A Dynamic Theory of Fidelity Networks with an Application to the Spread of HIV/AIDS

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Abstract: We study the dynamic stability of fidelity networks, which are networks that form in a mating economy of agents of two types (say men and women), where each agent desires direct links with opposite type agents, while engaging in multiple partnerships is considered an act of infidelity. Infidelity is punished more severely for women than for men. We consider two stochastic processes in which agents form and sever links over time based on the reward from doing so, but may also take non-beneficial actions with small probability. In the first process, an agent who invests more time in a relationship makes it stronger and harder to break by his/her partner; in the second, such an agent is perceived as weak. Under the first process, only egalitarian pairwise stable networks (in which all agents have the same number of partners) are visited in the long run, while under the second, only anti-egalitarian pairwise stable networks (in which all women are matched to a small number of men) are. Next, we apply these results to find that under the first process, HIV/AIDS is equally prevalent among men and women, while under the second, women bear a greater burden. The key message is that anti-female discrimination does not necessarily lead to higher HIV/AIDS prevalence among women in the short run, but it does in the long run.

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1 Introduction

We study the dynamic stability of networks among agents of two types. Each agent enjoys having direct links with opposite type agents –the benefit of each relationship–, but establishing a link is costly in some dimension. In our leading application, which we shall refer to as *fidelity networks*, the two sets of agents are men and women who may consider establishing relationships with agents of the opposite sex. Each agent derives utility from having more direct links, but engaging in multiple partnerships is considered an act of infidelity and is punished if detected by the cheated partner. If one assumes that the benefit function is the same for all agents, but the punishment after detection of infidelity is more severe for women than for men, this results in women having a smaller optimal number of partners than do men. (As it turns out, this assumption on the asymmetry in the number of optimal links for both sides is one of the keys for our analysis.)[3]

There may be other uses of fidelity networks. For instance, countries as purchasers of latest-generation military equipment and their suppliers may want to keep the number of established links small, fearful of information leakages. Here, while in principle one would like to have more suppliers/customers, having more links has the potential of undesired transmission of confidential information. But our results extend to other non-fidelity networks as well. Buyers and sellers in a market can be modeled in this way, and in this case, one can argue that typically the optimal number of buyers for each seller exceeds the optimal number of stores each buyer purchases from. Graduate students writing a doctoral dissertation and their faculty advisors is another example, in which the number of optimal links for students is usually lower than it is for professors.

Apart from the theoretical interest of the dynamic analysis of such relationship networks, we also use it to shed new light on the effects of networks on communication transmission. Given that our leading application will be fidelity networks among men and women, we will highlight the use of our results in the understanding of the mechanisms of HIV/AIDS transmission due to different configurations of sexual networks.[4] We will

[3]This asymmetry is supported by a long standing literature on anti-female discrimination in most societies (see, e.g., Wollstonecraft (1792), Nussbaum and Glover (1995), Sen (1999)). One of the manifestations of this asymmetric treatment often appears in household surveys where it is claimed that women generally underreport their sexual activity (Fenton et al (2001), Zaba et al (2004), Mensch, Hewett, and Erulkar (2003), Jaya et al (2008)), consistent with the notion that women find it more difficult to admit having experienced sex outside a socially sanctioned relationship (Dare and Cleland (1994)).

[4]The implications of the spread of HIV/AIDS for the world are dramatic and serious. Having said that, the epidemic may have some unintended positive consequences in terms of a higher future per-capita output and consumption (the "Black Death effect") through lowering fertility rates, as argued in A. Young (2005) or Bloom and Mahal (1997); in contrast, Over (1992), Bell, Devarajan and Gersbach (2003), and Arndt and Lewis (2000) offer a more pessimistic assessment, by not emphasizing the drop in fertility rates. Perhaps consistent with these latter studies, more recent papers by Fortson (2009) and Juhn, Kalemli-Ozcan and Turan (2008) have found little effect of community-level HIV/AIDS prevalence on fertility. Also, see Galor (2005) for an appraisal of the different theories that connect fertility rates and growth. In particular, the arguments concerning mortality rates and different choices in terms of human capital investment may be of interest to this controversy.

also discuss some implications of our findings for models of union formation in different societies.

1.1 Fidelity

Fidelity is an important quality and an ethical principle in most types of social and economic relationships. In people's private lives, partners' faithfulness to each other is essential in sustaining a marriage relationship. In a competitive economic and political environment, the security of confidential information is crucial to the survival and success of such various organizations as firms, governments, intelligence services, military, political parties, research labs, or financial institutions. In all these cases, employees or members' loyalty is essential, whereas improper leaks of vital information (regarding technology, R&D programs, marketing strategy, political or military secrets, and so on) to rivals or to the media by disloyal employees or members can be very damaging.[5]

Infidelity in intimate relationships is also prevalent, and has dramatic social and economic consequences. Psychiatrist Frank Pittman documents several forms of betrayal, and argues that infidelity is behind 90% of first time divorces in the United States (Pitman (1990, 1999)). A study of DNA tests revealed that 10-15% of children were conceived as a result of an affair in Australia (ALRC (2003)), and in the United States, the father was not the true biological parent in 30% of paternity tests conducted by the American Association of Blood Banks (AABB (2003)). Globally, 33 million people live with the AIDS virus today, and infidelity in sexual relationships is advanced as the single most important driver of this epidemic (UNAIDS (2008)).

Despite playing an essential role in the determination of crucial social, epidemiological and economic outcomes, the notion of fidelity and its importance in the formation of links between self-interested agents who otherwise have a *prima facie* duty of loyalty to each other have received little attention in the economic literature.[6]

We study bipartite graphs, which means that each link connects a member of one of the two sets of agents (e.g., a man) with a member of the other set (e.g., a woman).[7] Our contribution in this study is twofold. First, we identify which networks are likely to arise in a bipartite environment in which opposite type agents

[5] Employees' loyalty has been identified as a major factor in a firm's growth (Reichheld (2003)), while leakage of technological information and its various economic consequences have been also documented (see, e.g., Mansfield et al. (1982), Mansfield (1985), Helpman (1993), Aghion et al. (2001)).

[6] Note however that networks have been used to study a wide variety of topics including job search through contact information (Boorman (1975), Montgomery (1991), Calvó-Armengol (2004), Ioannides and Loury (2004)), purchasing behavior and consumer products information (Frenzen and Davis (1990), Ellison and Fudenberg (1995)), technology diffusion and adoption (Coleman (1966)), friendship (Jackson and Rogers (2007a)), and community insurance (Fafchamps and Lund (2000), Kohler and Hammel (2001)).

[7] There is a long tradition of using bipartite environments to study matching problems, including for instance the marriage problem, the hospital-intern problem, the college admissions problem, buyer-seller networks, and the employee-employer problem (see, e.g., Hall (1935), Gale and Shapley (1962), Roth and Sotomayor (1989), Echnique and Oviedo (2006), Kranton and Minehart (2001), Sotomayor (2003)).

who desire fidelity from each other form and sever ties over time. Specifically, we shall study the dynamic and long-run stability of these networks. A unique aspect of the networks we study stems from the fact that a priori, agents do not know the other partners of their partners, and do not gain anything from being indirectly linked to them.[8] Thus, in particular, one can assume that links are not observable. Second, we examine the implications of these networks for gender differences in sexually transmitted diseases, with a focus on HIV/AIDS.

1.2 Overview of the Dynamic Model and Theoretical Results

Our economic environment consists of a finite population of two equal-size exogenously determined sets of individuals, say men and women. Each individual derives utility from the number of direct links with opposite sex agents, while engaging in multiple links is an act of infidelity, and is punished if detected by the cheated partner. Detection occurs with positive probability, and it is assumed that a woman whose infidelity is detected is more severely punished than a man in a similar situation. These considerations result in each agent having a single-peaked utility function, which implies that each agent has a desired or optimal number of partners. Due to gender asymmetry in the punishment of infidelity, this number is strictly greater for each man than for each woman.

We characterize the *pairwise stable or equilibrium networks* of this mating economy. In a mating economy such as the one we are describing, individuals form new links or sever existing links based on the reward that the resulting network offers them relative to the current network. We say that a network is pairwise stable or in equilibrium if (*i*) no individual has an incentive to sever an existing link he or she is involved in, and (*ii*) no pair of a man and a woman have an incentive to form a new link while at the same time severing some of the existing links they are involved in.

We shall assume that our population is sufficiently large, which allows for a simple characterization result of equilibrium networks.[9] In particular, we find that a network is pairwise stable if and only if each woman has exactly her optimal number of partners, and each man has at most his optimal number of partners. Women supply a smaller number of links than the ones demanded by men, which in turn results in only men competing for female partners while each woman is sure of having the number of male partners she desires.

The center of our analysis is a dynamic matching process for this economy, more precisely a Markov process,

[8]The extension of our analysis to the case in which an agent's well-being is affected by indirect links is important, but beyond the scope of this paper.

[9]In a companion study of fidelity networks, Pongou (2009a) provides a full characterization of stable networks without the "large populations" assumption made here. The basic static framework of that study has been extended to multi-ethnic societies in Pongou (2009b), yielding testable predictions for the effects of ethnic diversity on multiple sexual partnerships and the spread of HIV/AIDS.

based on the incentive that agents have to form new links or sever existing ones. The *unperturbed Markov process* assumes discrete time, and is defined as follows. In each period, a male-female pair chosen at random with arbitrary positive probability is given the opportunity to sever or add a link based on the improvement that the resulting network offers to each of them relative to the current network. If they are already linked in the current network, the decision is whether to sever the link; severance is a unilateral decision. Otherwise, the decision is whether to form a new link. While forming a new link, for simplicity, each agent is allowed to sever at most one of the links he/she is involved in in the current network. Link formation is a bilateral decision. The *long run predictions* of this process coincide with the set of equilibrium networks, a very large set.

To gain determinacy in our analysis, the matching process is perturbed in two ways, each perturbation consisting of allowing a small probability of forming new links or severing existing ones when this action is not beneficial. We study the long-run predictions of these *perturbed processes* –their *stochastically stable networks*–, these predictions being the networks that are visited a positive proportion of time in the very long run.[10]

In both perturbed dynamic processes, if a link formation is mutually beneficial or if a link severance is beneficial to its initiator, it occurs with probability 1. That is, this feature of the unperturbed dynamics is retained. However, the perturbed processes allow for more transitions. In both, an action that worsens its initiator, which we shall call a *mistake*, occurs with a small probability $\varepsilon > 0$. In between are actions that leave their initiators exactly indifferent. We shall refer to these as *neutral actions*. In the spirit of papers assuming that more serious mistakes are less likely, an agent's probability of taking a neutral action will always exceed ε. We now explain how.

In our models, neutral actions uniquely correspond to situations in which an agent severs an existing link with a current partner and forms a new link with another agent. We shall assume that the probability of taking such a neutral action is $\varepsilon^{f(\cdot)}$ (a number strictly greater than ε because the exponent will be smaller than 1). The exponent is the *strength of the existing link* so that stronger links –$f(\cdot)$ closer to 1– are harder to break.

