Pareto Improving Monetary Policy in Incomplete Markets

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Abstract

We show that for generic economies, every equilibrium admits Pareto improving monetary policy, even with multiple commodities per state.

The main assumption is that asset incompleteness be intermediate, in that household heterogeneity does not exceed the number of assets present and absent.

We argue this as a special case of the general framework in Turner (2003b) for proving the generic existence of Pareto improving taxes.

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1 Introduction

When asset markets are incomplete, there are almost always many Pareto improving policy interventions, if there are multiple commodities and households. Remarkably, these policies do not involve adding any new markets.

We show the generic existence of Pareto improving monetary policy, assuming its ability to set price levels. The argument follows the general framework of Turner (2004b) for the generic existence of Pareto improving taxation.

The protagonist is the price adjustment following an intervention. Its role is to improve on asset insurance by redistributing endowment wealth across states, as anticipated by Stiglitz (1982). The price adjustment is determined by how monetary policy and prices affect aggregate demand.

If monetary policy targeting current incomes is Pareto improving, then it must cause an equilibrium price adjustment, Grossman (1975). Conversely, if the price adjustment is sufficiently sensitive to risk aversion, then for almost all risk aversions and endowments, Pareto improving monetary policy exists, as we show. We then verify this sensitivity test with standard demand theory, which Turner (2003a) extends from complete to incomplete markets.

Turner (2003a) develops the Slutsky theory of demand for commodities and assets in incomplete markets. First, it shows how a Slutsky matrix decomposes into substitution and income effects the derivative of demand with respect to commodity prices and yield structure. Next, it identifies the Slutsky matrix's properties. The Slutsky matrix can be perturbed arbitrarily, subject only to preserving these properties, by perturbing the underlying utility's Hessian, while fixing point demand and marginal utility.

These Slutsky perturbations for possibly incomplete markets generalize the Slutsky perturbations for complete markets, first introduced by Geanakoplos and Polemarchakis (1980), who proved the latter two results in the complete setting. Then Geanakoplos and Polemarchakis (1986) introduced the study of generic improvements with incomplete markets, applying the Slutsky perturbations. Since they allowed the central planner to decide the agents' asset portfolios, they did not need to go beyond perturbing the Slutsky matrices of commodity demand. To show why weaker interventions may improve welfare, such as anonymous taxes or changes in asset payoffs, it became necessary to take into account how agents' portfolio adjustments caused a further price adjustment. Naturally, this required perturbing asset demand as well as commodity demand. The lack of a Slutsky theory for incomplete markets blocked contributions for over ten years[1], until a breakthrough by Citanna, Kajii, and Villanacci (1998), who analyzed first order conditions instead of Slutsky matrices. Researchers have extended the theory of generic improvements with incomplete markets to many policies by applying this first order approach; Cass and Citanna (1998), Citanna, Polemarchakis, and Tirelli (2001), Bisin et al. (2001), and Mandler (2003). Turner (2003b) is an alternative approach based on the original strategy of Slutsky perturbations, as is the present work on monetary policy.

Tirelli (2000) shows the generic existence of Pareto improving capital income taxation, when there is–unlike here–a single commodity per state hence no commodity price adjustment. The reason Pareto improvements are possible is that the intervention improves on the asset span. Although our intervention is monetary and not fiscal, it also changes the asset span, whose welfare impact we must add to the welfare impact of the change in commodity prices. Our argument shows that generically we can engineer these two welfare impacts toward a Pareto improvement, under the extra hypothesis that the asset incompleteness is intermediate–they need not cancel each other out.

The paper continues as follows. Section 2 presents a general model of policy, and details monetary policy as an example. Section 3 has the formula for the welfare impact of policy. Section 4 obtains the generic existence of Pareto improving policy from the sensitivity condition on price adjustment, which it then reinterprets in terms of the Reaction of Demand to Prices and to Policy. Section 5 summarizes the demand theory in incomplete markets necessary to check the sensitivity in terms of the Reactions, then section 6 checks it for monetary policy. Section 8 derives the welfare impact formula, and spells out the notation and the parameterization of economies.

[1] The sole one is Elul (1995).

2 GEIL[2] model

Households $h = 1, ..., H$ know the present state of nature, denoted 0, but are uncertain as to which among $s = 1, ..., S$ nature will reveal in period 1. They consume commodities $c = 1, ..., C$ in the present and future, and invest in assets $j = 1, ..., J$ in the present only. Each state has commodity C as unit of account, in terms of which all value is quoted. Markets assign to household h an income $w^h \in R^{S+1}_{++}$, to commodity $c < C$ a price $p_{\cdot c} \in R^{S+1}_{++}$, to asset j a price $q^j \in R$ and future yield $a^j \in R^S$. We call $(p_{\cdot c})_1^C = p = (p_{s\cdot})$ the spot prices, $q = (q^j)$ the asset prices, $(a^j) = a = (a_s)$ the asset structure, and $w = (w^h)$ the income distribution, $\mathbf{P} \equiv R^{(C-1)(S+1)}_{++} \times R^J$.[3] Taxes are $t \in T, T$ some Euclidean space, negative coordinates corresponding to subsidies. The set of **budget variables** is

$$b \equiv (P, a, w, t) \in B \equiv \mathbf{P} \times R^{J \times S} \times R^{(S+1)H}_{++} \times T$$

and has some distinguished nonempty relatively open subset $B' \subset B$. B_0 is B with $T = \{0\}$.

Demand for commodities and assets $d = (x, y) : B' \rightarrow R^{C(S+1)}_{++} \times R^J$ is a function on B'. The demand $d^h = (x^h, y^h)$ of household h depends on own income only, $(x^h, y^h)(P, a, w, t) = (x^h, y^h)(P, a, w', t)$ if $w^h = w'^h$. **Policy payment** $\tau : B'_0 \times codom(d) \rightarrow R^{S+1 \times \dim(T)}$ is a function such that $\tau(b_0, d)t$ is the actual payment, if demand and policy parameters are d, t. **Policy** $(\tau^h)_h$ is **anonymous** if τ^h is independent of h, and **policy revenue** τ is $\tau(b_0, (d^h)_h) \equiv \Sigma \tau^h(b_0, d^h)$.

An **economy** (a, e, t, t_*, d) consists of an asset structure a, endowments e, policy parameters t, distribution rates t_*, and demands d. For each household h, **endowments** specify a certain number $e^h_{sc} > 0$ of each commodity c in each state s, the **distribution rates** specify a fraction $t^h_* > 0$ with $\Sigma t^h_* = 1$, and **demands** specify a demand d^h. Let Ω be the set of (a, e, t, t_*, d).[4]

A list $(P, r; a, e, t, t^*, d) \in \mathbf{P} \times R^{S+1} \times \Omega$ is a **GEIL** $\leftrightarrow$

$$\sum(x^h(b) - e^h) = 0 \qquad \sum y^h(b) = 0 \qquad r - \tau(b_0, (d^h(b))_h)t = 0$$

$$\text{and } \; b \equiv (P, a, (w^h_s = e^{h\prime}_s \overline{p}_s + t^h_* r_s)^h_s, t) \in B'$$

[2] "L" stands for linearity in the policy's parameters of the implied transfers.

[3] The numeraire convention is that unity is the price of $sC,s \geq 0$, which $\mathbf{P}$ therefore omits. The addition to p of the $sC,s \geq 0$ coordinates, bearing value unity, is denoted $\overline{p}$. We use the notation $P = (p, q) \in \mathbf{P}$.

[4] The appendix spells out the parameterization of demand d.

We say $(a, e, t, t^*, d) \in \Omega$ has **equilibrium** $(P, r) \in \mathbf{P} \times R^S$. A **GEI** is a GEIL with $t = 0$.

