

EXCLUSIVE INTERMEDIATION

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ABSTRACT. In this paper, we argue that an important function fulfilled by intermediaries is to facilitate trust by enabling social pressure towards the enforcement of informal agreements. To that end, we develop a new model that uses network theory to show that intermediaries who have exclusivity over a large enough number of interaction opportunities are able to exploit their position in the chains of interactions in the market to overcome incentive problems that would otherwise shut down the market. We derive conditions on the network structure under which intermediaries fulfill this function. Finally, we analyze two applications: (1) the market for short term apartment rentals; and (2) a financial market with investors and entrepreneurs. We provide additional examples suggesting that this paper uncovers an important channel through which intermediaries operate.

Key words: Networks, intermediation, long-term relationships, self-governance, community enforcement, trust, social capital, cooperation, strategic default, financial intermediation, Airbnb.com.

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1. INTRODUCTION

Much of our economic activity relies on intermediaries, from retailers and whole sellers, to financial intermediaries, to two-sided platforms. In a survey of economic work on intermediation, Spulber (1996) describes four major functions that intermediaries fulfill in the economy.¹

“[...] four of the most important actions of economic intermediaries: setting prices and clearing markets; providing liquidity and immediacy; coordinating buyers and sellers; and guaranteeing quality and monitoring performance.” (Spulber 1996)²

In *guaranteeing quality and monitoring performance*, Spulber has in mind a world in which either (1) a seller's or good's type is costly to observe; or (2) the efforts of a trading partner are costly to observe.

If the type of a seller or a good is observed to be low before the transaction takes place, a buyer could choose not to buy the good. Therefore, to explain the role of intermediaries in case (1) it may be sufficient to show that an intermediary has stronger incentives than the buyers to invest in obtaining the necessary information (and also has the proper incentives to reveal the information to her clients).³ On the other hand, to prevent one transaction partner from cheating the other, e.g., by choosing to exert low effort or not to repay a loan, monitoring must be combined with governance. This is true regardless of whether effort is costly to observe or not.⁴ If cheating (once observed) is verifiable, legal enforcement fills the gap (e.g., Diamond 1984). Focusing on lawless economic environments, Dixit (2003) suggests that intermediaries who have exogenous abilities to punish, provide the governance that is needed for transactions to take place. Dixit's leading example is the Sicilian mafia. However, governance problems are not exclusive to lawless economies—in lawful economies, issues of verifiability make legal enforcement less effective, whereas 'wild-west' type intermediaries run into legal difficulties and are less effective. Thus, for many (if not most) transactions to take place, market participants are required to establish trust relying on social pressure and reputation (e.g., not lending again

¹Spulber argues that even according to a conservative estimate, intermediation activities account for about one-quarter of gross domestic product.

²An additional role not mentioned in the survey is executing inter-generational transfers, see also McGill and Quinzii (2002).

³This could be the case if the intermediary serves many clients, or sells a variety of goods. E.g., Biglaiser (1993), and Biglaiser and Friedman (1994).

⁴Dixit (2003) notes that “All economic transactions, except spot exchanges of goods or services with objectively known attributes, offer opportunities for one or both or all of the parties to cheat for their own gain at the expense of the others. In turn, the expectation of suffering a loss due to such cheating can make all prospective participants unwilling to enter into a transaction that would benefit them all if the cheating could be checked. Therefore almost all economic transactions need governance.”

to a borrower who strategically defaulted, and not repurchasing from a seller who provided poor quality).⁵

In this paper, we argue that an important function fulfilled by intermediaries is to facilitate trust by enabling social pressure (via repeated interactions) towards the enforcement of informal agreements. More specifically, we develop a new model that uses network theory to show that intermediaries are able to exploit their position in the chains of interactions in the market to overcome incentive problems that would otherwise shut down the market. We derive conditions on the network structure under which intermediaries can fulfill this function effectively, and provide two applications and additional examples suggesting that this is an important channel through which intermediaries operate.