In the first perturbed process, from the point of view of the agent who initiates a neutral action with the severance of an existing link, its strength $f(\cdot)$ is *inversely* proportional to the number of partners that the old partner had in the existing network. The interpretation is that this link is as strong as the amount of time

[10] The notion of stochastic stability has been applied to studying a number of problems in the economic literature (see, e.g., Freidlin and Wentzell (1984), Foster and H.P. Young (1989), Kandori, Mailath and Rob (1993), H.P. Young (1993, 1998), Ellison (1993), Noldeke and Samuelson (1993), Vega-Redondo (1997), Hart (2002), Jackson and Watts (2002), Alós-Ferrer and Weidenholzer (2007), etc.)

invested in it by the other partner. We study the long-run predictions of this perturbed process, and find that networks are visited a positive proportion of time in the very long run if and only if they are *egalitarian pairwise stable networks*. In these networks, men and women have the same number of partners, which is the optimal number of partners for women. Monogamous networks are a salient particular case, if such a number is 1.[11]

In contrast, the second perturbed process assumes that, in evaluating the probability of taking a neutral action, and the consequent severance of an existing link, its strength $f(\cdot)$ is *directly* proportional to the number of partners that the old partner had in the existing network. Here, the interpretation is that in a relationship, the partner who invests more time is perceived as "weak" or dominated by the other partner; and thus, it is easier for the dominant partner than for the dominated partner to break the relationship. For this case, we find that *anti-egalitarian pairwise stable networks*, which are networks in which each woman has her optimal number of partners, and a smallest possible set of men is matched, will be the only ones visited a positive proportion of time in the very long run. Each non-isolated man is matched to his optimal number of partners (except for at most one man, who will be matched to the remaining women). In the special case when each woman optimally has one partner, polygynous networks are selected.[12]

Note how what was taken to define the strength of a link in the first perturbed process is in the second process utilized to define the domination status of a party to a partnership. The key implication is that, while in the first process it is harder for an agent to break a relationship in which his/her partner invests too much time, in the second process it is much easier to break such a link. There are several advantages to considering both approaches. From a theoretical view point, both approaches, being polar opposites in the assumptions behind neutral actions, offer a more complete study of the problem being investigated. From an empirical view point, the two approaches correspond to different sociological realities.

1.3 HIV/AIDS

Our findings shed new light on the origins of gender differences in HIV/AIDS. Globally, the share of women among HIV infected adults has grown from 43% in 1990 to 50% in 2001 when it stabilized (UNAIDS (2008)). In sub-Saharan Africa, this figure has grown from 53% in 1990 to 60% in 2007 (UNAIDS (2008)). A recent study based on Demographic and Health Surveys and AIDS Indicator Surveys, which are household surveys

[11] Actually, given the random nature of our model, our prediction in this case would correspond quite closely to the practice widely known as "serial monogamy", which consists of a succession of short or long-term monogamous relationships. This practice is especially prevalent in some sectors of Western societies, and is associated with high divorce rates (Schoen et al. (1985), Glick et al. (1986), Cherlin (1992), Macura et al. (1994)). We discuss these issues further in our concluding section.

[12] Polygyny is common in many non-Western societies (see, e.g., Garenne and van de Walle (1989) on Senegal). In our concluding section, we show a simple example in which a polygynous network is associated with a low divorce rate. This contrasts with the high divorce rates that characterize serial monogamy.

commissioned by the United States Agency for International Development through the MEASURE DHS, shows that in most developing countries, women bear a disproportionate share of the HIV/AIDS burden (Mishra et al. (2009)).

Early on, it was hypothesized that the male-to-female transmission rate of the AIDS virus is greater than the female-to-male transmission rate, which was proposed as an explanation for the higher prevalence of HIV/AIDS among women. The argument generally put forth to support this hypothesis is speculative, and rests on the claim that women have larger exposed surface area of mucous membrane during sexual intercourse, as well as a larger quantity of potentially infectious fluids than men (WHO (2003)). But as pointed out in the same WHO's report, the evidence on this subject is incomplete. In fact, the hypothesis advanced for higher female vulnerability received its first empirical tests in the African context in two ground-breaking studies conducted in Uganda among a sample of monogamous heterosexual, HIV-discordant couples (Quin et al. (2000), Gray et al. (2001)). These couples were identified retrospectively from a population cohort in Rakai, Uganda. Frequency of intercourse within couples and HIV-1 seroconversion in the uninfected partners were assessed prospectively. Men and women independently reported similar frequencies of sexual intercourse.[13] The first study found that the male-to-female transmission rate of the AIDS virus was 12.0 per 100 person-years, while the female-to-male transmission rate was 11.6 per 100 person-years. But both figures were not found to be significantly different from each other. The second study found that the probability of the virus transmission per coital act from infected women to their initially uninfected male partners was 0.0013, compared with a transmission probability of 0.0009 per act from infected men to their initially uninfected female partners, but these figures were not statistically different from each other either. These findings run contrary to the early hypothesis and explanation for gender asymmetry in transmission rate. Also, in several Western countries where the prevalence of HIV/AIDS is low, women are not significantly more infected than men (UNAIDS (2008)). The question of the origins of gender differences in HIV infections therefore remains open.

Despite an increasing interest in understanding the role of gender discrimination in the higher vulnerability of women (WHO (2003)), how discrimination really plays out is still not well understood. The complexity of this topic partly stems from the fact that discrimination does not necessary lead to HIV/AIDS being more prevalent among women in the short run (Pongou (2009a)). The current study, however, offers a possible explanation: we show that anti-female discrimination always causes societies in which HIV/AIDS is more prevalent among men to progress toward ones in which women are more at risk, which means that in such

[13] This is a good feature of these data, as it seems to reflect that partners were faithful to each other, and thus infected individuals who were initially uninfected contracted the AIDS virus through intercourse with their initially infected partners. This makes it possible to assess gender differential transmission rates.

societies, the proportion of infected women grows over time.

A theoretical framework useful to the study of gender differences in HIV infection is proposed in Pongou (2009a). Assume that an agent is drawn at random from a network to receive a piece of information (interpreted here as an instance of getting infected by the HIV virus due to a random event). He/she then communicates it to his/her partners, who in turn communicate it to their other partners, and so on. If that agent has no partner, the information does not spread. Under the assumption that each agent is drawn with equal probability, Pongou (2009a) defines the *communication or contagion potential* of that network, which is the expected proportion of agents who will receive the information, and provides a formula for this notion. The study also derives a formula for *gender difference in contagion potential* in a network.

We show that under the first stochastic process, gender difference in contagion potential in any of the stochastically stable networks is zero, which implies that gender difference in HIV infection is small. Under the second stochastic process, women's contagion potential is greater than men's, which shows the higher vulnerability of women to HIV/AIDS.

When it comes to understanding HIV/AIDS spread, there are two key messages of the current study. (*i*) In the long run, men do not bear a higher burden of HIV/AIDS under neither process. And, (*ii*) in the second process, women are more severely affected by HIV/AIDS than men. The first explanation that underlies both facts, which seem to suggest that women are the "weak side", is our assumption of the greater punishment for infidelity that we pose holds for women. But to understand the second finding is slightly more subtle. Theoretically, the "female subjugation" in the stochastically stable networks of the second process is surprising, given that the definition of the stochastic process itself is "gender neutral". Indeed, that it is easier for the dominant partner than for a dominated partner to break the relationship always applies, whether the dominant partner is a man or a woman. However, in combination with our infidelity punishment gender asymmetry assumption, societies under the second perturbed process can be termed *male dominant*. Given the structure of our long run predictions, all the key transitions involve a woman severing a link to form a new one, and in doing so, the cost of breaking that link is a direct function of the dominant role of her old male partner, measured by the number of his links. The result is then that in such male-dominant societies, the long run predictions are anti-egalitarian pairwise stable networks, which leads to a contagion potential that is always greater for women than for men.

1.4 Related Literature

Jackson and Wolinsky (1996) were among the first to propose a general framework for the study of the stability and efficiency of social and economic networks, taking into account the incentive that self-interested agents have to form and sever links with each other.[14] They develop a notion of pairwise stability of networks, and study its relationship with efficiency. The analysis in the Jackson-Wolinsky paper is however based on a static approach of stability, leaving unanswered the questions pertaining to dynamics. Such questions are subsequently addressed in Jackson and Watts (2002) using the notion of dynamic and stochastic stability.

Our study shares several features with these two papers. First, our analysis of statically stable networks is based on a notion of pairwise stability that allows for simultaneous link formation and severance. While this definition of stability differs from the one proposed in Jackson and Wolinsky (1996), it is close to the one underlying the analysis of the marriage problem in Jackson and Watts (2002). It can however be shown that the set of stable networks is the same under the two definitions. Second, our analysis of the dynamic stability of fidelity networks draws on the theoretical framework proposed in Jackson and Watts (2002). But our study differs from theirs in some important respects. First, our focus is on analyzing fidelity networks; and second, our dynamic analysis rests on the notion that more severe mistakes in link formation or severance are less likely (our distinction between mistakes and neutral actions).[15]

Our study also shares the idea underlying Bala and Goyal (2000) in that both papers study the dynamics of network formation. The focus in Bala and Goyal (2000) is however on directed networks, in which an agent can connect to another agent without the consent of the latter, whereas we study undirected networks where forming a link requires the mutual consent of the two parties involved. Our models therefore end up having very different applications. Second, the dynamic analysis of network formation used in Bala and Goyal is based on a repeated game to which learning is applied to characterizing equilibrium networks, an approach which is quite different from the stochastic stability of mating, adopted here.

Finally, we have acknowledged that the analysis of gender difference in information concentration relies on the theoretical framework proposed in Pongou (2009a). In that paper, the computation of information

[14] This important study was preceded by Aumann and Myerson (1988). Aumann and Myerson (1988) study a two-stage game in which in the first stage, players form bilateral links resulting in a communication and cooperative structure, to which the Myerson value (Myerson (1977)) is applied to determine the payoff to each player in the second stage. Extensions and variants of this game have been considered in Dutta, van den Nouweland and Tijs (1996), Slikker and van den Nouweland (2001a), and Slikker and van den Nouweland (2001b). Following the pioneering works of Aumann and Myerson (1988) and Jackson and Wolinsky (1996), a number of studies on strategic network formation have been conducted (see, e.g., Dutta and Mutuswami (1997), Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Jackson and van den Nouweland (2005), Page, Wooders and Kamat (2005), Dutta, Ghosal and Ray (2005), Bloch and Jackson (2007), etc.).

[15] In this regard, we draw on a recent literature that has used stochastic stability and the "more serious mistakes are less likely" assumption in other problems: see, e.g., Blume (1993, 1997), Durlauf (1997), H.P. Young and Burke (2001), Ben-Shoham, Serrano and Volij (2004), Sandholm (2007), Kandori, Serrano and Volij (2008), Serrano and Volij (2008), Myatt and Wallace (2006).

concentration in a given network relies on the assumption that information travels the network via word-of-mouth or neighbors' contagion, and so does not spread if received by an isolated agent. This assumption also underlies some analyses of how network structure affects the spread or diffusion of certain diseases or behaviors (see, e.g., Pastor-Satorras and Vespignani (2000, 2001), Jackson and Rogers (2007b), Jackson and Yariv (2007), and Lopez-Pintado (2008)). The different approaches used in these studies to analyzing diffusion generally assume a distribution of links or connections in the population, and/or a payoff function whose arguments include an individual's and her neighbors' choice of a certain behavior, and often rely on the mean-field approximation theory, which consists of solving a particular differential equation, to identify equilibria. Our approach makes no assumptions on the connectivity distribution of the population, but relies only on the knowledge of the number of components and their size.

1.5 Plan of the Paper

The remaining of this paper unfolds as follows. Section 2 introduces the model that forms the basis for our analysis. We characterize pairwise stable networks in Section 3. In Section 4, we define the unperturbed Markov process and characterize its recurrent states. This process is perturbed in Section 5 and Section 6 respectively, and a characterization result of stochastically stable networks is provided for each of the two perturbed systems. In section 7, we study the implications of our results for gender differences in HIV/AIDS, which is followed by the conclusion in Section 8.