Under neoclassical assumptions $(a, e, 0, t_*, d) \in \Omega$ has an equilibrium[5], and then the implicit function theorem gives conditions for a neighborhood of $(a, e, 0, t_*, d)$ to have an equilibrium.

2.1 Neoclassical demand

Consider the **budget** function $\beta^h : B_0 \times R^{C(S+1)} \times R^J \to R^{S+1}$

$$\beta^h(b, x, y) \equiv (\overline{p}'_s x_s - w^h_s)^S_{s=0} - \begin{bmatrix} -q' \\ a' \end{bmatrix} y$$

Demand $d^h = (x^h, y^h)$ is **neoclassical**$_0$ if $T = \{0\}$ and there is a **utility** function $u : R^{C(S+1)}_+ \to R$ with

$$u(x^h(b)) = \max_{X^h_0(b)} u \ \text{ throughout } \ B' \qquad X^h_0(b) \equiv \{x \in R^{C(S+1)}_+ \mid \beta^h(b, x, y) = 0, \text{ some } \ y \in R^J\}$$

More generally, demand $d^h = (x^h, y^h)$ is **neoclassical** if there is a **utility** function $u : R^{C(S+1)}_+ \to R$ with

$$u(x^h(b)) = \max_{X^h(b)} u \ \text{ throughout } \ B' \qquad X^h(b) \equiv \{x \in R^{C(S+1)}_+ \mid \beta^h(b_0, x, y) + \tau^h(b_0, x, y)t_b = 0, \text{ some } \ y \in R^J\}^6$$

If policy $t_b = 0$ is inactive, $X^h(b) = X^h_0(b)$. Thus neoclassical demand restricts to neoclassical$_0$ demand.

Neoclassical welfare is $v : B' \to R^H, v(b) = (v^h(b)) \equiv (u^h(x^h(b)))$.

The interpretation of X is that the cost of consumption x in excess of income w is financed by some portfolio $y \in R^J$ of assets, net of policy payments. A **portfolio** specifies how much of each asset to buy or sell ($y_j \gtrless 0$), and a^j_s how much value in state s an asset j buyer is to collect, a seller to deliver.

2.2 Monetary policy as an example

As in Magill and Quinzii (1992), monetary policy is able to deflate the price level $\tilde{p}_s = \frac{p_s}{1+t_s}$ in each state from the original $t_s = 0$ to a new $t_s \neq 0$. At a GEI, this amounts to being able to change the asset structure $\tilde{a} = a\,(I + [t])$ from the original $t = 0$ to a new one $t \in R^S$, where throughout $[]$ converts a vector $*$ into a diagonal matrix $[*]$.

[5] Geanakoplos and Polemarchakis (1986).

[6] The functions $b \to b_0, \to t_b$ are $(p, q, a, w, t) \to (p, q, a, w, 0), \to t$. Here y is defined by x, if a is full rank.

If portfolios are y^h at a GEI, then they payoff $a'y^h$. More conveniently, the change in the payoff as a linear function of monetary policy $t \in R^S$ is

$$\begin{bmatrix} 0_{1\times S} \\ \left[a'y^h\right] \end{bmatrix} t$$

We shall need to change t only in the first H coordinates–assuming $S \geq H$, i.e. $t_s = 0$ for $s > H$. Thus the above $t_S \in \mathbf{R}^S$ reads in terms of the new $t_H \in R^H$ as

$$t_S = \begin{bmatrix} I_H \\ 0 \end{bmatrix} t_H$$

and the change in the payoff as a linear function of monetary policy $t \in R^H$ is

$$\begin{bmatrix} 0_{1\times S} \\ \left[a'y^h\right] \end{bmatrix} \begin{bmatrix} I_H \\ 0 \end{bmatrix} t$$

with $t \in R^H$. We may view this change in payoff as the policy payment, on switching its sign; the *monetary* policy payment as a special case of the above model is the $S + 1 \times H$ matrix

$$\tau^h =_{def} - \begin{bmatrix} 0_{1\times S} \\ \left[a'y^h\right] \end{bmatrix} \begin{bmatrix} I_H \\ 0 \end{bmatrix}$$

Debreu's smooth preferences imply neoclassical demand exists, and is smooth in a neighborhood of b.

3 Welfare impact of policy

We think of a smooth path $t = t(\xi)$ of policy through $t = 0$, and of *infinitesimal policy* as its initial velocity $\dot{t} = \dot{t}(0)$. Suppose the active GEI $(P, r; a, e, 0, t_*, d)$ is **regular** in that such a path lifts locally to a unique path $(P, r; a, e, t, t_*, d) = (P(\xi), r(\xi); a, e, t(\xi), t_*, d)$ of GEIL through the GEI. Then welfare is $v(b(\xi))$ with $b(\xi) = (P(\xi), r(\xi); a, (w_s^h = e_s^{h\prime}\overline{p}_s(\xi) + t_*^h r_s(\xi))_s^h, t(\xi))$. Thus policy impacts welfare only via the budget variables it implies. By the fundamental theorem of calculus the welfare impact is the integral of $D_b v^h \cdot \dot{b}$, which by abuse we call the *welfare impact.* We compute this product in the appendix, using the envelope theorem for $D_b v^h$ and the chain rule for $\dot{b}$, where the details of the notation appear.$\lambda^{h\prime}\left((t_*^h\tau - \tau^h) + \overline{y}_{\mathbf{1}}^h D_t a - \underline{z}^h dP\right)$

Proposition 1 (Envelope) *The **welfare impact** $\dot{v} \in R^H$ of infinitesimal policy $\dot{t}$ at a regular GEI is*

$$\dot{v} = (\lambda)'\dot{m} \qquad \dot{m} = \underbrace{(t_*^h \dot{r} - \tau^h \dot{t} + \Psi^h \dot{a})_h}_{PRIVATE} \quad \underbrace{-\underline{z}\dot{P}}_{PUBLIC}$$

Here $(\lambda)'$ collects the households' marginal utilities of income across states, and $\dot{m}$ the impact on their incomes, private and public. The private one is the impact $\dot{r}$ on revenue distributed at rate $t_* \in R^H$ net of the impact $\tau^h \dot{t}$ on policy payments, plus the impact on portfolio payoffs, and the public one is the impact on the value of their excess demands $\underline{z}$ in all nonnumeraire markets, that implied by the impact $\dot{P}$ on prices.

Policy targeting welfare must account for the equilibrium price adjustment it causes. The equilibrium price adjustment undoes the excess aggregate demand that policy causes, and depends on the reactions of aggregate demand to both policy and prices.

Proposition 2 (Revenue Impact) *At a regular GEI $\dot{r} = \tau\dot{t}$.*

This follows from $r = \tau t$, the chain rule, and $t = 0$ at a GEI. At a regular GEI there is a **price adjustment** matrix dP, smooth in a neighborhood of it, such that $\dot{P} = dP\dot{t}$. Thus the welfare impact is

$$\boxed{dv = (\lambda)' \left((t_*^h \tau - \tau^h + \Psi^h D_t a)_h - \underline{z} dP\right)}$$

A policy targeting current incomes is (first order) Pareto improving only if policy causes a price adjustment. For if $\tau^h_{s\geq 1}\dot{t} = 0, dP\dot{t} = 0$ then $\Sigma \frac{1}{\lambda_0^h}\dot{v}^h = \Sigma \frac{1}{\lambda_0^h}\lambda^{h\prime}\dot{m}^h = \Sigma \dot{m}_0^h = \Sigma(t_*^h \tau_0 - \tau_0^h + 0) = 0$ so $\dot{v} \gg 0$ is impossible. Next we prove a converse.

4 Framework for generic existence of Pareto improving policy

We prove the generic existence of Pareto improving policy, stressing the role of changing commodity prices over the role of the particular policy. Existence follows directly from a hypothesis on price adjustment. Thus the policy is relevant only insofar as it meets the hypothesis on price adjustment. Then we reinterpret this hypothesis on dP in terms of primitives, the Reaction of Demand to Prices and the Reaction of Demand to Policy.

