economic Review* 87, 192-205.

[11] Hintermaier, T., 2003, "On the minimum degree of returns to scale in sunspot models of the business cycle," *Journal of Economic Theory* 110, 400-409.

[12] Perli, R., 1998, "Indeterminacy, home production, and the business cycle: A calibrated analysis," *Journal of Monetary Economics* 41, 105-125.

[13] Nakajima, T., 2001, "Indeterminacy with non-separable, concave utility," mimeo. Brown University.

[14] Nakajima, T., 2003, "Involuntary unemployment, sunspots, and the business cycle," mimeo. Brown University.

[15] Shapiro, C., and J. E. Stiglitz, 1984, "Equilibrium unemployment as a worker discipline device," *American Economic Review* 74, 434-44.

[16] Solow, R. M., 1979, "Another possible source of wage stickiness," *Journal of Macroeconomics* 1, 595-618.

[17] Wen, Y., 1998, "Capacity utilization under increasing returns to scale," *Journal of Economic Theory* 81, 7-36.

Table 1: The Parameter Values

Parameter	Value	Description
ρ	0.05	Discount rate
δ	0.1	Depreciation rate
α	0.3	Capital share
η	$[1, 1.5]$	Returns to scale
S_C	0.8	Steady-state share of consumption
S_I	0.2	Steady-state share of investment
$\overline{N}$	0.93	Steady-state employment rate
χ	$(1, \infty)$	Ratio of consumption of employed to that of unemployed

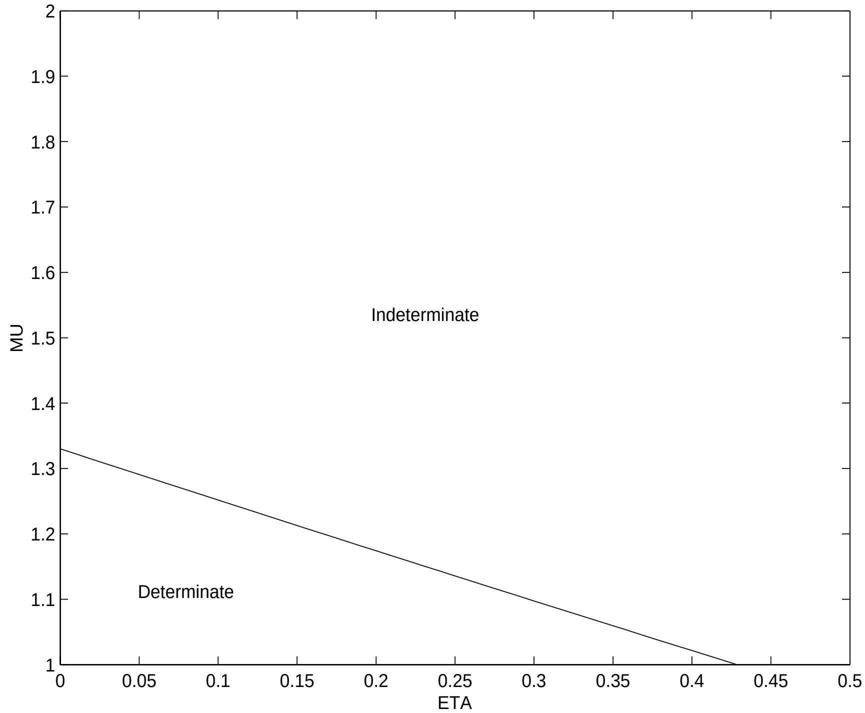

Figure 1: Conditions on μ and η for indeterminacy