2 The Model

The economic environment consists of a finite set of individuals $N = \{1, 2, \ldots, n\}$, partitioned into a set of men M and a set of women W, each of equal size. Each individual derives utility from direct links with opposite sex agents, but engaging in multiple links is an act of infidelity, and is punished if detected by the cheated partner. Detection occurs with positive probability. It is assumed that a woman whose infidelity is detected is more severely punished than a man in a similar situation. Networks that arise from this environment are called fidelity networks.

2.1 Utility Functions

Let $M \times W$ denote the cartesian product of M and W. A network is a subset of $M \times W$. Let g be a network. Since we are dealing with undirected graphs, if $(i, j) \in g$, we will abuse notation and consider that $(j, i) \in g$

(in fact, (i,j) and (j,i) represent the same relationship). Let $i \in N$ be an individual, and $s_i(g)$ the number of opposite sex partners that i has in the network g. The utility that i derives from g is expressed by the following function:

$$u_i(g) = v(s_i(g)) - c(s_i(g))$$

where $v(s_i(g))$ is the utility derived from direct links with opposite sex partners in g, and is concave and strictly increasing in $s_i(g)$; and $c(s_i(g))$ the cost of infidelity.

Let us define the cost function more precisely. Let $j,k \in N$ be such that $(i,j) \in g$ and $(i,k) \in g$. Let π be the probability that j detects the liaison (i,k), and c the cost incurred by i if j detects that liaison. Because i has $s_i(g)$ partners, he/she will be detected $s_i(g)(s_i(g)-1)$ times with probability π, incurring an average cost of $s_i(g)(s_i(g)-1)\pi c$. So we define the cost function as:

$$c(s_i(g)) = s_i(g)(s_i(g)-1)\pi c$$

Assuming that i is an expected utility maximizer, he/she will thus maximize the following utility function:

$$u_i(g) = v(s_i(g)) - s_i(g)(s_i(g)-1)\pi c$$

We denote the extension of u_i to the non-negative reals as $\overline{u_i}(s_i)$. Without loss of generality, let $\overline{u}_i$ be twice continuously differentiable. The following remark is straightforward:

Remark 1 *(1)* $\exists s^* \in [1,+\infty[$ *such that* $\overline{u}'(s^*) = 0$, $\forall s \in [0,s^*[$, $\overline{u}'(s) > 0$, *and* $\forall s \in]s^*,+\infty[$, $\overline{u}'(s) < 0$.

(2) $\frac{\partial s^*}{\partial c} \leq 0$

Remark 1 implies that u_i is single-peaked. Given that the cost incurred per detection is equal for all individuals of the same sex, they have the same optimal number of partners. Further, the optimal number of partners for women is smaller than the optimal number of partners for men because the former are more severely punished than the latter if their infidelity is detected. Note that if s^* is not an integer, then the optimal number of partners will be either the largest integer smaller than s^* $\lfloor s^* \rfloor$ or the smallest integer greater than s^* $\lceil s^* \rceil$. We also postulate that for no $s \geq 0$, $u_i(s) = u_i(s+1)$. These considerations motivate our first assumption, which we make explicit as follows:

Assumption A1. Denoting by s_m^* and s_w^* the unique optimal integer number of partners for men and women, respectively, we assume that $s_m^* > s_w^*$.

In order to derive our results, we will make a "large populations" assumption. As it turns out, this will be a strenghthening of Assumption A1. Specifically:

Assumption A2. We assume $|M|$ to be large enough so that $\frac{|M|-(s_w^*-1)}{|M|} > \frac{s_w^*}{s_m^*}$.

Note how A2 is stronger than A1 whenever $s_w^* > 1$. If the optimal number of partners for women is exactly 1, A2 reduces to A1. For the kinds of applications we have in mind, this is an appropriate assumption. On the other hand, Pongou (2009a) studies a related problem without assuming this.

2.2 Fidelity Networks

Let g be a fidelity network. The elements of N are called vertices. A path in g connecting two vertices i_1 and i_n is a set of distinct nodes in $\{i_1, i_2, \ldots, i_n\} \subset N$ such that for any k, $1 \leq k \leq n-1$, $(i_k, i_{k+1}) \in g$.

Let i be an individual. We denote by $g(i) = \{j \in N : (i,j) \in g\}$ the set of individuals who have i as a partner in the network g. The cardinality of $g(i)$ is called the degree of i. If a set A is included either in M or W, then the image of A in the network g is $g(A) = \bigcup_{i \in A} g(i)$.

We denote respectively by $M(g) = \{i \in M : \exists j \in W, (i,j) \in g\}$ and by $W(g) = \{i \in W : \exists j \in M, (i,j) \in g\}$ the set of men and women who are matched in the network g. We pose $N(g) = M(g) \cup W(g)$.

A subgraph $g' \subset g$ is a component of g if for any $i \in N(g')$ and $j \in N(g')$ such that $i \neq j$, there is a path in g' connecting i and j, and for any $i \in N(g')$ and $j \in N(g)$ such that $(i,j) \in g$, $(i,j) \in g'$. A network g can always be partitioned into its components. This means that if $C(g)$ is the set of all components of g, then $g = \bigcup_{g' \in C(g)} g'$, and for any $g' \in C(g)$ and $g'' \in C(g)$, $g' \cap g'' = \emptyset$.

A fidelity network is said to be egalitarian if all agents have the same degree.

3 Equilibrium Networks

In a society such as the one we are describing, individuals form new links or sever existing links based on the improvement that the resulting network offers them relative to the current network. We say that a network g is pairwise stable or in equilibrium if (i) no individual has an incentive to sever an existing link he/she is involved in, and (ii) no pair of a man and a woman have an incentive to form a new link while at the same time severing some of the existing links they are involved in.

More formally, given a profile of utility functions $u = (u_i)_{i \in N}$, a network g is pairwise stable with respect to u if:

(i) $\forall i \in N$, $\forall (i,j) \in g$, $u_i(g) > u_i(g \setminus \{(i,j)\})$

(*ii*) $\forall (i,j) \in (M \times W) \setminus g$, if network g' is obtained from g by adding the link (i,j) and perhaps severing other links involving i or j, $u_i(g') > u_i(g) \Longrightarrow u_j(g') \leq u_j(g)$ and $u_j(g') > u_j(g) \Longrightarrow u_i(g') \leq u_i(g)$.

To illustrate this definition, consider the following examples. A network in which a woman is matched to $s > s_w^*$ men is not an equilibrium as she can unilaterally sever $s - s_w^*$ links. A network in which a man is matched to $s_m^* + 2$ women and a woman not matched to him is matched to fewer than s_w^* men is not stable, as they could form a link while the man could sever three of his former links. Finally, a network in which a man and a woman who are unmatched have fewer than their optimal partners is not pairwise stable either, as they could form a link without severing any other.

3.1 Characterization of the Equilibrium Networks

In this subsection, under our "large populations" assumption, we characterize the equilibrium networks. This characterization will be useful in our dynamic analysis later on.

Theorem 1 *Assume A2, and let g be a network. Then, (1) and (2) are equivalent.*

(1) g is pairwise stable

(2) $\forall (m,w) \in M \times W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$.

Proof. $(1) \Longrightarrow (2)$: Let g be a pairwise stable network. It is straightforward that $\forall (m,w) \in M * W$, $0 \leq s_m \leq s_m^*$ and $0 \leq s_w \leq s_w^*$. In fact, if an agent has more than his/her optimal number of partners, he/she will be better off by unilaterally severing one link, thus implying that g is not pairwise stable, a contradiction.

Therefore, it only remains to show that $\forall w \in W$, $s_w = s_w^*$. By contradiction, suppose that there exists a woman w_0 with $s_{w_0} < s_w^*$. First, it should be clear that for every man m not matched with w_0, $s_m = s_m^*$. This is because, if at least one such man were matched with fewer women, that man and w_0 would improve by forming a new link, implying that g is not pairwise stable, which is a contradiction.

It then follows that the number of links coming from the men side is at least $(|M| - s_{w_0})s_m^*$, which is greater than or equal to $[|M| - (s_w^* - 1)]s_m^*$, which by Assumption A2 is greater than $|M|s_w^* = |W|s_w^*$, an upper bound on the number of links coming from the women side. Since the number of links coming from the men side must exactly equal the number of links coming from the women side, this is impossible. We conclude that $\forall w \in W$, $s_w = s_w^*$.

$(2) \Longrightarrow (1)$: Let g be a network. Assume that $\forall (m,w) \in M * W$, $0 \leq s_m \leq s_m^*$ and $s_w = s_w^*$, and let us show that g is pairwise stable. A man alone cannot improve by severing a link since he is at the upward sloping part of his utility function. He cannot form a new link with another woman since each woman has her optimal number of partners. And a woman cannot be part of any blocking move (either by herself or with a man) since she is at her peak. Therefore, g is a pairwise stable network. ∎

Let us illustrate Theorem 1 with the following examples.

Example 1 *Consider a mating economy in which there are 10 men and 10 women. Assume that their utility functions are such that* $s_w^* = 2$ *and* $s_m^* = 4$. *The three networks represented respectively by Figure 1-1, Figure 1-2 and Figure 1-3 are pairwise stable. In fact, in each graph, each woman has 2 partners (the optimal number of partners for each woman), and each man has at most 4 partners. In the first network component configuration* $[(2,2);(5,5);(3,3)]$[16], *all agents have 2 partners, thus this network is egalitarian; in the second network component configuration* $[(7,6);(2,4);(1,0)]$, *2 men have 1 partner each, 5 men have 2 partners each, 2 men have 4 partners each, and 1 man has no partner; in the third network component configuration* $[(2,4);(2,2);(2,4),(1,0),(1,0),(1,0),(1,0)]$, *2 men have 2 partners each, 4 men have 4 partners each, and 4 men have no partner. An interesting feature of the last two graphs is the uneven share of female partners among men, which reveals a sharp competition in the latter group.*

In Example 1, note that the "large populations" condition (Assumption A2) is satisfied. Now, consider the following example in which that condition is violated. We show that our characterization does not hold then.

Example 2 *Consider a mating economy in which there are 7 men and 7 women;* $s_w^* = 4$ *and* $s_m^* = 5$. *Assumption 2 clearly does not hold. The network component configuration* $[(3,2);(4,5)]$ *represented by Figure 2-1 is pairwise stable. In it, the first component has 3 men and 2 women, each woman is matched to 3 men (that is less than* s_w^**) and each man is matched to 3 women (also less than* s_m^**). The second component has 4 men and 5 women, each women matched to* $s_w^* = 4$ *men, and each man matched to* $s_m^* = 5$ *women. Note that although there exists a woman* w_0 *such that* $s_{w_0} < s_w^*$, *men who are linked to fewer than* $s_m^* = 5$ *women are already matched to her in the first component. The argument is similar for men who have fewer partners than their optimal number. It follows that this network is pairwise stable.*

We also note that the network component configuration $[(7,7)]$ *represented by Figure 2-2 is pairwise stable. In this network, each woman has 4 partners, 6 men have 4 partners each, and 1 man has 3 partners. This network therefore meets the characterization of Theorem 1. One can show that even if Assumption 2 does not hold, all networks that meet the characterization of Theorem 1 are pairwise stable (Pongou (2009a)), which implies that in small populations, the set of such networks is included in the set of all pairwise stable networks.*

[16] $[(2,2);(5,5);(3,3)]$ refers to a network component configuration with 3 components, the first component containing 2 men and 2 women, the second component 5 men and 5 women, and the third component 3 men and 3 women. This notation is a simplification that abstracts from the complete network structure as represented by the graph.

4 A Dynamic Network Formation Process

In this section we turn to dynamics. First, we shall define a Markov process for any given mating economy as previously defined, to describe the formation and severance of links over time. Later on, given the lack of predictive power of this process, we shall resort to perturbing it in two different ways, leading to two perturbed Markov processes.