Consider a client and an agent who have repeated opportunities to engage in an interaction that requires the client to trust the agent. Examples include the provision of services, uncollateralized loans and investments, and even apartment rentals (see section 2). If the frequency with which the client and agent get to interact with each other is low, then “bad” behavior (henceforth defection) by the agent cannot be deterred by a threat of the client to never interact with him again in the future. However, an intermediary (e.g., an agency connecting service providers and consumers, an investment bank, or an online platform for sales / exchanges / rentals) who exclusively represents an agent, or at least a large pool of clients in their transactions with that agent, can still enforce “good” behavior (henceforth cooperation) by the agent. The intermediary can do that by threatening to eliminate the agent’s access to future interactions with many clients.

Clearly, we are not the first to study game theoretic foundations for the enforcement of informal contracts through social pressure. The literatures on community enforcement and self-governance offer two enforcement mechanisms to explain the prevalence of informal contracts in the presence of incentives problems. One mechanism is *ostracism*.⁶ Ostracizing a borrower requires coordination. In some markets coordination is achieved by tight social groups, i.e., families or ethnic groups (e.g., Greif 1993 and Munshi 2011). When a market is not dominated by social groups, coordination requires common observations and knowledge of the patterns of interactions between individuals. A second mechanism suggested in the literature is *contagion*

⁵Macaulay (1963) points out that social pressure and reputation are perhaps more widely used than formal contracts and enforcement. Karlan et al. (2009) document a growing body of research demonstrating the importance of trust for economic outcomes in both lawless and lawful economies. For example, trust has been linked with outcomes including economic growth (Knack and Keefer 1997), judicial efficiency and lack of corruption (LaPorta et al. 1997), international trade and financial flows (Guiso, Sapienza, and Zingales 2009), and private investment (Bohnet, Herrmann, and Zeckhauser 2010). Additional examples of trust intensive interactions in which verifiability is an issue are available in section 2.

⁶See Greif (1993), and in the context of social networks also Babus (2010), Ali and Miller (2012b), and Fainmesser and Goldberg (2012).

– any individual who observes a defection reacts by defecting (if an agent) or by avoiding interactions (if a client), independent of the identity of their trading partner.⁷ Contagion is hard to motivate in large markets (see Kandori 1992, Ellison 1994), and requires implicit coordination in order to provide the incentives to spread “bad” behavior throughout the entire population.

This paper proposes a third mechanism—*intermediation*. If each of a group of clients agrees to interact with a given agent only via a given intermediary, the intermediary can single-handedly ‘cut off’ a defecting agent’s access to interactions with a large group of clients. We capture the patterns of repeated interactions using a network structure, and show that well positioned intermediaries can enforce repayment in environments in which ostracism and contagion cannot. We also present suggestive evidence from markets which were created by intermediaries consciously taking on this role (see section 2).

A novel feature of our model is that individuals are not assumed to observe the network structure directly; instead, individuals observe their own interactions, and their knowledge of the network is derived as an upper bound on what they would be able to learn about the network structure based on their observations in many such interactions.

Given that individuals receive information only on parts of the network that affect their own interactions, some forms of community enforcement are infeasible. If the agent defects, his link with the client or intermediary, say k , who interacted with him is lost. However, for any additional intermediary or client, say j , to disconnect her link to the agent, two conditions need to be fulfilled: [1] j observes the defection (or the elimination of the link between k and the agent); and [2] given her observations and beliefs, j has the incentives to eliminate her link to the agent rather than “pretend” not to have observed the defection (or the elimination of the link between k and the agent). For example, j may prefer to “cover-up” for the agent’s defection if j believes that other clients or intermediaries did not observe the defection (or the elimination of the link between k and the agent) and that the agent has sufficient incentives not to defect in an interaction with j as long as no additional links are eliminated.

Our main theoretical result offers a complete characterization of the set of networks that are *robust* – networks that can be sustained in equilibria of the infinitely repeated game given any belief from a large set of beliefs that we consider. Our characterization shows that in all robust networks *all* interactions are intermediated. Moreover, there exists a mapping from the parameters of the model to a positive integer m^* such that in robust networks any active intermediary is m^* -*local monopoly*—the intermediary is connected to at least m^* clients who are not connected to the agent in any other way, either directly or via another intermediary. That is, any intermediary exclusively represents the agent in interactions with at least m^* clients. Figure 1.1 demonstrates the notion of local monopolism.

⁷See Kandori (1992), Ellison (1994), and in the context of social networks also Haag and Lagunoff (2006), Ali and Miller (2012a), Mihm, Toth, and Lang (2009), Lippert and Spagnolo (2011), and Nava and Piccione (2011).