Pareto improving policy parameters exist if there exists a solution to $dv\dot{t} \gg 0$. In turn this exists if $dv \in R^{H \times \dim T}$ has rank H, which in turn implies that policy parameters outnumber household types $\dim T \geq H$. The key idea is that if $dv = (\lambda)'(t_*^h \tau - \tau^h)_h - (\lambda)'\underline{\overline{z}} dP$ is rank deficient, then a perturbation of the economy would restore full rank by preserving the first summand but affecting the second one. Namely, if some economy's dP is not appropriate, then almost every nearby economy's dP is.

We have in mind a perturbation of the households' **risk aversion** $(D^2 u^h)_h$, which affects nothing but dP in the welfare impact dv. Now, to restore the rank the risk aversion must map into $(\lambda)'\underline{\overline{z}} dP$ richly enough. Since this map keeps $(\lambda)'\underline{\overline{z}}$ fixed, we require that $(\lambda)'\underline{\overline{z}}$ have rank H and that dP be sufficiently sensitive to risk aversion. Cass and Citanna (1998) gift us the first requirement:

Fact 1 (Full Externality of Price Adjustment on Welfare) *Suppose asset incompleteness exceeds household heterogeneity $S - J \geq H > 1$. Then generically in endowments every GEI has $(\lambda_s^h z_{s1}^h)_{s \leq H-1}^{h \leq H}$ invertible.*

Fact 2 *At a regular GEI, dP is locally a smooth function of risk aversion; the marginal utilities λ^i, policy payments τ^i, and excess demands z^i are locally constant in risk aversion.*

For $k \in R^{(S+1)(C-1)+J}$ we say that a *commodity coordinate* is one of the first $(S+1)(C-1)$.

Definition 1 *At a regular GEI, dP is k-**Sensitive to risk aversion** if for every $\alpha \in R^{\dim(T)}$ there is a path of risk aversion that solves $k' d\dot{P} = \alpha'$.*[7] *It is **Sensitive to risk aversion** if it is k-Sensitive to risk aversion for all k with a nonzero commodity coordinate.*

Figure 1

Assumption 1 (Generic Sensitivity of dP) *If $H > 1$, then generically in endowments and utilities, at every GEI dP is Sensitive to risk aversion.*

Figures 2, 3

[7] The appendix spells out a path of risk aversion. Here the dot denotes differentiation with respect to the path's parameter.

This assumption banishes the particulars of the policy, leaving only its imprint on dP. Of course, dP is defined only at regular GEI, so implicitly assumed is that regular GEI are generic in endowments.

Theorem 1 (Logic of Pareto Improvement) *Fix the policy and the desired welfare impact* $\dot{v} \in R^H$. *Grant the Generic Sensitivity of* dP *under* $\dim(T), S - J \geq H > 1, C > 1$. *Then generically in utilities and endowments, at every GEI* $\dot{v}$ *is the welfare impact of some* $\dot{t} \in T$. *Hence a nearby Pareto superior GEIL exists.*

Proof. We fix generic endowments, utilities from the fact, assumption, and apply transversality to

$$
\begin{array}{ll}
1 & \text{nonnumeraire excess demand equations} \\
2 & \gamma'(\lambda)'\left((t_*^h \tau - \tau^h + \Psi^h D_t a)_h - \underline{z} dP\right) = 0 \\
3 & r - \tau t = 0 \\
4 & \gamma'\gamma - 1 = 0
\end{array}
$$

Suppose this is transverse to zero and the natural projection is proper. By the transversality theorem, for generic endowments and utilities, this system of $(\dim p + \dim q) + \dim(T) + \dim r + 1$ equations is transverse to zero in the remaining endogenous variables, which number $\dim p + \dim q + \dim r + H$. By hypothesis $\dim(T) \geq H$, so for these endowments and utilities the preimage theorem implies that no endogenous variables solve this system–every GEI has dv with rank H.

This is transverse to zero. As is well known, we can control the first equations by perturbing one household's endowment. For a moment, say that we can control the second equations and preserve the top ones. We then perturb the third equations and preserve the top two, by perturbing r as well as numeraire endowments–to preserve incomes $w_s^h = e_s^{h\prime}\overline{p}_s + t_*^h r_s$. We control the fourth equation and preserve the top three, by scalar multiples of γ. So transversality obtains if our momentary supposition on $\gamma' dv$ holds:

Write $k' \equiv \gamma'(\lambda)'\underline{z}$. Differentiating $\gamma' dv$ with respect to the parameter of a path of risk aversion,

$$\alpha' =_{def} \frac{d}{d\xi}\gamma'(\lambda)'\left((t_*^h \tau - \tau^h + \Psi^h D_t a)_h - \underline{z} dP\right) = -\gamma'(\lambda)'\underline{z}\frac{d}{d\xi}(dP) = -k' d\dot{P}$$

since λ, τ^i, z are locally constant. We want to make α arbitrary, and we can if dP is k-sensitive, which holds by assumption if k has a nonzero commodity coordinate. It has: Full Externality of Price Adjustment on Welfare, $C > 1, \gamma \neq 0$ imply $\gamma'(\lambda)'\underline{z}$ is nonzero in the coordinate $m = s1$ for some $s \leq H - 1$.

That the natural projection is proper we omit. (The numeraire asset structure is fixed.) ■

We have seen that policy targeting current incomes, such as monetary policy, supports a Pareto improvement only if there is a price adjustment. Conversely, policy generically supports a Pareto improvement if the price adjustment is sufficiently sensitive to risk aversion. Therefore price adjustment is pivotal.

4.1 Expression for Price Adjustment

Before we can check whether a particular policy meets the Sensitivity of dP to Risk Aversion, we need an expression for dP. We express dP in terms of the Reaction of Demand to Prices and the Reaction of Demand to Policy, notions which are well defined at an active GEI.

Let an underbar connote the omission of the numeraire in each state, define

$$d : B' \to R_{++}^{(C-1)(S+1)} \times R^J \qquad d = \Sigma \underline{d}^h$$

and the **aggregate demand** of $(a, e, t, t_*) \in \Omega$

$$d_{a,e,t,t_*}(p, q, r) \equiv d(p, q, a, (w_s^h = e_s^{h\prime} \overline{p}_s + t_*^h r)_s^h, t)$$

with domain $\mathbf{P}_{a,e,t,t_*} \equiv \{(p, q, r) \in \mathbf{P} \times R^{S+1} \mid (p, q, a, (w_s^h = e_s^{h\prime} \overline{p}_s + t_*^h r_s)_s^h, t) \in B'\}$.[8]

Now define

$$\begin{aligned} \nabla &\equiv D_{p,q} d_{a,e,t,t_*} && \text{the } \textbf{Reaction of Demand to Prices} \\ \Delta &\equiv D_r d_{a,e,t,t_*} \cdot \tau + D_t d_{a,e,t,t_*} && \text{the } \textbf{Reaction of Demand to Policy}^{9} \end{aligned} \tag{1}$$

Suppose a path of GEIL $(P(\xi), r(\xi), a, (e_s^{h\prime} \overline{p}_s(\xi) + t_*^h r_s(\xi))_s^h, t(\xi))$ through an active GEI. Then

$$d_{a,e,t,t_*}(P, r) = \begin{bmatrix} \sum \underline{e}^h \\ 0 \end{bmatrix}$$

is an identity in the path's parameter ξ. Differentiating with respect to it,

$$\nabla \dot{P} + D_r d_{a,e,t,t_*} \cdot \dot{r} + D_t d_{a,e,t,t_*} \cdot \dot{t} = 0$$

[8] P_{a,e,t,t_*} is open, as the preimage by a continuous function of the open B'. Recall the notation $P' = (p', q')$.