The unperturbed Markov process P^0 is as follows. Time is discrete. In each period, a male-female pair chosen at random with arbitrary positive probability is given the opportunity to sever or add a link based on the improvement that the resulting network offers to them relative to the current network. If they are already linked in the current network, the decision is whether to sever the link. Otherwise, the decision is whether to form a new link. While forming a new link, for simplicity, each agent is allowed to sever at most one of the links he/she is involved in in the current network. Link severance is unilateral, while link formation is bilateral.

Let g and g' be two networks. They are said to be adjacent if there exist $i \in M$ and $j \in W$ such that $g' \in \{g+ij, g+ij-ik, g+ij-ik-jm, g+ij-jm, g-ij\}$.[17] Let x and y be two networks. An $(x,y)-path$ is a finite sequence of networks $(g^0, g^1, \ldots, g^k)$ such that $g^0 = x$, $g^k = y$ and for any $t \in \{0, 1, \ldots, k-1\}$, g^t and g^{t+1} are adjacent.

An improving path from x to y is a finite sequence $g^0, g^1, \ldots, g^k$ such that for any $t \in \{0, 1, \ldots, k-1\}$:

- *(i)* $g^{t+1} = g^t - ij$ for some ij such that $u_i(g^{t+1}) > u_i(g^t)$ or $u_j(g^{t+1}) > u_j(g^t)$; or
- *(ii)* $g^{t+1} \in \{g^t+ij, g^t+ij-ik, g^t+ij-ik-jm, g^t+ij-jm\}$ for some ij such that $u_i(g^{t+1}) > u_i(g^t)$ and $u_j(g^{t+1}) > u_j(g^t)$.

Recurrent classes of a Markov process are those sets of states such that, if reached, the process cannot get out of them. We next characterize the recurrent classes of the unperturbed markov process P^0:

Theorem 2 *The recurrent classes of the unperturbed markov process P^0 are singletons, each of which containing each pairwise stable network.*

Proof. The proof is straightforward and left to the reader. ■

Thus, the set of long run predictions of the unperturbed dynamics is quite large (recall the characterization in Theorem 1). We proceed by perturbing this process in the sequel. We shall define below two such perturbed processes.

[17] We simplify notation here and write ij instead of (i,j), $g+ij$ instead of $g \cup \{(i,j)\}$, and $g-ij$ instead of $g \setminus \{(i,j)\}$, etc.

5 The First Perturbed Markov Process P_1^{ε} or "If You are not Committed, I May Leave You"

In each period, the revision opportunity offered at random to a male-female pair is the same as described in the process P^0. However, now agents may make decisions that do not necessarily lead to an immediate individual improvement. We describe these events in detail.

- If the two agents are linked in the current network:
  - Link severance takes place with probability 1 if it benefits either of the two agents, just as before.
  - Otherwise, while in the unperturbed process no severance of this link was taking place, now if it makes the two agents worse off, severance takes place with probability ε (note that in our model, link severance cannot make an agent indifferent). Recall that link severance is a unilateral decision, and thus it takes one "mistake" to sever such a good link: an agent making a mistake with probability $\varepsilon > 0$.
- If the two agents are not linked in the current network, the decision is whether to form a new link:
  - This link formation takes place with probability 1 if it is mutually beneficial, just as before. All other transitions did not happen in the unperturbed process, while now they will.
  - If forming the link makes one agent worse off and the other better off –one "mistake"–, it occurs with probability ε.
  - If the link formation makes the two agents worse off –two "mistakes"–, it occurs with probability ε^2.
  - If the transition makes one agent better off and the other agent, say j, indiffirent, agent j may take this "neutral action" and looks at considerations other than his/her well-being. Indifference in the transition happens because, while forming a new link with i, j severs an existing link, say with agent k in the current network. Then, the resistance of this transition amounts essentially to the strength of the severed link. Specifically, we assume that the transition occurs with probability $\varepsilon^{f(\frac{1}{s_k})}$ where the link strength f is a strictly increasing function of $\frac{1}{s_k}$ mapping into $(0, 1)$. Here, s_k is the number of partners that k has in the current network. We offer an interpretation below, at the end of the description of the process.

– If the transition makes one agent worse off and the other agent indifferent (one "mistake" and one "neutral action"), the transition occurs with probability $\varepsilon * \varepsilon^{f(\frac{1}{s_k})} = \varepsilon^{1+f(\frac{1}{s_k})}$.

– Finally, if it makes the two agents indifferent (two "neutral actions"), meaning that while forming a new link, i and j severed links with, say h and k, respectively in the current network, it occurs with probability $\varepsilon^{f(\frac{1}{s_h})} * \varepsilon^{f(\frac{1}{s_k})} = \varepsilon^{f(\frac{1}{s_h})+f(\frac{1}{s_k})}$.

We emphasize our assumption on the resistance of transitions involving indifferences or "neutral actions." The function $f(\frac{1}{s_k})$ can be viewed as the strength of the link that is being severed by j. If we assume for instance that each agent is endowed with 1 unit of time that he/she splits equally among all his/her partners, then it makes sense to assume that the strength of a link is inversely proportional to the number of partners.[18]

5.1 Resistance of a Path

For any adjacent networks g and g', the resistance of the transition from g to g' , $r(g, g')$, is the weighted number of agents directly involved in the transition who do not find this change profitable; it is the exponent of ε in the corresponding transition probability. We explicitly define $r(g, g')$ in the table below, as a function of the possible frictions –"mistakes" or "neutral actions"– found in a random chosen pair (i, j). To read the table, note that there are only three actions that either i or j can take, some combinations of which might not be possible:

A- Forming a new link without severing an existing link.

B- Forming a new link while severing an existing link.

C- Severing an existing link.

Let (a_i, a_j) be the pair of actions taken by i and j respectively. Then $(a_i, a_j) \in \{(A, A), (A, B), (B, B), (C, C)\}$. A pair of actions (a_i, a_j) might made either agent better off (b), lose (l), or indifferent (i). Transition probabilities and resistances are summarized in Table 1 below.

[18] Although for simplicity we assume that j observes s_k, given the nature of fidelity networks, note that we could assume that j doesn't, but can evaluate the strength $f(\frac{1}{s_k})$, for instance through a noisy signal, such as the amount of time spent by the partner out of the house, etc. A similar comment applies to the process in the next section.

Table 1

(a_i, a_j)	*Outcomes*	*Probability*	$r(g, g')$
(A, A)	(b, b)	1	0
(A, A)	(b, l)	ε	1
(A, A)	(l, l)	ε^2	2
(A, B)	(b, i)	$\varepsilon^{f(\frac{1}{s_k})}$	$f(\frac{1}{s_k})$
(A, B)	(l, i)	$\varepsilon^{1+f(\frac{1}{s_k})}$	$1 + f(\frac{1}{s_k})$
(B, B)	(i, i)	$\varepsilon^{f(\frac{1}{s_h})+f(\frac{1}{s_k})}$	$f(\frac{1}{s_h}) + f(\frac{1}{s_k})$
(C, C)	(b, b)	1	0
(C, C)	(b, l)	1	0
(C, C)	(l, l)	ε	1

The resistance of an (x, y)-path $q = (g^0, g^1, \ldots, g^k)$ is the sum of the resistances of its transitions: $r(q) = \sum_{t=0}^{k-1} r(g^t, g^{t+1})$.

Let $Z^0 = \{g^0, g^1, \ldots, g^l\}$ be the set of absorbing states of the unperturbed process.[19] Consider the complete directed graph with vertex set Z^0, denoted ∇. The resistance of the edge (g^i, g^j) in ∇ is the minimum resistance over all the resistances of the $(g^i, g^j) - paths : r(g^i, g^j) = minimum\{r(q) \mid q$ is an (g^i, g^j)-path$\}$.

Let g be an absorbing state. A g-tree is a tree whose vertex set is Z^0 and such that from any vertex g' different from g, there is a unique directed path in the tree to g. The resistance of a g-tree is the sum of the resistances of the edges that compose it. The stochastic potential of g, denoted $r(g)$, is the minimum resistance over all the $g - trees$.

The set of stochastically stable networks is the set $\{g \mid r(g) \leq r(g')$ for all $g'\}$ (H.P. Young (1993), Kandori, Mailath and Rob (1993)).

5.2 The Result

We shall now characterize the set of stochastically stable states of the perturbed process P_1^{ε}. The following definitions and lemmas are needed.

Let g be a network. We shall say that g is egalitarian if all vertices have the same degree.

Pose $I(g) = \{i \in M : s_i(g) \geq s_j(g) \; \forall j \in M\}$, i.e., the set of men that are matched with the highest number of women in the network g.

[19] Absorbing states are those in singleton recurrent classes.

Let $J(g) = \{i \in M : s_i(g) \leq s_j(g)\ \forall j \in M\}$, i.e., the set of men who are matched with the smallest number of women in the network g.

And call $I^*(g) = \{i \in M : s_i(g) \geq s_w^*\}$, i.e., the set of men who have at least a number of partners no less than the women's optimal number.

It is obvious that, if g is pairwise stable, $I(g)$, $J(g)$ and $I^*(g)$ are non-empty. Let $L(g) = \sum_{i \in I^*(g)}(s_i(g) - s_w^*)$.

Lemma 1 *Assume A2, and let g be a non-egalitarian pairwise stable network. Then, $\forall (i,j) \in I(g) \times J(g)$, $s_i(g) > s_w^* > s_j(g)$ (and therefore, $s_i(g) \geq s_j(g) + 2$).*

Proof. Appealing to the characterization of pairwise stable networks in Theorem 1 and using the definition of egalitarian networks, the proof is straightforward and left to the reader. ■

The following lemma describes a simple way to reach egalitarian networks:

Lemma 2 *Let g be a pairwise stable network. Then, there exists a finite sequence of pairwise stable networks $(g^0, g^1, \ldots, g^k)$ such that $g^0 = g$, $g^k = g^{L(g)}$, and g^k is egalitarian.*

Proof. Let g be a pairwise stable network. Pose $g^0 = g$. If g is egalitarian, then $\forall i \in M \cup W$, $s_i(g) = s_w^*$. Thus $L(g) = \sum_{i \in I^*(g)}(s_i(g) - s_w^*) = 0$, implying that the sequence searched for is (g). If g is non-egalitarian, then it is obvious that $L(g) > 0$ since from Lemma 1, at least one man has more than s_w^* partners. There exists a pair of men $(i_0, j_0) \in I(g) * J(g)$. Again by Lemma 1, since $s_{i_0}(g) \geq s_{j_0}(g) + 2$, there exists a woman k_0 such that $(i_0, k_0) \in g$ and $(j_0, k_0) \notin g$. Sever the link (i_0, k_0), and add the link (j_0, k_0); call the resulting network g^1. It is easy to check that g^1 is pairwise stable and that $L(g^1) = L(g) - 1$. Then, either g^1 is egalitarian and we are done, or not. That is, repeating the same operation $L(g) - 1$ more times induces a sequence $(g^1, \ldots, g^{L(g)})$ of pairwise stable networks. We have $L(g^{L(g)}) = L(g) - L(g) = 0$. Therefore, in the network $g^{L(g)}$, no man has more than s_w^* partners. But given that each woman has s_w^* partners in $g^{L(g)}$, that $|M| = |W|$, and that $\sum_{i \in M} s_i(g^{L(g)}) = \sum_{j \in W} s_j(g^{L(g)}) = s_w^* |W|$, it is necessarily the case that $\forall i \in M$, $s_i(g^{L(g)}) = s_w^*$. Thus $g^{L(g)}$ is pairwise stable and egalitarian. ■

In addition, any two egalitarian pairwise stable networks are "connected". This is shown in the following connectivity lemma:

Lemma 3 *Let g and g' be two distinct egalitarian pairwise stable networks. Then, there exists a finite sequence of pairwise stable networks $(g^0, g^1, \ldots, g^{2k})$ such that $g^0 = g$, $g^{2k} = g'$, and for any t such that $0 \leq t \leq k$, g^{2t} is egalitarian.*

Proof. Let g and g' be two distinct egalitarian pairwise stable networks. Pose $g^0 = g$. Pose $g' \setminus g = \{(m,w) : (m,w) \in g' \text{ and } (m,w) \notin g\}$. Since g and g' are different, $g' \setminus g$ is non-empty. Thus, there exists a pair (m_0, w_0) such that $(m_0, w_0) \in g'$ and $(m_0, w_0) \notin g$. Since g and g' are egalitarian, this implies that there exists a man m_0' such that $(m_0', w_0) \in g$ and $(m_0', w_0) \notin g'$. (In fact, if we assumed by contradiction that the latter statement were wrong, then it would mean that for any pair $(m_0', w_0) \in g$, then $(m_0', w_0) \in g'$; and since $(m_0, w_0) \in g'$ and $(m_0, w_0) \notin g$, this would imply that w_0 has more than s_w^* in the network g', contradicting the fact that g' is egalitarian and pairwise stable.)