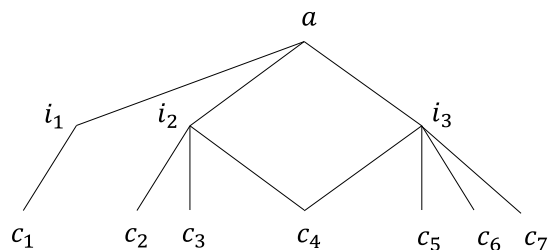


FIGURE 1.1. Local monopolism. Intermediary i_1 is a 1-local monopoly, i_2 is k -local monopoly for any $k \leq 2$, and i_3 is k -local monopoly for any $k \leq 3$.

This result explains the presence of intermediaries even in markets in which there is no exogenous cost advantage to intermediation, nor are there any exogenous “punishing” abilities. The requirement that an intermediary provide the agent with unique access to at least m^* clients highlights that the important factor is not the absolute size of an intermediary or the overall number of interactions that she intermediates, but rather the exclusivity over a sufficient number of *interaction paths*. Such exclusivity can be achieved by a large intermediary, but it can also be achieved by an intermediary who specializes and focuses on a small (but not too small) number of clients and/or agents that cannot interact otherwise. For example, an intermediary can focus on local businesses, or can provide a connection between otherwise disconnected communities. In that sense, our results provide new insights that are related to the discussion of the optimal size of intermediaries in financial and other markets.

We provide two applications of our result. First, we consider interactions between hosts and guests in the market for short term apartment rentals. By showing that well positioned intermediaries can help prevent hosts from appropriating the guests’ security deposits, our results provide an explanation for the success of Airbnb.com—an online intermediary which succeeded in a market already dominated by an incumbent two-side platform, Craigslist.org. To be specific, we argue that by taking part in the interaction and committing to “police” the market, Airbnb.com essentially created a new market with many hosts and guests who did not participate in the market facilitated by the passive platform of Craigslist.com.

A second application is to financial markets with investors and entrepreneurs. We follow much of the financial literature and consider an environment in which limited verifiability and/or limited liability make it possible for an entrepreneur to strategically default by not repaying investors even when the projects they invest in yield positive returns (see also Bolton and Scharfstein 1990). By relating m^* to the parameters of the market, we are able to show that the minimal level of exclusivity an intermediary is required to hold (as captured by m^*) decreases in the frequency of arrival of investment opportunities and the expected return on investments, and increases in the entrepreneur’s discount rate and in the return on capital demanded by investors. From a macroeconomic perspective, the characterization predicts that in

times of economic booms (high frequency of arrival of investment opportunities and high expected returns on investments) there is room for a large number of intermediaries and markets in which no single intermediary has significant levels of exclusivity (as captured by m^*). On the other hand, in times of economic downturn, especially ones that are triggered by liquidity crunches, the model predicts a smaller number of intermediaries, each with significant level of exclusivity over investment paths.

We also find that requiring that an entrepreneur self-finance a positive fraction of an investment opportunity reduces the level of exclusivity required by intermediaries in order to enforce repayment, and that the same is true for partial collateral and for the presence of strict bankruptcy laws. On the other hand, introducing the possibility of pledging full collateral increases the level of exclusivity that intermediaries are required to have in order to enforce repayment of uncollateralized investments. Therefore, if the cost of pledging full collateral is sufficiently low, a market may revert to simple debt contracts even when equity contracts are more efficient. In particular, full collateral contracts are likely to undermine the role of intermediaries for the **riskiest and safest investments**, and intermediaries are likely to continue trading the **intermediately risky assets** without collateral. Finally, we show that central credit information agencies (a generalized version of credit rating agencies) may relax the requirement of exclusivity, but do not eliminate the need for intermediaries.