[9] Clearly $D_r d_{a,e,t,t_*} = \Sigma D_{w^h} \underline{d}^h t_*^h$.

Substituting for $\dot{r} = \tau \dot{t}$ from the Revenue Impact proposition,

$$\nabla \dot{P} + \Delta \dot{t} = 0$$

An active GEI is **regular** if ∇ is invertible. By the implicit function theorem, a regular GEI lifts a local policy through $t = 0$ to a path of GEIL through itself, such as the one just above.

Proposition 3 (Price Adjustment) *At a regular GEI the Price Adjustment to infinitesimal policy is*

$$dP = -\nabla^{-1} \Delta \qquad (dP)$$

where the Reactions ∇, Δ *are defined in (1).*

4.2 Primitives for the Sensitivity of Price Adjustment to Risk Aversion

Given the Logic of Pareto improvement, we want to check whether a policy meets the Generic Sensitivity of dP. We provide primitives for the Sensitivity of dP, thanks to expression (dP)[10]:

$$d\dot{P} = -\nabla^{-1} \dot{\Delta} + \nabla^{-1} \dot{\nabla} \nabla^{-1} \Delta$$

Recall equation $k' d\dot{P} = \alpha'$ from definition 1. If $\dot{\Delta} = 0$ and $\tilde{k}' \equiv_{def} k' \nabla^{-1}$ then the equation reads $\tilde{k}' \dot{\nabla} \nabla^{-1} \Delta = \alpha'$. If Δ has rank $\dim(T)$ then there is a solution β to $\beta' \nabla^{-1} \Delta = \alpha'$ so it suffices to solve $\tilde{k}' \dot{\nabla} = \beta'$. Thus dP is k-Sensitive if (1) Δ has rank $\dim(T)$, (2) $\tilde{k}$ is nonzero everywhere, (3) whenever $\tilde{K}$ is nonzero everywhere and $\beta \in R^{(S+1)(C-1)+J}$, there is a path of risk aversion that solves $\dot{\Delta} = 0, \tilde{K}' \dot{\nabla} = \beta'$. (Take $\tilde{k} = \tilde{K}$.) Thus Generic Sensitivity of dP follows from the following (independently of the $\tilde{k}$ defined):

Lemma 1 (Activity) *If* $H > 1$, *generically in endowments every GEI is active and regular.*[11]

Assumption 2 (Full Reaction of Demand to Policy) *If* $C > 1$, *generically in utilities and endowments, at every GEI* Δ *has rank* $\dim(T)$.

[10] Applying the chain rule to $JJ^{-1} = I$ gives $\frac{d}{d\xi} J^{-1} = -J^{-1} (\frac{d}{d\xi} J) J^{-1}$.

[11] We do not argue this relatively simple statement. For these endowments, both Δ and dP are defined.

Lemma 2 (Mean Externality of Price Adjustment on Welfare is Regular) *Generically in utilities, at every regular GEI, whenever* k *is nonzero in some commodity coordinate,* $\tilde{k}' \equiv k'\nabla^{-1}$ *is nonzero everywhere.*

Assumption 3 (Sufficient Independence of Reactions) *If* $H > 1$*, then generically in endowments and utilities, whenever* $\tilde{k} \in R^{(S+1)(C-1)+J}$ *is nonzero everywhere and* $\beta \in R^{(S+1)(C-1)+J}$*, at every GEI there is a path of risk aversion that solves* $\dot{\Delta} = 0, \tilde{k}'\dot{\nabla} = \beta'$.

These primitives for the Generic Sensitivity of dP and the Logic of Pareto Improvement yield

Theorem 2 (Test for Pareto Improvement) *Fix the policy and the desired welfare impact* $\dot{v} \in R^H$*. Say the policy passes the Full Reaction of Demand to Policy and the Sufficient Independence of Reactions under* $\dim(T), S - J \geq H > 1, C > 1$*. Then generically in utilities and endowments, at every GEI* $\dot{v}$ *is the welfare impact of some* $\dot{t} \in T$*. Hence there is a nearby Pareto superior GEIL.*

Next we illustrate how to check whether a policy passes this test via demand theory in incomplete markets, as developed by Turner (2003a). We show that the monetary policy in the introduction pass this test, and therefore generically admit Pareto improving parameters, owing to the unifying logic of a sensitive price adjustment. At a GEI ∇ will turn out to be independent of the policy, so we will verify the lemma on the Mean for one and all policies.

5 Summary of demand theory in incomplete markets

We must check whether each policy meets the Full Reaction of Demand to Policy and the Sufficient Independence of Reactions. For this we report the theory of demand in incomplete markets as developed by Turner (2003a). The basic idea is to use decompositions of Δ, ∇ in terms of Slutsky matrices, and then to perturb these Slutsky matrices by perturbing risk aversion, while preserving neoclassical demand at the budget variables under consideration. We stress that this theory is applied to, but independent of, equilibrium.

5.1 Slutsky perturbations

Define $H : R^{C^* \times C^*} \to R^{C^*+J+(S+1) \times C^*+J+(S+1)}$ as

$$H(D) = \begin{bmatrix} D & 0 & -[\overline{p}] \\ 0 & 0 & W \\ -[\overline{p}] & W' & 0 \end{bmatrix}$$

where $p, W = [-q : a] \in R^{J \times S+1}$ of rank J are given, and $C^* = C(S+1)$. In other notation,

$$H(D) = \begin{bmatrix} M(D) & -\rho \\ -\rho' & 0 \end{bmatrix} \qquad \text{where } M(D) = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}, \rho = \begin{bmatrix} [\overline{p}] \\ -W \end{bmatrix}$$

In showing the differentiability of demand, the key step is the invertibility of $H(D^2u)$. **Slutsky matrices** are $H(D^2u)^{-1}$. If D is symmetric, so are $H(D), H(D)^{-1}$ when defined. Thus we write

$$H(D)^{-1} = \begin{bmatrix} S & -m \\ -m' & -c \end{bmatrix}$$

where S, c are symmetric of dimensions $C^* + J, S+1$ and $m = (m_x, m_y)$ is $C^* + J \times S + 1$. A **Slutsky perturbation** is $\nabla = H(D)^{-1} - H(D^2u)^{-1}$, for some symmetric $D \approx D^2u$ that is close enough for the inverse to exist. A Slutsky perturbation is a perturbation of Slutsky matrices rationalizable by some perturbation of the Hessian of utility. Being symmetric, we write

$$\nabla = \begin{bmatrix} \dot{S} & -\dot{m} \\ -\dot{m}' & -\dot{c} \end{bmatrix}$$

and view a Slutsky perturbation as a triple $\dot{S}, \dot{m}, \dot{c}$. We identify Slutsky perturbations, without reference to the inversion defining them, in terms of independent linear constraints on ∇ :

on $\dot{S}$	$\rho'\dot{S} = 0$ and $\dot{S}$ is symmetric	
on $\dot{m}$	$\rho'\dot{m} = 0$ and $\dot{m}_x W' = 0$	(constraints)
on $\dot{c}$	$\dot{c}W' = 0$ and $\dot{c}$ is symmetric	

Theorem 3 (Identification of Slutsky perturbations, Turner 2003a) *Given u smooth in Debreu's sense and b in B' with $t = 0$, consider the Slutsky matrices $H(D^2u)^{-1}$. Every small enough Slutsky*

perturbation ∇ *satisfies (constraints). Conversely, every small enough perturbation* ∇ *that satisfies (constraints) is Slutsky:* $H(D^2u)^{-1}+\nabla$ *is the inverse of* $H(D)$ *for some* D *that is negative definite and symmetric.*

We use only Slutsky perturbations with $\dot{m},\dot{c}=0$ by choosing $\dot{S}$ as follows. A matrix $\underline{\dot{S}} \in R^{(C-1)(S+1)+J\times(C-1)(S+1)+J}$ is extendable in a unique way to a matrix $\dot{S} \in R^{C^*+J\times C^*+J}$ satisfying $\rho'\dot{S}=0$; we call $\dot{S}$ the **extension** of $\underline{\dot{S}}$. It is easy to verify that if $\underline{\dot{S}}$ is symmetric, so is its extension. In sum, any symmetric $\underline{\dot{S}}$ defines a unique Slutsky perturbation with $\dot{m},\dot{c}=0$.