Then, in g, add the link (m_0, w_0) and delete the link (m_0', w_0) (this is equivalent to woman w_0 severing her link with m_0' to form a new link with m_0), and call the resulting network g^1. In g^1, m_0 and m_0' have respectively $s_w^* + 1$ and $s_w^* - 1$ partners, and each woman has s_w^* partners as in g. Thus g^1 is pairwise stable, but it is not egalitarian. Also, note that g^1 is (one step) closer to g' than $g^0 = g$ (that is, $g' \setminus g^1 \subset g' \setminus g$).

We now want to construct g^2. Let $g^1(m_0) = \{w \in W : (m_0, w) \in g^1\}$. There exists a woman $w_0' \in g^1(m_0)$ such that $w_0' \neq w_0$, $(m_0', w_0') \notin g^1$ and $(m_0, w_0') \notin g'$ (in fact, since $|g^1(m_0)| = s_w^* + 1 > 1$ and $w_0 \in g^1(m_0)$, there exists $w_0' \in g^1(m_0)$ such that $w_0' \neq w_0$; now, if by contradiction, we assume that for any such w_0', $(m_0', w_0') \in g^1$, then it will turn out that $|g^1(m_0')| = s_w^*$, which is a contradiction since we know from the last paragraph that m_0' has exactly $s_w^* - 1$ partners in g^1; finally, if by contradiction, we assume that for any such w_0', $(m_0, w_0') \in g'$, then it will turn out that $g'(m_0) = g^1(m_0)$, implying that $|g'(m_0)| = s_w^* + 1$, thereby contradicting the fact that g' is egalitarian). Therefore, sever the link (m_0, w_0'), add the link (m_0', w_0'), and call the resulting network g^2. It is easy to check that in g^2, each man and each woman has exactly s_w^* partners. Thus g^2 is egalitarian and pairwise stable.

We also note that g^2 is at least 1 step closer to g' (in fact, since $(m_0, w_0') \notin g'$, severing this link in g^1 does not take us 1 step further from g'; also, if possible, one can choose w_0' in such a way that $(m_0', w_0') \in g'$, and in that case, g^2 will be 2 steps closer to g'; if not, g^2 will be 1 step closer to g').

If $g^2 = g'$, we are done; if not, repeat the same operation as previously by replacing g^0 with g^2. That will induce g^3 and g^4, and will take us at least one step closer to g'. In general, since $|g' \setminus g|$ is finite, repeating this operation a finite number of times (at most $\left\lceil \frac{|g' \setminus g|}{2} \right\rceil$ times) induces a finite sequence of pairwise stable networks $(g^0, g^1, \ldots, g^{2k})$ that ends at $g^{2k} = g'$ and satisfying that for any t such that $0 \leq t \leq k$, g^{2t} is egalitarian. ■

We are now ready to state and prove the main result of the section:

Theorem 3 *Assume A2. A network is stochastically stable in the perturbed process P_1^ε if and only if it is egalitarian and pairwise stable.*

Proof. The proof is divided in two steps, as follows:

Step 1: Let g be a non-egalitarian pairwise stable network. We shall show that g is not stochastically stable. It suffices to show that there exists a network g' such that $r(g') < r(g)$.

Call $T(g)$ the g-tree on which the calculation of $r(g)$ is based. There exists a pair of men $(i_0, j_0) \in I(g) * J(g)$. Since from Lemma 1, $s_{i_0}(g) \geq s_{j_0}(g) + 2$, there exists a woman k_0 such that $(i_0, k_0) \in g$ and $(j_0, k_0) \notin g$. Sever the link (i_0, k_0), and add the link (j_0, k_0), and call the resulting network g^1.

Consider now the tree $T(g)$. Let $S(g^1, T(g))$ be the successor of g^1 in the tree. Now, in $T(g)$, delete the edge $(g^1, S(g^1, T(g)))$ that leads away from g^1 and add the edge (g, g^1). This results in a g^1-tree that we denote by $T(g^1)$.

Since $T(g^1)$ is not necessarily optimal for g^1, we have $r(g^1) \leq r(g) - r(g^1, S(g^1, T(g))) + r(g, g^1)$. Because $\forall i \in I(g^1)$, $s_i(g) \leq s_{i_0}(g)$, we have $r(g^1, S(g^1, T(g))) \geq f(\frac{1}{s_{i_0}(g)}) = r(g, g^1)$. This is because the cheapest way of getting away from g^1 (which is pairwise stable) is for a pair of a man and a woman to undertake an action that benefits one of them and leaves the other indifferent; such an action is taken with probability at least equal to $\varepsilon^{f(\frac{1}{s_{i_0}(g)})}$. This implies that $r(g^1) \leq r(g)$.

If g^1 is egalitarian, then $r(g^1, S(g^1, T(g))) = f(\frac{1}{s_w^*}) > r(g, g^1)$, implying $r(g^1) < r(g)$. If g^1 is non-egalitarian, repeat the same operation $L(g) - 1$ more times. From lemma 2, that will induce a sequence of pairwise stable networks $(g^1, \ldots, g^{L(g)})$ where $g^{L(g)}$ is an egalitarian network. The induced sequence of g^ℓ-trees, $1 \leq \ell \leq L(g)$, $(T(g^1), \ldots, T(g^{L(g)}))$ will be such that for any $\ell \in \{2, \ldots, L(g)\}$, $r(g^\ell) \leq r(g^{\ell-1})$ with $r(g^{L(g)}) < r(g^{L(g)-1})$. This obviously implies $r(g^{L(g)}) < r(g)$, and therefore, g is not stochastically stable.

Recall that in any perturbed finite Markov process the set of stochastically stable states is always non-empty. Step 1 has therefore established that the set of stochastically stable networks of the perturbed process P_1^ε is a non-empty subset of the set of egalitarian pairwise stable networks.

Step 2: We shall next show that the set of stochastically stable networks of P_1^ε coincides with the set of egalitarian pairwise stable networks. It suffices to show that all egalitarian pairwise stable networks have the same stochastic potential.

Let g and g' be any two egalitarian pairwise stable networks, and $r(g)$ and $r(g')$ their respective stochastic potentials. Call $T(g)$ the g-tree on which the calculation of $r(g)$ is based. From Lemma 3, we know that there exists a finite sequence of pairwise stable networks $(g^0, g^1, \ldots, g^{2k})$ such that $g^0 = g$, $g^{2k} = g'$, and for any t such that $0 \leq t \leq k$, g^{2t} is egalitarian.

Construct g^1 from g as in the proof of Lemma 3, and consider the g-tree $T(g)$. In it, delete the edge $(g^1, S(g^1, T(g)))$ that leads away from g^1 and add the edge (g, g^1). This results in a g^1-tree that we denote by

$T(g^1)$. Note that $r(g^1, S(g^1, T(g))) \geq f(\frac{1}{s_w^*+1})$ and $r(g, g^1) = f(\frac{1}{s_w^*})$.

Next, construct g^2 from g^1 as in the proof of Lemma 3, and consider the g^1-tree $T(g^1)$. In it, delete the edge $(g^2, S(g^2, T(g^1)))$ and add the edge (g^1, g^2). This results in a g^2-tree that we denote by $T(g^2)$. We have $r(g^2, S(g^2, T(g^1))) = f(\frac{1}{s_w^*})$ and $r(g^1, g^2) = f(\frac{1}{s_w^*+1})$.

Therefore, noting that $T(g^2)$ is not necessarily optimal as a g^2-tree, we have that $r(g^2) \leq r(g) - r(g^1, S(g^1, T(g))) + r(g, g^1) - r(g^2, S(g^2, T(g^1))) + r(g^1, g^2) = r(g) - r(g^1, S(g^1, T(g))) + f(\frac{1}{s_w^*+1}) \leq r(g)$ since $r(g^1, S(g^1, T(g))) \geq f(\frac{1}{s_w^*+1})$. This establishes that $r(g^2) \leq r(g)$, and by symmetry, going back from g^2 to g, that $r(g) \leq r(g^2)$. Therefore, $r(g) = r(g^2)$.

If $g' = g^2$, then we have shown that $r(g') = r(g)$. If $g' \neq g^2$, repeat the same exercise as previously, constructing g^ℓ from $g^{\ell-1}$ as in Lemma 3, until g' is obtained. This induces a sequence of $g^t - trees$ $(T(g), T(g^1), T(g^2), T(g^3), \ldots, T(g^{2k}) = T(g'))$ satisfying that for any t such that $1 \leq t \leq k$, $r(g^{2t}) \leq r(g^{2(t-1)})$. This implies $r(g') \leq r(g)$. By symmetry, going back in the opposite direction, we also have $r(g) \leq r(g')$, thus implying $r(g) = r(g')$, which completes the proof. ■

To illustrate how Theorem 3 works, consider the following example:

Example 3 *There are 10 men and 10 women; $s_m^* = 3$ and $s_w^* = 2$. The network component configuration represented by Figure 3-1 is as follows:*

$$[(7,6),(3,4)].$$

Call that network g. In the first component of g,

- *m_1 is matched with w_1,*
- *m_2 with w_1 and w_2,*
- *m_3 with w_2 and w_3,*
- *m_4 with w_3 and w_4,*
- *m_5 with w_4 and w_5,*
- *m_6 with w_5 and w_6,*
- *and m_7 with w_6.*

The matches in the second component of g are as follows:

- *m_8 is matched with w_7 and w_{10};*

- *m_9 with w_7, w_8, w_9;*
- *and m_{10} with w_8, w_9, w_{10}.*

Note that g is pairwise stable. But the uneven distribution of partners among men in g implies that g is not stochastically stable (Theorem 3).