1.1. **Related literature.** This paper contributes to the literature on social capital that studies the ability of a society to sustain trust and cooperation. Broadly speaking, the literature emphasizes two structural elements that generate social capital: on the one hand, the importance of social pressures for fostering cooperation dates back to sociological work by Simmel (1950) and Coleman (1988). This literature emphasizes the importance of closure—in order to facilitate a *strong tie* between two individuals, they are required to share many acquaintances. More recent economics literature provides more rigorous theoretical underpinning (see also Raub and Weesie 1990, Haag and Lagunoff 2006, Ali and Miller 2012a, Mihm, Toth, and Lang 2009, Jackson, Rodriguez-Barraquer, and Tan 2011, and Lippert and Spagnolo 2011). On the other hand, seminal work by Burt (1992) suggests that social capital requires intermediaries that bridge across communities and facilitate interactions across otherwise disconnected individuals. However, the reason for the emergence of such ‘structural holes’ that can be exploited by well positioned intermediaries to extract surplus remains mostly unexplored in the economics literature (an exception is Goyal and Vega-Redondo 2007). We study an economy in which market participants must rely on well positioned intermediaries to enforce cooperation. To that extent our analysis contributes to the literature on social capital by showing why individuals may have to limit their direct relationships and rely on a small number of intermediaries to execute transactions on their behalf.

Methodologically, this paper is related to the growing literature on social and economic networks. A comprehensive review of this literature is beyond the scope of this paper.⁸ Instead, we just note that most of the economic literature on networks makes the assumption that individuals have complete knowledge of the network structure, and that they perfectly observe any change made to the network structure throughout the game. The perfect observability approach is often justified as a good approximation for setups in which observability may be imperfect and knowledge of the network structure incomplete. This is not true in our setup: we show that robustness provides a significant refinement relative to the set of networks that can be sustained in a subgame perfect equilibrium of the game with perfect observability of the network structure.

Recently, several papers take an imperfect observability approach: Caballero and Simsek (2010) assume that individuals (in their case banks) observe the networks structure up to a permutation on the identities of individuals. McBride (2006), Jackson and Yariv (2007), Galeotti et al. (2010), Fainmesser and Goldberg (2012), Fainmesser (2012), and Fainmesser (2013) assume that each individual observes the network structure up to a constant geodesic distance from her in the network. An individual's belief on parts of the networks that are farther away from her may be grounded in a random process that all individuals believe to have generated the network (e.g. Jackson and Yariv 2007, Fainmesser and Goldberg 2012, Fainmesser 2012, Fainmesser 2013, and implicitly also Galeotti et al. 2010), or it is allowed to be any belief that corresponds to the information available to the individual (e.g. McBride 2006).

Considering individuals who observe the network structure up to a fixed geodesic distance provides a mathematically appealing setup and is a novel approximation for social networks and other markets that consist of ex-ante homogeneous individuals. However, it is not always obvious what is the most reasonable geodesic distance to assume. Moreover, in markets in which individuals have different roles and activity is asymmetric, there are good reasons to expect that individuals' observations of the network structure depend on their different experiences in the marketplace.

Endogenizing individuals' knowledge of the network structure in which they are embedded is a known open challenge. Given any realistic dynamic interaction, characterizing the mapping from individuals' observations in their own bilateral interactions to their knowledge of the network structure is an intractable exercise. To overcome the hurdle, we offers a dynamic setup in which the following question is much more tractable: what can individuals learn about the network structure given what they would observe in their interactions over an infinitely long period of time? The answer provides an upper bound on the knowledge that individuals may have at

⁸See Goyal (2007) and Jackson (2008) for extensive surveys of the literature on social and economics networks, and Gale and Kariv (2007), Gofman (2011), Condorelli and Galeotti (2011), and Nava (2010) for work analyzing the efficiency of intermediated trade in exogenously determined networks.

any point in time.⁹ We derive this upper bound on individuals' knowledge in our setup and endow individuals with the corresponding knowledge at any point in time. We then characterize networks that are robust in the sense that they can be sustained indefinitely in an equilibrium of the game given the aforementioned knowledge and (almost) any belief profile.

The robustness requirement allows us to provide sharp predictions in a repeated games setup by ruling out networks that are 'too sensitive' to the underlying individuals' beliefs. This ties back to a well known challenge in the study of games of incomplete information—weeding out equilibria that rely on 'unreasonable' or 'unrealistic' beliefs.¹⁰ Instead of approaching directly the problem of defining reasonable versus unreasonable beliefs in our setup, we suggest a criterion that rules out any network that can be sustained only given highly specific belief profiles. In this sense, this paper offers predictions based on an incomplete information refinement of a repeated games model.