5.2 Decomposition of demand

The relevance of Slutsky perturbations is that they allow us to perturb demand functions directly, while preserving their neoclassical nature, without having to think about utility. This is because Slutsky matrices appear in the **decomposition** of the reaction $D_{p,q}\underline{d}$ of demand to prices at b with $t=0$:

$$D_{p,q}\underline{d}^h = \underline{S}^h L_+^h - \underline{m}^h \cdot ([\underline{x}^h]' : \overline{y}_0^h) \tag{dec}$$

Here L_+^h a diagonal matrix displaying the marginal utility of contingent income

$$L_+^h \equiv \begin{bmatrix} L^h & 0 \\ 0 & \lambda_0^h I_J \end{bmatrix} \qquad L^h \equiv \begin{bmatrix} \cdot & & 0 \\ & \lambda_s^h I_{C-1} & \\ 0 & & \cdot \end{bmatrix}$$

$m^h = D_{w^h} d^h$, and $([\underline{x}^h]' : \overline{y}_0^h)$ is the transpose of $\underline{d}^h$:[12]

$$[\underline{x}^h]' = \begin{bmatrix} \cdot & 0 & 0 \\ 0 & \underline{x}_s^{h\prime} & 0 \\ 0 & 0 & \cdot \end{bmatrix}_{(S+1)\times(C-1)(S+1)} \qquad \overline{y}_0^h = \begin{bmatrix} y^{h\prime} \\ 0 \end{bmatrix}_{S+1\times J}$$

Writing $(e_s^{h\prime}\overline{p}_s)_s$ as $[e^h]'\overline{p}$, we have $D_{p,q}[e^h]'\overline{p} = ([\underline{e}^h]' : 0)$, so from (1) we have

$$\nabla = \Sigma D_{p,q}\underline{d}^h + \underline{m}^h \cdot ([\underline{e}^h]' : 0)$$

[12] We view p as one long vector, state by state, and p,q as an even longer one; $(* : \#)$ denotes concatenation of $*,\#$.

Inserting decomposition (dec),

$$\nabla = \Sigma \underline{S}^h L_+^h - \underline{m}^h \cdot ([\underline{x}^h - \underline{e}^h]' : \overline{y}_0^h)$$

Writing $\underline{z}^{h\prime} \equiv ([\underline{x}^h - \underline{e}^h]' : \overline{y}_0^h)$ this reads

$$\boxed{\nabla = \Sigma \underline{S}^h L_+^h - \underline{m}^h \cdot \underline{z}^{h\prime}} \tag{∇}$$

This **decomposition** of the aggregate demand of $(a, e, t, t_*) \in \Omega$ generalizes Balasko 3.5.1 (1988) to incomplete markets.

One implication of the decomposition is that ∇ is independent of the policy. So let us now provide

Proof that Mean Externality of Price Adjustment on Welfare is Regular. See Turner (2003b).

■

There is another decomposition of the reaction $D_a \underline{d}^h$ of demand to the asset structure:

$$D_a \underline{d}^h = -\underline{S}^h \begin{bmatrix} 0 \\ \Lambda^h \end{bmatrix} + \underline{m}_{\mathbf{1}}^h \Psi^h \tag{2}$$

where

$$\Lambda^h \equiv [\lambda_1^h I_J : ... : \lambda_S^h I_J]_{J \times JS} \qquad \Psi^h \equiv \begin{bmatrix} y^{h\prime} & & 0 \\ & . & \\ 0 & & y^{h\prime} \end{bmatrix}_{S \times SJ}$$

and a is parameterized as a long column vector

$$a = \begin{bmatrix} a^1 \\ . \\ a^S \end{bmatrix}$$

6 Pareto improving monetary policy

We check for each policy the Full Reaction of Demand to Policy and the Sufficient Independence of Reactions. In computing

$$\Delta = D_t d_{a,e,t,t_*} + (\Sigma D_{w^h} \underline{d}^h t_*^h) \cdot \tau$$

we use the following notation for $\underline{S}^h$, where A^h, B^h are symmetric of dimensions $(C-1)(S+1), J$:

$$\underline{S}^h = [\underline{S}_p^h : \underline{S}_q^h] = \begin{bmatrix} A^h & P^h \\ P^{h\prime} & B^h \end{bmatrix} \qquad (\underline{S}^h)$$

We can perturb P^h arbitrarily and get a Slutsky perturbation.

Remark 1 *In checking the Sufficient Independence of Reactions, the* $\dot{\underline{S}}^h$ *Slutsky perturbations affect only the Jacobian* $\dot{\nabla} = \Sigma \dot{S}^h L_+^h$ *in* (∇)*. Also, we solve* $\tilde{k}'\dot{\nabla} = \beta'$ *piecemeal, solving* $\tilde{k}'\dot{\nabla}_p = \beta_p', \tilde{k}'\dot{\nabla}_q = \beta_q'$ *by splitting* $\dot{\nabla} = [\dot{\nabla}_p : \dot{\nabla}_q]$.

6.1 Monetary policy

Theorem 4 *Fix the desired welfare impact* $\dot{v} \in R^H$*. Assume intermediate incompleteness,* $S-J, J \geq H$*, and* $H, C > 1$*. Then generically in utilities and endowments, at every GEI* $\dot{v}$ *is the welfare impact of some* $\dot{t} \in T$*. Hence there is a nearby Pareto superior GEIL with active monetary policy.*

Proof. The next lemmas and $\dim(T) = H$ enable theorem 2. ■

To compute $\Delta = D_t d_{a,e,t,t_*} + (\Sigma D_{w^h} \underline{d}^h t_*^h) \cdot \tau$, note that $\tau = 0$ for monetary policy, thanks to $\Sigma y^h = 0$:

$$\tau = \Sigma \tau^h = -\Sigma \begin{bmatrix} 0_{1\times S} \\ [a'y^h] \end{bmatrix} \begin{bmatrix} I_H \\ 0 \end{bmatrix} = 0$$

So $\Delta = D_t d_{a,e,t,t_*}$. Monetary policy's effect on budget variables is only on the asset structure. Thus $D_t d_{a,e,t,t_*} = \Sigma D_a \underline{d}^h D_t a$. To compute $D_t a$, we write the new asset structure as

$$\tilde{a} = \begin{bmatrix} a^1 & . & . & . & . & t_1 \\ 0 & . & 0 & . & . & . \\ . & . & a^H & . & . & t_H \\ . & . & 0 & . & 0 & 0 \\ . & . & . & . & a^S & 0 \end{bmatrix}$$

to see

$$D_t a = \begin{bmatrix} a^1 & . & . \\ 0 & . & 0 \\ . & . & a^H \\ 0 & 0 & 0 \end{bmatrix}_{SJ \times H}$$

Further, from decomposition (2)

$$D_a \underline{d}^h = \Sigma - \underline{S}^h \begin{bmatrix} 0 \\ \Lambda^h \end{bmatrix} + \underline{m}_{\mathbf{1}}^h \Psi^h$$

This simplifies slightly if we let $\Lambda_H^h, \Psi_H^h, a_H$ be Λ^h, Ψ^h, a truncated of the coordinates corresponding to the last $S - H$ states. Then

$$\begin{aligned} \Delta^m &= \Sigma \left(-\underline{S}^h \begin{bmatrix} 0 \\ \Lambda_H^h \end{bmatrix} + \underline{m}_{\mathbf{1}}^h \Psi_H^h \right) \begin{bmatrix} a^1 & . & . \\ 0 & . & 0 \\ . & . & a^H \end{bmatrix} && (\Delta^m) \\ &= \Sigma \left(-\underline{S}^h \begin{bmatrix} 0 \\ \Lambda_H^h \end{bmatrix} + \underline{m}_{\mathbf{1}}^h \Psi_H^h \right) [a_H] && (3) \end{aligned}$$

Lemma 3 (Full Reaction of Demand to Policy) *If* $C > 1, J \geq H \geq 1,$ *generically in utilities and endowments, at every GEI* Δ^m *has rank* $\dim(T)$.