Consider next the following alternative component configuration represented by Figure 3-3:

$$[(10, 10)].$$

Call that network g'. The only differences between g and g' are that:

- *m_1 is linked to w_1, w_8 in g', but to only w_1 in g;*
- *m_7 is matched to w_6, w_{10} in g', but to only w_6 in g;*
- *m_9 is matched to w_7, w_9 in g', but to w_7, w_8, w_9 in g;*
- *and m_{10} is matched to w_8, w_9 in g', but to w_8, w_9, w_{10} in g.*

Note that g' is egalitarian pairwise stable. Let us explain why g is not stochastically stable by constructing a path between g and g' such that the overall resistance of going from g to g' is smaller than the resistance of going back from g' to g. Without loss of generality, we measure the strength of an existing link with the function $f(s_k) = 1/s_k$:

- *First, w_8 severs her link with m_9 and links with m_1 (with a resistance of $1/3$, and $1/2$ in the opposite direction) (the resulting network is represented in Figure 3-2).*
- *Second, w_{10} severs her link with m_{10} and links with m_7 (with a resistance of $1/3$, and $1/2$ in the opposite direction – Figure 3-3).*

Adding up, $(1/3) + (1/3) < (1/2) + (1/2)$. Thus, given this section's assumption on the cost of taking "neutral actions," the system gravitates towards the egalitarian pairwise stable networks.

6 The Second Perturbed Process P_2^{ε} or "If You are Weak, I May Leave You"

The second perturbed process is defined as the first one, the only difference being the definition of the probability of a "neutral action," an action that leaves an agent indifferent. Recall that that probability was based on the strength of the link to be broken to form the new link. Now, the strength of such a link is inversely proportional to the amount of time invested in it. This corresponds to a situation in which an agent who invests too much time in a relationship might be perceived as weak or dominated in that relationship. We describe next more formally the only change in assumptions with respect to the previous perturbed process:

- A person who is indifferent in a particular transition, and in it, breaks an existing link with another person who has s_k partners in order to form a new link looks at the strength of the link he/she severs. That strength $f(s_k)$ is strictly increasing in s_k and strictly bounded between 0 and 1.[20]

6.1 Resistance of a Path

- All the definitions of resistance provided earlier apply to this section as well. For completeness, for each adjacent transition in the perturbed process P_2^{ε}, its probability and resistance are summarized in Table 2 below. It uses the same notation employed in Table 1:

Table 2

(a_i, a_j)	*Outcomes*	*Probability*	$r(g, g')$
(A, A)	(b, b)	1	0
(A, A)	(b, l)	ε	1
(A, A)	(l, l)	ε^2	2
(A, B)	(b, i)	$\varepsilon^{f(s_k)}$	$f(s_k)$
(A, B)	(l, i)	$\varepsilon^{1+f(s_k)}$	$1 + f(s_k)$
(B, B)	(i, i)	$\varepsilon^{f(s_h)+f(s_k)}$	$f(s_h) + f(s_k)$
(C, C)	(b, b)	1	0
(C, C)	(b, l)	1	0
(C, C)	(l, l)	ε	1

6.2 The Result

We shall now characterize the set of stochastically stable states of the perturbed process P_2^{ε}. The following definition is needed.

Let g be a network. We shall say that g is anti-egalitarian if $\left\lfloor \frac{s_w^*}{s_m^*} |M| \right\rfloor$ men are matched to s_m^* women each, at most one man is matched to the remaining women (if there is such a remaining), and all other men

[20]Note that the strength $f(s_k)$ is actually taken to measure the domination status of agent k in the relationship with that person, so that it is easier or less costly to break a relationship with a weak or a dominated partner.

have no partner.

To understand this definition, the idea is that all women are matched to a set of men that is as small as possible; hence the name "anti-egalitarian." Thus, if $\frac{s_w^*}{s_m^*}|M|$ happens to be an integer, each of those men is matched to s_m^* women and the rest of men are unmatched. Note that if $\frac{s_w^*}{s_m^*}|M|$ is not an integer, one can assign the remaining women to only one man and have a pairwise stable network. This is because, if one calls K the integer part of that fraction, the total number of links from the set of men not matched to their optimal number must be less than s_m^*: otherwise, the number of links coming from the men side would be at least $Ks_m^* + s_m^*$, but this number is strictly greater than $s_w^*|M|$, the number of links coming from the women side, and both numbers must always be equal.

Equipped with this definition, we state our next result:

Theorem 4 *Assume A2. A network is stochastically stable in the perturbed process P_2^ε if and only if it is anti-egalitarian and pairwise stable.*

Proof. The proof is again organized in two steps, as follows:

Step 1: Let g be a pairwise stable network that is not anti-egalitarian. We shall show that g is not stochastically stable. It suffices to show that there exists a network g' such that $r(g') < r(g)$.

Consider $T(g)$, the g-tree on which the calculation of $r(g)$ is based. We claim that, if g^λ and $g^{\lambda+1}$ are two pairwise stable networks such that for some m, m', w, $g^\lambda \setminus g^{\lambda+1} = \{(m, w)\}$ and $g^{\lambda+1} \setminus g^\lambda = \{(m', w)\}$, the underlying transition does not involve non-pairwise stable networks: if it did, at least one agent directly involved in it would decrease his or her utility, which implies that the resistance of such a transition would exceed 1, whereas the resistance of the direct transition between the two (being adjacent) is strictly less than 1. A simple induction argument shows that this is still true even if two pairwise stable networks are not adjacent (by constructing a path going from one to the other consisting of direct transitions between pairs of adjacent networks).

Therefore, in any transition described in $T(g)$, only pairwise stable networks are visited. By Theorem 1, we know that each pairwise stable network contains exactly the same number of links, i.e., $s_w^*|W|$. It follows that each transition described in the tree involves a woman w who severs a link with a man m and replaces it with another link with man m'. Specifically, the pair (m', w) is offered the opportunity to revise their situation, and as a result, woman w severs (m, w) and gets matched with m'.

But then, in describing the transition between any two pairwise networks in $T(g)$, one can, without loss of generality, list the transitions that are required going through each individual woman. That is, starting with the woman with the lowest index who has a different set of men to which she is matched in the two networks, one can describe the required severance/creation of links that takes her from her configuration of men in the original network to the one in the final network, and one can proceed like these with each such woman until the full transition is complete.

Consider then the network g, and recall it is not anti-egalitarian. We propose the following algorithm. Without loss of generality, label the men so that $s_{m_1}(g) \geq s_{m_2}(g) \geq \ldots \geq s_{m_{|M|}}(g)$. Let m be the lowest index such that $s_m(g) < s_m^*$. If there exists w who is matched in g to $m' > m$, sever the link (m', w) and replace it with (m, w). Call the resulting network g^1. We can have two cases. Either g^1 is anti-egalitarian, or not. If it is, let $g' = g^1$. If not, repeat the same step. Note how this algorithm always ends after a finite number of steps, say k, in a network $g' = g^k$ that is anti-egalitarian.

Consider the g-tree $T(g)$, and without loss of generality (as the first paragraphs of the proof showed), suppose that the transition $g' = g^k \rightarrow g^{k-1} \rightarrow \ldots \rightarrow g^1 \rightarrow g^0 = g$ constitutes a path of directed links in $T(g)$. Change the direction of this path and consider the transition $g = g^0 \rightarrow g^1 \rightarrow \ldots \rightarrow g^{k-1} \rightarrow g^k = g'$. It is obvious that the rest of edges of $T(g)$, along with these new edges (in which the only change introduced is the direction change of previous links in $T(g)$), constitute a g'-tree, which we call $T(g')$.

We claim that $r(g') < r(g)$. Indeed, $r(g')$ is no greater than the resistance of $T(g')$, which is equal to $r(g) + \sum_{\alpha=0}^{k-1}[r(g^\alpha, g^{\alpha+1}) - r(g^{\alpha+1}, g^\alpha)]$. And note that, by construction of the algorithm described, each bracketed term is negative. Indeed, in the transition $g^\alpha \rightarrow g^{\alpha+1}$, let m' be the man who loses a link in favor of man m. We know that $s_{m'}(g^\alpha) < s_m(g^{\alpha+1})$, and therefore, $r(g^\alpha, g^{\alpha+1}) = f(s_{m'}(g^\alpha)) < f(s_m(g^{\alpha+1})) = r(g^{\alpha+1}, g^\alpha)$.

We have therefore established that, if g is pairwise stable but it is not anti-egalitarian, it is not stochastically stable in the perturbed process P_2^ε. Given that the set of stochastically stable networks is non-empty, we just proved that this set is a non-empty subset of the set of pairwise stable and anti-egalitarian networks.

Step 2: We shall now prove that the set of stochastically stable networks of P_2^ε coincides with the set of pairwise stable and anti-egalitarian networks. It suffices to prove that all of them have the same stochastic potential.

Let g and g' be any two such networks. Assume for simplicity that, in each of them, exactly $\frac{s_w^*}{s_m^*}|M|$ men are matched with s_m^* each. Obviously, this must hold for both g and g'.[21]

It is easy to see that there must exist $m, m' \in M, m \neq m'$ and $w, w' \in W, w \neq w'$ such that $(m, w) \in g \setminus g'$ and $(m', w') \in g' \setminus g$. We propose the following algorithm that transforms g into g'. For each such pair of

[21]If, instead, the number $\frac{s_w^*}{s_m^*}|M|$ is not an integer, and one man is matched to the remaining women, the argument is the same, but the notation is slightly more complicated. Again, in this case, both g and g' have the same structure of having only one man matched to the remaining women.

links, we describe the following steps:

- First, woman w severs her link to man m and gets matched to man m_0, where $s_{m_0}(g) = 0$ –we know such a man exists in g.
- Second, woman w' severs her link to man m' and gets matched to man m.
- And third, woman w severs her link to man m_0 and gets matched to man m'.

And to go back, travel the same steps in reverse.

Consider now an optimal g'-tree, and call it $T(g')$. In it, focus on the collection of directed edges connecting g to g'. By arguments similar to those at the beginning of Step 1 of this proof, one can argue that the transition outlined in the previous algorithm must be part of any optimal tree. (We know that transitions in optimal trees do not go through non-pairwise stable networks. In addition, a resistance of $f(s_m^*)$ must be paid every time a link with a man matched to his optimal number is broken, and aside from that, a resistance of $f(1)$ that comes from breaking a link with a man who was unmatched in g and remains unmatched in g' is the smallest possible positive resistance in this perturbed process.)

Thus, without loss of generality, let the directed path from g to g' in $T(g')$ be the set of transitions outlined. Now, change the direction of the edges in this path, and let that be the only change introduced to the directed edges of $T(g')$. Observe that the result is a g-tree, which we call $T(g)$.

We will now argue that the stochastic potentials of g and g' are the same:

$r(g) = r(g') + \sum_{\beta=0}^{k-1}[r(g^\beta, g^{\beta+1}) - r(g^{\beta+1}, g^\beta)] = r(g')$ because $\sum_{\beta=0}^{k-1}[r(g^\beta, g^{\beta+1}) - r(g^{\beta+1}, g^\beta)] = 0$. This can be easily established, by induction on the number of links that are different between g and g'.

Indeed, suppose that g and g' differ in the smallest possible number of links, which is two, i.e., there exist $m \neq m'$ and $w \neq w'$ such that $g \setminus g' = \{(m, w)\}$ and $g' \setminus g = \{(m', w')\}$. Consider the transition $g \rightarrow g'$ in $T(g')$. By our previous arguments, such a transition is as follows:

- First, woman w severs her link to man m and gets matched to man m^0, where $s_{m_0}(g) = 0$ –we know such a man exists in g; the resistance of this step is $f(s_m^*)$.
- Second, woman w' severs her link to man m' and gets matched to man m; again, the resistance of this step is $f(s_m^*)$.
- And third, woman w severs her link to man m_0 and gets matched to man m'; the resistance of this step being $f(1)$.

The resistance of the whole transition is thus $2f(s_m^*) + f(1)$. But notice that travelling the same steps backwards takes us back from g' to g, with exactly the same resistance.