2. TRUST, INTERMEDIATION, AND EXCLUSIVITY: SUGGESTIVE EVIDENCE

This section presents evidence about the role of intermediaries in trust-intensive exchanges. We focus on anecdotal evidence about the mechanism through which intermediaries create the trust necessary for these transactions, including the practices of such intermediaries of observing the transactions they intermediate and of maintaining exclusivity over transaction paths.

We begin with a detailed example from the market for short term apartment rentals. Throughout the years, home-owners and short-term renters have mainly transacted using classified ads, with the posting forum (local newspapers or websites such as Craigslist.com) acting as little more than a message board. Yet the entry to the market of more active intermediaries such as Homeaway Inc. (controlling Homeaway.com, VRBO.com, VacationRentals.com, and more) and more recently Airbnb.com, led to a significant expansion of the market. Since properties rented through Homeaway.com are to a great extent run by property managers, we focus on Airbnb.com in which many of the properties are rented out by private owners.

Founded in August 2008, Airbnb.com is an online service that provides a platform for "hosts" to rent short-term lodging to "guests." As of November 2012, Airbnb.com includes over 250,000 listings in 30,000 cities and 192 countries. Listings include private rooms, entire apartments, castles, boats, manors, tree houses, tipis, igloos, private islands and other properties. Given the existence of large scale online classifieds websites such as Craigslist.com, and the fact that they are free (whereas Airbnb.com charges a per rental percentage fee) it is unconvincing to attribute the success of Airbnb.com to the search capabilities of the platform. Looking at the Airbnb.com website provides insights into its role in the market:

⁹In our setup it is also true that once the network structure stabilizes for a sufficiently long time period, individuals' knowledge is bound to reach arbitrarily close to this level of knowledge.

¹⁰See also the discussion of equilibrium refinements in chapter 8 of Fudenberg and Tirole (1991a), and the literature on robust partial implementation, e.g. Bergemann and Morris (2005).

“We believe that providing the right environment allows you to make safe, informed decisions. [...] We give every member of the community the tools to help us police our marketplace.” (Airbnb.com)

There are several aspects of short term apartment rental transactions that pose safety issues. For example, a guest could damage a property. To protect themselves, hosts could require a security deposit. However, given the short term nature of the rental, effective security deposits are large relative to the size of the transaction, raising a safety issue on the side of the guest—would the host keep the security deposit by falsely claiming damages. Airbnb.com acknowledges the problem and suggests that the solution lies in intermediation.

“Although it is primarily up to the host to determine the extent of the damage, Airbnb tracks every claim that is made, and if a host develops a trend of claiming damages in order to keep the security deposit, the host may be removed from Airbnb.” (Airbnb.com)

The mechanism suggested is clear: a host might not be worried about cheating a short-term, perhaps infrequently repeated, guest. However, cheating a guest in an interaction observed by a large intermediary risks being excluded from a large market. Such exclusion is effective as long as the host has no alternative channels through which he can transact with the same pool of guests. Thus, to make this exclusion effective, Airbnb.com takes actions to secure its exclusivity over transaction channels. The following also appears in the safety section of Airbnb.com:

“All contact info - email, phone number, and address - will be automatically exchanged between guest and host after the room is booked.” (Airbnb.com)

The enforcement is strict. Email addresses and URLs are detected and erased from messages between hosts and guests on Airbnb.com. Therefore, it is cumbersome for a host to extract a large number of contacts and overcome the need to transact via Airbnb.com in the future.

Airbnb.com provides a stark example of trust enabling intermediation and of the importance of exclusivity over transaction channels. However, it is far from being a special example for the practice. Real estate agents have their clients (both sellers/homeowners and buyers/renters) sign exclusivity contracts with them, and hedge funds lock investors funds for long periods of time. Other businesses facilitate exclusivity by introducing paid membership subscriptions that reduce purchase prices and ex-post transaction costs for buying and selling (e.g., Amazon.com introduces Amazon prime, Amazon mom, and more, which facilitate higher levels of exclusivity for Amazon marketplace). Yet an additional form of exclusivity is facilitated by focusing on market niches which are not served by the large market makers.