Proof. Apply transversality to

nonnumeraire excess demand equations

$$\hat{\Delta}^m \phi = 0$$

$$\phi' \phi - 1 = 0$$

where the hat omits the last J rows of Δ^m. This is transverse to zero. The burden of the argument is to control the middle equations independently of the top ones. We set $\dot{m}^h, \dot{\underline{A}}^h = 0$ so that

$$\frac{d}{d\xi} \hat{\Delta}^m = -\Sigma \left[\dot{\underline{A}}^h : \dot{\underline{P}}^h \right] \begin{bmatrix} 0 \\ \Lambda_H^h \end{bmatrix} [a_H] = -\Sigma \dot{\underline{P}}^h \Lambda_H^h [a_H]$$

We want to make $\frac{d}{d\xi}\hat{\Delta}^m\phi$ arbitrary by varying the $\underline{\dot{P}}^h$. Write $\theta^h =_{def} \Lambda_H^h [a_H] \phi$, so $\frac{d}{d\xi}\Delta^m\ \phi = -\Sigma\underline{\dot{P}}^h\theta^h$. Suppose for a moment that $\theta^h \neq 0$ for some h, say $\theta_j^h \neq 0$. Then we set $\underline{\dot{P}}^{i\neq h} = 0$ and $\underline{\dot{P}}^h$ to zero in every column but $-\frac{\alpha}{\theta_j^h}$ in column j, for an arbitarary α. Then $\frac{d}{d\xi}\Delta^m\ \phi = -\Sigma\underline{\dot{P}}^h\theta^h = -\underline{\dot{P}}_j^h\theta_j^h = \alpha$ can be made arbitary, as desired.

Now we justify that $\theta^h \neq 0$ for some h –better, any h. Since a has rank $J \geq H, [a_H]$ has rank H, hence so does $\Lambda_H^h [a_H]$, recalling $\lambda^h \gg 0$. Therefore $\theta^h \neq 0$ whenever $\phi \neq 0$, true since $\phi'\phi - 1 = 0$.

By the transversality theorem, generically in endowments and utilities the system of $\dim p + \dim q + (S+1)(C-1)+1$ equations is transverse in the remaining $\dim p + \dim q + H$ variables. Since $(S+1)(C-1) > S > J \geq H$, by the preimage theorem, for these every GEI has $\hat{\Delta}^m$ (a fortiori Δ^m) with linearly columns. ■

Definition 2 *We say a is **regular** if every $S-1$ rows have rank J.*

Lemma 4 (Sufficient Independence of Reactions) *If $H > 1$ and a is regular, then generically in endowments and utilities, whenever $\tilde{k} \in R^{(S+1)(C-1)+J}$ is nonzero everywhere and $\beta \in R^{(S+1)(C-1)+J}$, at every GEI there is a path of risk aversion that solves $\dot{\Delta} = 0, \tilde{k}'\dot{\nabla} = \beta'$.*

Proof. Fix generic endowments from the next lemma. We follow remark 1. Setting $\dot{B}^h = 0, \underline{\dot{m}}^h = 0$ we see

$$\begin{aligned}
\dot{\Delta} &= \Sigma\left(-\begin{bmatrix} \underline{\dot{A}}^h & \underline{\dot{P}}^h \\ \underline{\dot{P}}^{h\prime} & \dot{B}^h \end{bmatrix}\begin{bmatrix} 0 \\ \Lambda_H^h \end{bmatrix} + \underline{\dot{m}}_{\mathbf{1}}^h \Psi_H^h\right)[a_H] \\
&= -\Sigma\begin{bmatrix} \underline{\dot{P}}^h \Lambda_H^h \\ 0 \end{bmatrix}[a_H]
\end{aligned}$$

So solving $\dot{\Delta}=0$ amounts to solving (a) $\Sigma\underline{\dot{P}}^h\Lambda_H^h[a_H]=0$. We also see

$$\begin{aligned}
\dot{\nabla} &= \Sigma\underline{\dot{S}}^h L_+^h - \underline{\dot{m}}^h\cdot\underline{z}^{h\prime} = \Sigma\underline{\dot{S}}^h L_+^h \\
&= \Sigma\begin{bmatrix}\underline{\dot{A}}^h & \underline{\dot{P}}^h \\ \underline{\dot{P}}^{h\prime} & 0\end{bmatrix}\begin{bmatrix}L^h & 0 \\ 0 & \lambda_0^h I_J\end{bmatrix} \\
&= \Sigma\begin{bmatrix}\underline{\dot{A}}^h L^h & \underline{\dot{P}}^h\lambda_0^h \\ \underline{\dot{P}}^{h\prime}L^h & 0\end{bmatrix}
\end{aligned}$$

So solving $\tilde{k}'\dot{\nabla}=\beta'$ amounts to solving (b) $\beta_p'=\tilde{k}'\dot{\nabla}_p=\tilde{k}_p'\Sigma\underline{\dot{A}}^h L^h+\tilde{k}_q'\Sigma\underline{\dot{P}}^{h\prime}L^h$ and (c) $\beta_q'=\tilde{k}'\dot{\nabla}_q=\tilde{k}_p'\Sigma\underline{\dot{P}}^h\lambda_0^h$ by splitting $\dot{\nabla}=[\dot{\nabla}_p:\dot{\nabla}_q]$. To begin, suppose the $\underline{\dot{P}}^h$ have been chosen to solve (a), (c). Then to solve (b) $\beta_p'=\tilde{k}_p'\Sigma\underline{\dot{A}}^h L^h+\tilde{k}_q'\Sigma\underline{\dot{P}}^{h\prime}L^h$, we choose $\underline{\dot{A}}^{h>1}=0$ and solve $\beta_p'=\tilde{k}_p'\underline{\dot{A}}^1L^1+\tilde{k}_q'\underline{\dot{P}}^{1\prime}L^1$ or $\alpha'=_{def}\beta_p'-\tilde{k}_q'\underline{\dot{P}}^{1\prime}=\tilde{k}_p'\underline{\dot{A}}^1$ by setting $\underline{\dot{A}}^1$ as follows. It must be symmetric, to be a Slutsky perturbation, so let it be the diagonal matrix with entries $\frac{\alpha_i}{\tilde{k}_i}$, recalling by assumption $\tilde{k}$ is nonzero in every coordinate.