If g and g' differ by more links (note this must always be an even number), we use the fact that the path going from g to g' and the same path travelled in the opposite direction are "mirror images" of one another. Thus, since the cheapeast transition must always involve establishing links with unmatched men –like m_0 in the previous paragraph– (because $f(1)$ is the smallest resistance to be added to the $f(s_m^*)$ terms, which must be always there), a replication of the argument detailed in the previous paragraph establishes that the total resistance of travelling from g to g' is exactly the same as the one travelling backwards on the same path. This completes the proof. ■

To illustrate how Theorem 4 works, consider the following example:

Example 4 *There are 13 men and 13 women;* $s_m^* = 4$ *and* $s_w^* = 3$*. The anti-egalitarian network component configuration represented by Figure 4-5 is as follows:*

$$[(3,4),(3,4),(4,5),(1,0),(1,0),(1,0)].$$

For instance, the following network g *is in this class: men* $m_1 - m_3$ *are matched each to women* $w_1 - w_4$ *in the first component; men* $m_4 - m_6$ *are matched each to women* $w_5 - w_8$ *in the second component; men* $m_{11} - m_{13}$ *are isolated; and the matches in the third component are as follows:*

- m_7 *is linked with* $w_9, w_{10}, w_{11}, w_{12}$*;*
- m_8 *with* $w_{10}, w_{11}, w_{12}, w_{13}$*;*
- m_9 *with* w_9*,* w_{11}, w_{12}, w_{13}*;*
- *and* m_{10} *with* w_9, w_{10}, w_{13}*.*

That is, only one man (m_{10} *in this case) gets matched to some women, but not to his optimal number.*

Consider next the following alternative component configuration represented by Figure 4-1:

$$[(3,4),(3,4),(3,4),(3,1),(1,0)].$$

For instance, the following network g' *is in this class: as before, men* $m_1 - m_3$ *are matched each to women* $w_1 - w_4$ *in the first component; men* $m_4 - m_6$ *are matched each to women* $w_5 - w_8$ *in the second component; man* m_{13} *is isolated; and the matches in the third and fourth components are as follows:*

- $m_7 - m_9$ *are linked each with* $w_9, w_{10}, w_{11}, w_{12}$*;*

- *and* $m_{10} - m_{12}$ *are matched each with* w_{13}*.*

Note that g' *is pairwise stable, but it is not anti-egalitarian. In it, there is one non-isolated component in which the number of men exceeds the number of women. We explain why* g' *is not stochastically stable in the perturbed process of this section by constructing a path between* g *and* g' *such that the overall resistance of going from* g' *to* g *is smaller than the resistance of going back from* g *to* g'*.*

We replicate some of the steps of the proof of Theorem 4. Going woman by woman, the only women that have different matches in g *and* g' *are* w_9, w_{10} *and* w_{13}*. To be precise, we list the links that are different below:*

- w_9 *is linked to* m_8 *in* g' *and to* m_{10} *in* g*;*

- w_{10} *is linked to* m_9 *in* g' *and to* m_{10} *in* g*;*

- *and* w_{13} *is linked to* m_{11} *and* m_{12} *in* g'*, and to* m_8 *and* m_9 *in* g*.*

Then, we describe the transitions, from g' *to* g *(and we can travel back the same way), and without loss of generality, we measure the strength of an existing link with the function* $f(s_k) = \frac{S_k}{n}$*:*

- *First,* w_9 *severs her link with* m_8 *and links with* m_{10} *(with a resistance of* $4/n$*, and* $2/n$ *in the opposite direction) (the resulting network is represented in Figure 4-2).*

- *Second,* w_{10} *severs her link with* m_9 *and links with* m_{10} *(with a resistance of* $4/n$*, and* $3/n$ *in the opposite direction – Figure 4-3).*

- *Third,* w_{13} *severs her link with* m_{11} *and links with* m_8 *(with a resistance of* $1/n$*, and* $4/n$ *in the opposite direction– Figure 4-4).*

- *And finally,* w_{13} *severs her link with* m_{12} *and links with* m_9 *(with a resistance of* $1/n$*, and* $4/n$ *in the opposite direction – Figure 4-5).*

Adding up, $(4/n) + (4/n) + (1/n) + (1/n) < (2/n) + (3/n) + (4/n) + (4/n)$*. Thus, given this section's assumption on the cost of taking "neutral actions," the system gravitates towards the anti-egalitarian pairwise stable networks.*

Note also how the characterization in Theorem 4 is in terms of the number of partners that each matched man has, but not in terms of the component configuration, which is not unique within anti-egalitarian networks.

For example, the following component configuration is consistent with anti-egalitarian pairwise stable networks in the example (Figure 5):

$$[(10, 13), (1, 0), (1, 0), (1, 0)].$$

That is, there is a single non-isolated component in which nine men are matched each to four women, and the tenth is matched to only three. It can be shown that such a network is also stochastically stable in the process of this section. Complexities like these are responsible for making the proof of Theorem 4 far from trivial.

7 Gender and HIV/AIDS

In this section, we study the implication of our analysis for gender difference in communication or contagion potential in stochastically stable networks, with a particular focus on HIV/AIDS. In doing so, we draw on the theoretical framework proposed in Pongou (2009a).

Let g be a network. Assume that an agent $i \in N$ is drawn at random to receive a piece of information γ that he/she communicates to his/her partners in $g(i)$, who in turn communicate it to their other partners, and so on. This "piece of information" might also be becoming infected with the HIV/AIDS virus through blood transfusion or any other random event. If i is not matched with any agent, the information does not spread. Suppose that with equal probability, $\frac{1}{|N|}$, each agent receives the information (i.e., is infected due to a random event). We define the communication or contagion potential of g as the expected proportion of agents who will receive the information. We also define gender difference in contagion potential as the difference in the expected proportion of men and women who will receive the information. To formally define these notions, we first need a few definitions.

Let $i \in N$ be an agent such that $g(i) = \emptyset$. We say that i is isolated in the network g. We abuse language and call $\{i\}$ an isolated component of g, thus consisting only of one agent. We denote by $\mathcal{I}(g)$ and $\mathcal{J}(g)$ respectively the set of isolated and non-isolated components of g. Clearly, the set of components of g $C(g) = \mathcal{I}(g) \cup \mathcal{J}(g)$.

Assume that g is a k-component network, and let $C(g) = \{g_1, \ldots, g_k\}$ be the set of its components. Pose $I_k = \{1, \ldots, k\}$. To simplify notation, we write $N(g_i) = N_i$, $M(g_i) = M_i$, $W(g_i) = W_i$, and $|N_i| = n_i$, $|M_i| = m_i$, and $|W_i| = w_i$ for $i \in I_k$. We associate each component g_i with the number n_i and its bipartite component vector (m_i, w_i), and g with the vector $[(n_i)]_{i \in I_k}$ and its bipartite vector $[(m_i, w_i)]_{i \in I_k}$. Also, if g_i is an isolated component, its associated vector is either $(1, 0)$ or $(0, 1)$.

Denote by $\rho(z, \gamma)$ the status of an agent z with respect to the information γ. We pose $\rho(z, \gamma) = 1$ if z has received the information and 0 if he/she has not. For any set $B = N, M, W$, let $\Pr(\gamma|B) = \frac{|\{z \in B : \rho(z,\gamma)=1\}|}{|B|}$ be the proportion of agents who have received the information in the population B. We provide below a formula for the expected value of $\Pr(\gamma|N)$ and $\Pr(\gamma|M) - \Pr(\gamma|W)$. We have the following result.

Claim 1 (Pongou 2009a)**:**

- $E[\Pr(\gamma|N)] = \frac{1}{n^2}\sum_{i\in I_k} n_i^2$.
- $E[\Pr(\gamma|M) - \Pr(\gamma|W)] = \frac{2}{n^2}\sum_{i\in I_k}(m_i^2 - w_i^2)$.

This result provides the foundation for the following definition:

Definition 1 *Let g be a k-component network with the corresponding component vector $[(n_i)]_{i\in I_k}$.*

(1) The communication or contagion potential of g is defined as

$$\mathcal{P}(g) = \frac{1}{n^2}\sum_{i\in I_k} n_i^2.$$

(2) If g is a bipartite graph with the corresponding component vector $[(m_i, w_i)]_{i\in I_k}$, the gender difference in the contagion potential of g is defined as

$$\mathcal{F}(g) = \frac{2}{n^2}\sum_{i\in I_k}(m_i^2 - w_i^2).$$

Consider the following illustrative example of this definition.

Example 5 *Consider the networks given in Example 1 and represented respectively by Figure 1-1, Figure 1-2 and Figure 1-3. Call them respectively g, g' and g''. The contagion potential of each of these networks is: $\mathcal{P}(g) = \frac{1}{20^2}(4^2 + 10^2 + 6^2) = \frac{152}{400} = 0.38$; $\mathcal{P}(g') = 0.515$; and $\mathcal{P}(g'') = 0.2$.*

The gender difference in the contagion potential of each of these networks is: $\mathcal{F}(g) = \frac{2}{20^2}[(2^2 - 2^2) + (5^2 - 5^2) + (3^2 - 3^2)] = 0$; $\mathcal{F}(g') = 0.01$; and $\mathcal{F}(g'') = -0.12$.

Note how the contagion potential varies across networks despite the fact that the number of links supplied by women and received by men is the same in all networks. This clearly shows the effect of network structure in the propagation of certain diseases like HIV/AIDS. We also note that network g is gender neutral in contagion potential; but in network g', men are more vulnerable to infection than women, while in network g'', it is the opposite. This again shows how network structure may cause a particular gender to be more vulnerable to a random infection shock.

This example also shows that anti-female discrimination does not necessarily cause women to be more vulnerable to infection, when one considers only statically stable networks. But we will show that in the networks that are visited a positive amount of time in the long run, the ones we are concerned with in the current paper, men are never more vulnerable than women.

We have the following useful Lemma:

Lemma 4 *(Pongou (2009a)) Let g be a network.*

(i) If $\forall g' \in \mathcal{J}(g)$, $|M(g')| = |W(g')|$, then $\mathcal{F}(g) = 0$.

(ii) If $\forall g' \in \mathcal{J}(g)$, $|M(g')| \leq |W(g')|$, then $\mathcal{F}(g) \leq 0$ with strict inequality if there exists some $g' \in \mathcal{J}(g)$ such that $|M(g')| < |W(g')|$.

We can now state the following, which is the main result of this section:

Theorem 5 *Assume A2.*

(1) For any stochastically stable network g of the perturbed process P_1^{ε}, $\mathcal{F}(g) = 0$.

(2) For any stochastically stable network g of the perturbed process P_2^{ε}, $\mathcal{F}(g) < 0$.

Proof. (1) The proof follows from the fact that in any egalitarian pairwise stable network g, there is an equal number of men and women in each component of g, from which it follows from Lemma 5 that $\mathcal{F}(g) = 0$.

(2) In any anti-egalitarian pairwise stable network g, it can be shown that the number of women strictly exceeds the number of men in each non-isolated component, from which it follows from Lemma 5 that $\mathcal{F}(g) < 0$.

■

Assuming that information is the AIDS virus, Theorem 5 implies that any initial network g will progress toward a network g' in which HIV prevalence is at least as high among women as among men, even if in the initial network g, the prevalence was higher among men. Furthermore, in the case of the second process, which together with our basic assumption A1, may be viewed as a description of male-dominant societies, the contagion potential for women exceeds that of men.

In Example 3 for instance, on notes that the gender difference in the contagion potential decreases from 0.03 in g to 0 in g'. This means that HIV/AIDS is more prevalent among men in the initial network g, but the number of infected women increases over time to reach the number of infected men in the very long run, resulting in equal prevalence of the disease in the two genders.

In Example 4, transiting from the non-stochastically stable network g' to the stochastically stable network g implies a reduction in gender difference in the contagion potential from -0.01775 to -0.02959, which implies that HIV/AIDS is more prevalent among women in g' and g; but more importantly, the number of infected women increases in the very long run.

Finally, one notes that anti-female discrimination does not necessary lead to higher HIV/AIDS prevalence among women in the short run, but women necessarily bear an equal or higher share of the burden in the long run. For instance, in India which has a long history of anti-female discrimination (see, e.g., Sen (1999)), the prevalence of HIV/AIDS is 0.2% among women and 0.4% among men (Mishra et al. (2009)). But our model predicts that this trend will reverse in favor of men in the very long run. This will reflect what is happening in all regions of the world, including those in which the prevalence of HIV/AIDS is lower among women such as Western Europe; the number of women with HIV/AIDS is increasing, and the gap between the two genders in this respect seems to be higher in societies where men have a dominant role (WHO (2003), UNAIDS (2008)).

8 Concluding Remarks

We have studied the dynamic stability of fidelity networks, which are networks that form in a mating economy of agents of two types (say men and women), where each agent derives satisfaction from the number of direct links with opposite type agents, while engaging in multiple partnerships is considered an act of infidelity and is punished if detected by the cheated partner. We have assumed that a woman whose infidelity is detected is more severely punished than a man in a similar situation. This results in that women's optimal number of partners is smaller than men's. In statically stable networks, each woman obtains her desired number of male partners while each man obtains at most his desired number of female partners, which reveals that the anti-female bias in infidelity punishment leads to men competing for female partners.

We have defined two dynamic and stochastic matching processes in which agents form new links and sever existing ones based on the reward from doing so, but possibly take actions that are not beneficial with small probability. Under the first process, only pairwise stable egalitarian networks, in which all agents have the same number of partners (the optimal for each woman), are visited in the long run; under the second process, which is more plausible in certain male-dominant societies, only anti-egalitarian pairwise stable networks are. In these networks, all women have their desired number of partners and are matched with a small number of men, each of which has his optimal number of partners except for possibly one man.

Relying on the approach proposed in Pongou (2009a) to study the diffusion of information in networks, we have found that under the first process, HIV/AIDS is equally prevalent among men and women. Under the second process, women bear a greater burden. The key implication is that even if the prevalence of HIV/AIDS is lower among women compared to men at some point in time, the number of infected women will grow over time to reach and possibly offset the number of infected men. This seems to confirm what is observed in data across countries, from Africa to the United States (UNAIDS (2008)). In understanding this trend, our analysis reveals that anti-female discrimination may be a key factor in the greater vulnerability of women to HIV/AIDS.

Our analysis also sheds light on patterns of union formation in some societies. Imagine that women's optimal number of partners is 1. Then, in the first process the model predicts a situation of serial monogamy. Theorem 3 shows for this case that only monogamous networks are stable in the long run. But note that this notion of stability does not mean that if the process reaches a monogamous network, it will stay there, since people might still make mistakes or be tempted by other potential partners. Indeed, if a woman moves from her only partner to another one, creating a non-monogamous network, the latter network will transit to another monogamous network which is not necessarily the initial one, and so on. Serial monogamy is associated with high divorce rates (e.g., Schoen and Standish (2001) and Goldstein (1999) document that the divorce rate in the U.S. is above 40%). In contrast, under the second process, the prediction of the model is polygyny, and then divorce rates may be low. Consider the following example. There are 3 men and 3 women, $s_w^* = 1$ and $s_m^* = 3$. Theorem 4 tells us that the only stochastically stable network (up to permutations) is the one in which the first man is matched to all three women. Assume that the process reaches that network. If a woman moves from the first man to another man, then considering that networks evolve following the path of least resistance, it is easy to see that that woman will return to the first man (so, there is reconciliation and no divorce). The model may be suggesting union formation patterns in non-Western societies: Pongou (2009b) observes that the divorce rate in a pooled sample of six sub-Saharan African countries is 2.7%.

Finally, we note that a distinctive feature of the fidelity networks is that a priori, individuals do not know their partners' other partners, and do not gain anything from being indirectly related to them. A natural extension of our analysis will be to consider the case in which an individual's well-being is affected by indirect links and their consequent externalities. In such an analysis, an agent's utility would be a function of the network of which he/she is a member. Then, it is possible that the internalization of such an externality when an agent chooses to join or depart from a certain network may have implications for the spread of HIV/AIDS.

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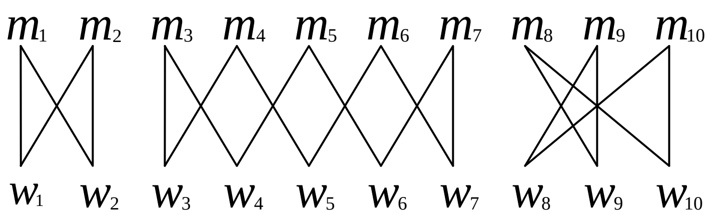

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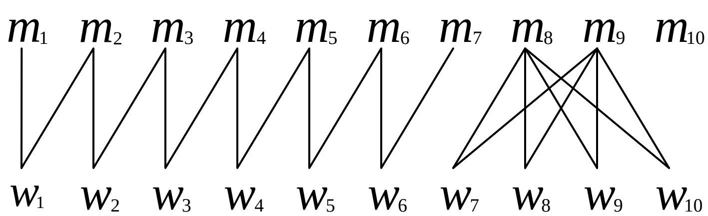

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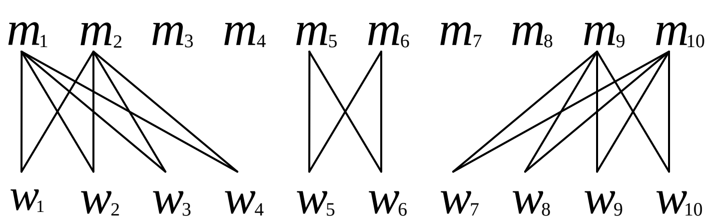

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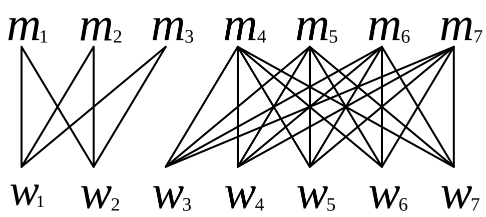

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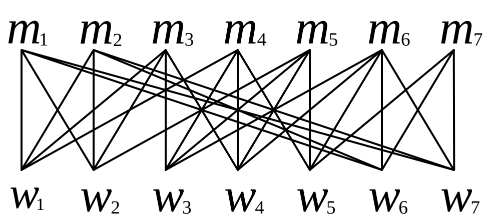

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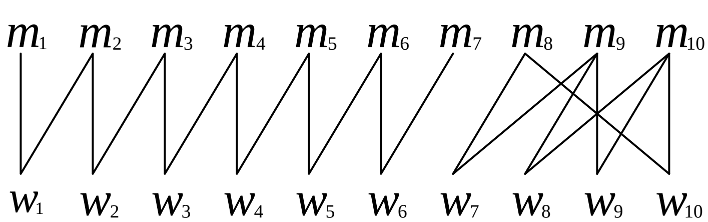

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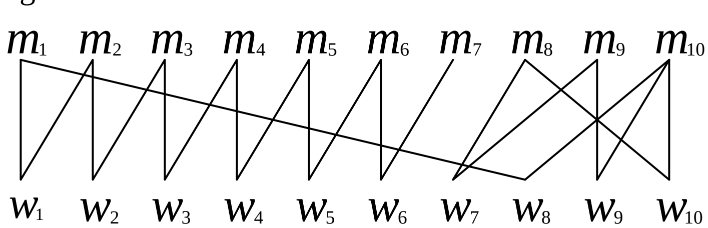

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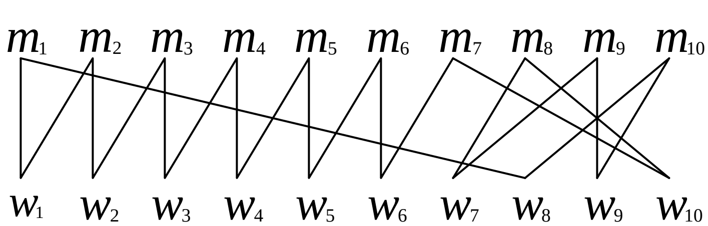

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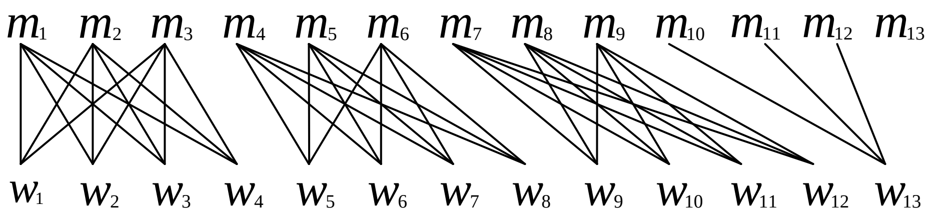

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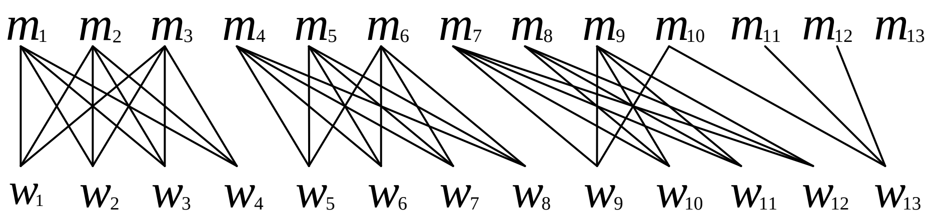

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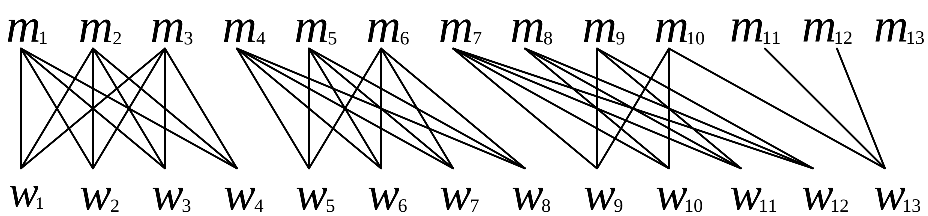

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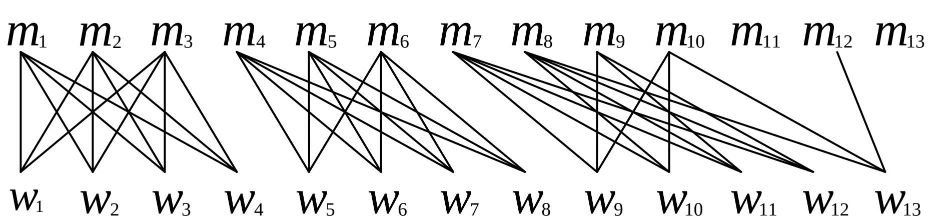

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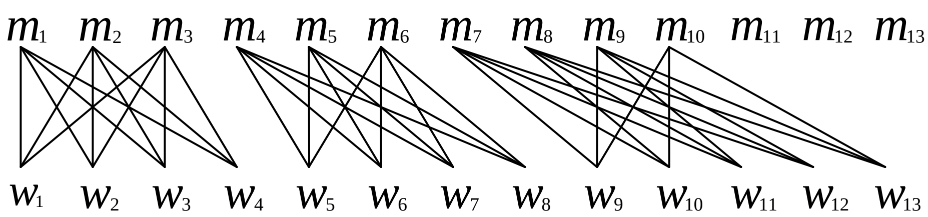

Figure 5