It remains to choose the $\underline{\dot{P}}^h$ to solve $\Sigma\underline{\dot{P}}^h\Lambda_H^h[a_H]=0,\beta_q'=\tilde{k}_p'\Sigma\underline{\dot{P}}^h\lambda_0^h$. Since $\tilde{k}_1\neq 0$, we perturb only the first row 1 of $\underline{\dot{P}}^h$ (letting the other rows be zero) and need only solve $\Sigma\underline{\dot{P}}_1^h\Lambda_H^h[a_H]=0,\frac{\beta_q'}{\tilde{k}_1}=\Sigma\underline{\dot{P}}_1^h\lambda_0^h$. Of course, given $\underline{\dot{P}}_1^{h>1}$, we can solve the latter equation by setting $\underline{\dot{P}}_1^1=_{def}\frac{1}{\lambda_0^1}\left(\frac{\beta_q'}{\tilde{k}_1}-\Sigma_{h>1}\underline{\dot{P}}_1^h\lambda_0^h\right)$. So it remains to choose the $\underline{\dot{P}}_1^{h>1}$ to solve $\Sigma\underline{\dot{P}}_1^h\Lambda_H^h[a_H]=0$ with $\underline{\dot{P}}_1^1$ so defined, namely, to solve

$$\begin{aligned}
0 &= \Sigma_{h>1}\underline{\dot{P}}_1^h\Lambda_H^h[a_H]+\underline{\dot{P}}_1^1\Lambda_H^1[a_H] \\
&= \Sigma_{h>1}\underline{\dot{P}}_1^h\Lambda_H^h[a_H]+\frac{1}{\lambda_0^1}\left(\frac{\beta_q'}{\tilde{k}_1}-\Sigma_{h>1}\underline{\dot{P}}_1^h\lambda_0^h\right)\Lambda_H^1[a_H] \\
&= \Sigma_{h>1}\underline{\dot{P}}_1^h\left(\Lambda_H^h-\frac{\lambda_0^h}{\lambda_0^1}\Lambda_H^1\right)[a_H]+\frac{1}{\lambda_0^1}\frac{\beta_q'}{\tilde{k}_1}
\end{aligned}$$

It suffices that $[\tilde{a}_H]=_{def}\left(\Lambda_H^h-\frac{\lambda_0^h}{\lambda_0^1}\Lambda_H^1\right)[a_H]$ have rank H, for then $\Sigma_{h>1}\underline{\dot{P}}_1^h[\tilde{a}_H]=-\frac{1}{\lambda_0^1}\frac{\beta_q'}{\tilde{k}_1}$ has a solution with $\underline{\dot{P}}_1^{h>2}=0$ and some $\underline{\dot{P}}_1^2$. Now $[\tilde{a}_H]$ is $[a_H]$ with the assets' payoff a_s in state $s\leq H$ rescaled by $l_{s>0}=_{def}\lambda_s^h-\frac{\lambda_0^h}{\lambda_0^1}\lambda_s^1$. Since $[a_H]$ has full rank $H\leq J$, so does $[\tilde{a}_H]$ if $l_s\neq 0$ for every s, which is the case given our choice of endowments from the next lemma. ■

Lemma 5 *Suppose* $J\geq H>1$ *and* a *is regular. Then generically in endowments,* $l_s=_{def}\lambda_s^h-\frac{\lambda_0^h}{\lambda_0^1}\lambda_s^1$ *is nonzero for every* $h>1$ *and* $s>0$.

The proof is standard, involving changes in the utilities' gradient through changes in the endowments.

7 Appendix

7.1 Derivation of formula for welfare impact

It is standard how Debreu's smooth preferences, linear constraints, and the implicit function theorem imply the smoothness of neoclassical$_0$ demand. In fact, the implicit function theorem implies smoothness of neoclassical demand in a neighborhood $\tilde{b} \approx b \in B$, if neoclassical$_0$ demand is active at $b \in B_0$. It is standard also that the envelope property follows from the value function's local smoothness, which is the case for v^h as the composition of smooth functions:

$$D_b v^h = D_b L(x, y, \lambda^h) \mid_{(x^h, y^h)(b)}$$

where $b = (p, q, a, w^h, t)$ and

$$L(x, y, \lambda^h) \equiv u^h(x) - \lambda^{h\prime} \left([\overline{p}]' x - w^h - \begin{bmatrix} -q' \\ a' \end{bmatrix} y + \tau^h(b_0, x, y) t \right)$$

Thus

$$\begin{aligned} D_b v^h &= -\lambda^{h\prime} \left([\underline{x}^h]' + D_p \tau^h t : \overline{y}_0^h + D_q \tau^h t : -\Psi^h : -I + D_{w^h} \tau^h t : \tau^h \right) \\ \text{where } \overline{y}_0^h &= \begin{bmatrix} y^{h\prime} \\ 0 \end{bmatrix} \qquad \Psi^h = \begin{bmatrix} 0 & 0 & 0 \\ y^{h\prime} & & 0 \\ & \cdot & \\ 0 & & y^{h\prime} \end{bmatrix}_{S+1 \times SJ} \end{aligned}$$

If $t = 0$

$$D_b v^h = -\lambda^{h\prime} \left([\underline{x}^h]' : \overline{y}_0^h : -\Psi^h : -I : \tau^h \right)$$

So much for demand theory. Recalling regular GEI from the subsection on the Expression for the Price Adjustment, $dP' = (dp', dq')$ exists and

$$\begin{aligned} w^h &= [\overline{p}]'e^h + t_*^h r \Rightarrow \\ dw^h &= [\underline{e}^h]'dp + t_*^h dr \\ &= ([\underline{e}^h]' : 0)dP + t_*^h \tau \end{aligned}$$

using $dr = \tau$ from the Revenue Impact proposition.

Thus the welfare impact at a regular GEI is

$$\begin{aligned} dv^h &= D_b v^h \cdot db \\ &= -\lambda^{h\prime}\left(([\underline{x}^h]' : \overline{y}_0^h) : -\Psi^h : -I : \tau^h\right) \cdot \left(dP : D_t a : ([\underline{e}^h]' : 0)dP + t_*^h \tau : I\right) \\ &= -\lambda^{h\prime}\left(([\underline{x}^h]' : \overline{y}_0^h)dP - \Psi^h D_t a - ([\underline{e}^h]' : 0)dP - t_*^h \tau + \tau^h\right) \\ &= -\lambda^{h\prime}\left(\underline{z}^{h\prime} dP - \Psi^h D_t a - t_*^h \tau + \tau^h\right) \end{aligned}$$

where $\underline{z}^{h\prime} \equiv ([\underline{x}^h - \underline{e}^h]' : \overline{y}_0^h)$ by definition and

$$D_t a = \begin{bmatrix} a^1 & . & . \\ 0 & . & 0 \\ . & . & a^H \\ . & . & 0 \end{bmatrix}_{SJ \times H}$$

In sum,

$$\boxed{dv^h = \lambda^{h\prime}\left(t_*^h \tau - \tau^h + \Psi^h D_t a - \underline{z}^h dP\right)}$$

7.2 Aggregate notation

We collect marginal utilities of contingent income, and denote stacking by an upperbar

$$(\lambda)' \equiv \begin{bmatrix} \cdot & & 0 \\ & \lambda^{h\prime} & \\ 0 & & \cdot \end{bmatrix}_{H \times H(S+1)} \qquad \overline{\underline{z}} \equiv \begin{bmatrix} \cdot \\ \underline{z}^{h\prime} \\ \cdot \end{bmatrix}_{H(S+1) \times (S+1)(C-1)+J}$$

Thus

$$dv = (\lambda)' \left(\left(t_*^h \tau - \tau^h + \Psi^h D_t a \right)_h - \underline{\overline{z}} dP \right)$$

To visualize the bracket notation $[\cdot]$ defined in footnote 7, it staggers state contingent vectors:

$$[p] \equiv \begin{bmatrix} \cdot & & & & \\ & p_{s-1} & & 0 & \\ & & p_s & & \\ & 0 & & p_{s+1} & \\ & & & & \cdot \end{bmatrix}_{C(S+1)\times S+1}$$

7.3 Transversality

A function $F : M \times \Pi \to N$ defines another one $F_\pi : M \to N$ by $F_\pi(m) = F(m, \pi)$. Given a point $0 \in N$ consider the "equilibrium set" $E = F^{-1}(0)$ and the natural projection $E \to \Pi, (m, \pi) \mapsto \pi$. A function is *proper* if it pulls back sequentially compact sets to sequentially compact sets.

Remark 2 (Transversality) *Suppose F is a smooth function between finite dimensional smooth manifolds. If 0 is a regular value of F, then it is a regular value of F_π for almost every $\pi \in \Pi$. The set of such π is open if in addition the natural projection is proper.*

A subset of Π is **generic** if its complement is closed and has measure zero. Write $C^* = C(S+1)$. Here the set of parameters is

$$\Pi = O \times O' \times (0, \epsilon)$$

where O, O' are an open neighborhoods of zero in $R^{C^* H}, R^{\frac{C^*(C^*+1)}{2} H}$ relating to endowments and symmetric perturbations of the Hessian of utilities. We have in mind a fixed assignment of utilities, which we perturb by $O' \times (0, \epsilon)$. Specifically, given an equilibrium commodity demand $\overline{x}$ by some household and $\Box \in R^{\frac{C^*(C^*+1)}{2}}, \alpha \in (0, \epsilon)$ we define $u_{\Box,\alpha}$ as

$$u_{\Box,\alpha}(x) \equiv u(x) + \frac{\omega_\alpha(\|x - \overline{x}\|)}{2}(x - \overline{x})' \Box (x - \overline{x})$$

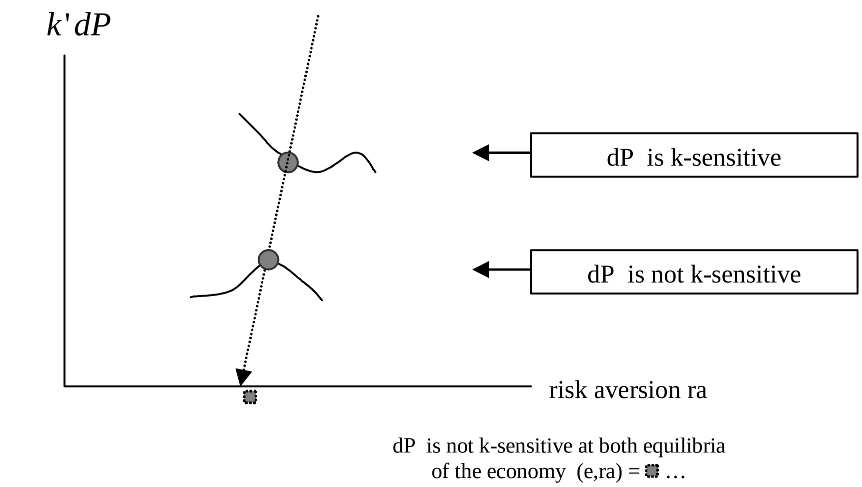
where $\omega_\alpha : R \to R$ is a smooth bump function, $\omega_\alpha \mid_{(-\frac{\alpha}{2},\frac{\alpha}{2})} \equiv 1$ and $\omega_\alpha \mid_{R\setminus(-\alpha,\alpha)} \equiv 0$. In a neighborhood $x \approx \overline{x}$ we have

$$\begin{aligned}
u_{\Box,\alpha}(x) &= u(x) + \frac{1}{2}(x-\overline{x})'\Box(x-\overline{x}) \\
Du_{\Box,\alpha}(x) &= Du(x) + (x-\overline{x})'\Box \Rightarrow Du_{\Box,\alpha}(\overline{x}) = Du(x) \\
D^2u_{\Box,\alpha}(x) &= D^2u(x) + \Box
\end{aligned}$$

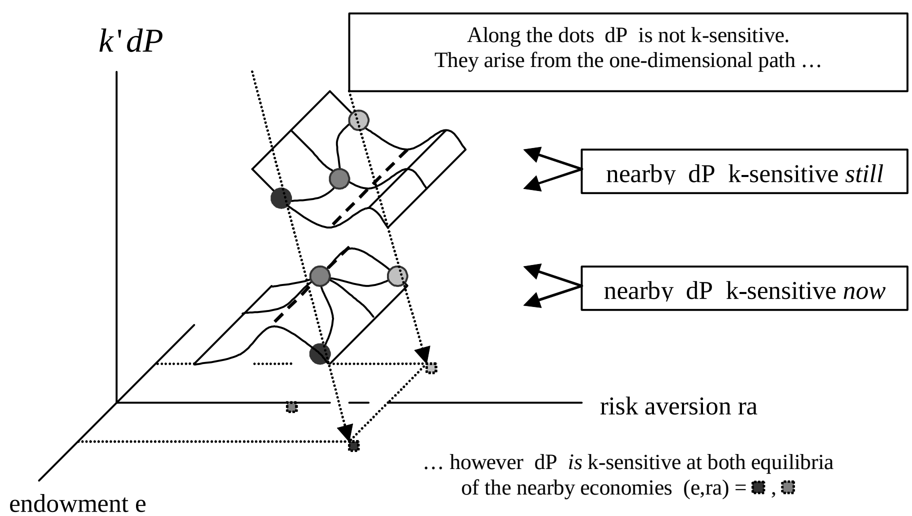
So in an α-neighborhood the Hessian changes, by $\Box$, but the gradient, demand do not. For small enough $\alpha, \Box$ this utility remains in Debreu's setting, so neoclassical demand is defined and smooth when active.

In the Sufficient Independence of Reactions, the path of risk aversion is identified with a linear path $(\Box^h, \alpha^h)(\xi) \equiv (\Box^h\xi, \frac{\|\overline{x}^h\|}{2})$ for each household, so that $\frac{d}{d\xi}D^2u^h_{\Box,\alpha}(x) = \Box^h$.

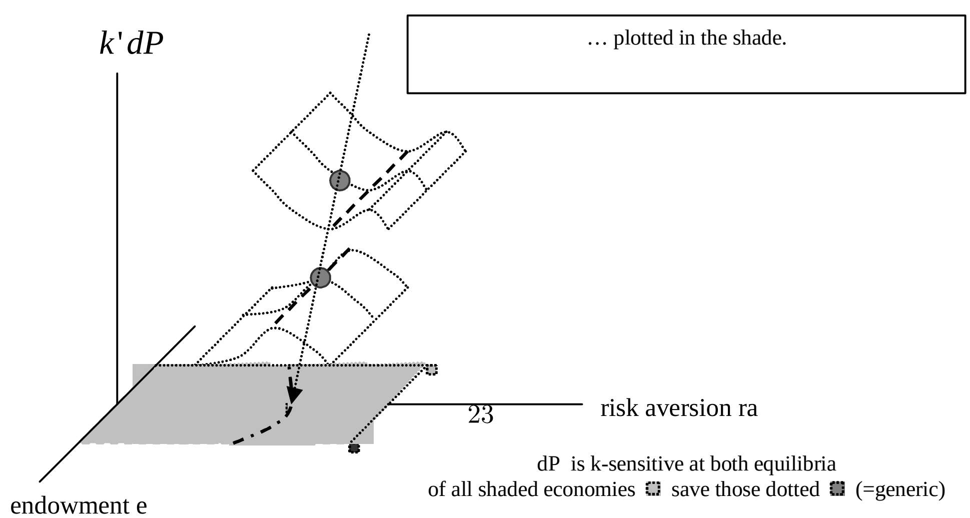
8 Figures



$k'dP$
dP is k-sensitive
dP is not k-sensitive
risk aversion ra
dP is not k-sensitive at both equilibria
of the economy (e,ra) = …



$k'dP$
Along the dots dP is not k-sensitive.
They arise from the one-dimensional path …
nearby dP k-sensitive *still*
nearby dP k-sensitive *now*
risk aversion ra
… however dP *is* k-sensitive at both equilibria
of the nearby economies (e,ra) = ,
endowment e



$k'dP$
… plotted in the shade.
risk aversion ra
dP is k-sensitive at both equilibria
of all shaded economies save those dotted (=generic)
endowment e

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