Some trust enforcing intermediaries, such as Airbnb.com and eBay.com, introduce reputation systems to replace or enhance their ability to police the market. If a reputation system

worked perfectly, it could make an intermediary’s work easier.¹¹ However, we show that unless the reputation system can convey to users sufficient information about the patterns of interactions in the market, it does not eliminate the role of intermediaries completely. That is, an intermediary’s role in the market goes beyond transmitting information on “bad” behavior of market participants. In other cases, the absence of a formal intermediary leads one side of the market to get organized into a virtual intermediary. Examples of this include lending groups, such as venture capital, bank consortia, and many informal lending arrangements in developing countries (see also Besley and Coate 1995).

After deriving our main result we apply our result and derive implications for the short-term apartment rentals market as well as to the structure of intermediation in financial markets.

3. THE ECONOMY

Consider a single agent a , and a finite set of clients C with an individual client denoted by c .¹² Time is continuous and individuals (i.e., clients and the agent) have a common discount rate ρ . Opportunities for clients to interact with the agent arrive stochastically over time—for any client c , the event that c has an interaction opportunity with a occurs according to a Poisson arrival process with parameter λ .¹³ An interaction opportunity takes the following form: the client chooses whether or not to engage with the agent in an interaction with an outcome that depends only on the actions of the agent—if the agent cooperates then the payoffs of the client and agent respectively are (π_C^{++}, π_A^+) , and if the agent defects then the payoffs are (π_C^-, π_A^{++}) , where for any individual $J \in C \cup \{a\}$, $\pi_J^{++} > \pi_J^+ > 0 > \pi_J^-$. Put differently, the client and agent engage in a *trust game*—by choosing to interact the client generates surplus for the agent and then the agent can decide whether to generate a higher surplus for herself while inducing a loss for the client, or to be satisfied with a lower surplus for herself, generating a surplus for the client. If the client chooses not to interact with the agent then both the client and the agent have a payoff of zero. An interaction and its outcome are observable only to the parties participating in the interaction.

In the apartment rental example, with homeowners requiring a large security deposit, we have that clients are renters and the agent is a homeowner. An interaction is then a rental with rent π_A^+ and security deposit equals $\pi_C^{++} - \pi_C^- = \pi_A^{++} - \pi_A^+$ (where π_C^{++} is the renter’s net utility from renting the apartment), and a defection by the homeowner means that she pretends to

¹¹There is a large literature studying the strengths and weaknesses of reputation systems. For a brief overview see Resnick et al. (2000) and references therein.

¹²Our results carry over to a model with multiple agents. For details, see a previous version of this paper titled “Intermediation and Exclusive Representations in Financial Networks” available on the author’s webpage.

¹³A Poisson arrival rate implies asynchronicity: at any given moment in time, the probability that more than one client has an interaction opportunity with the agent is zero. This allows us to focus on intertemporal relationships and abstract from other intratemporal considerations that may be second order to sustaining long term relationships.

find reasons not to refund the security deposit to the renter upon a successful completion of the rental period, thus making a profit of π_A^{++} instead of π_A^+ .

3.1. Preliminary analysis: hurdles for cooperation. If there was only one client in the economy, a well known observation is that an equilibrium in which on the equilibrium path there is an interaction in which the agent cooperates exists if and only if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ \geq \pi_A^{++}$. If the condition does not hold (that is, if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ < \pi_A^{++}$) one hopes that several clients could somehow join forces to enforce cooperation, for example by ostracizing a defecting agent. However, in this simple private monitoring environment the challenges to successful interactions persist when there are many clients.

Definition 1. We say that a Perfect Bayesian Equilibrium (PBE) is a *Cooperative Equilibrium* if on the equilibrium path in any interaction opportunity the client chooses to interact and the agent cooperates.

Proposition 1. *There exists a cooperative equilibrium if and only if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ \geq \pi_A^{++}$.*

Proposition 1 shows that enforcement of cooperation is independent of the number of clients. That is, enforcement is essentially bilateral. In fact, if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ < \pi_A^{++}$ then in all PBEs, no client ever chooses to interact and the agent never cooperates.¹⁴ The proof of Proposition 1 is trivial and is omitted.

For the remainder of the paper we focus on environments in which bilateral enforcement is not effective and assumption 1 holds.

Assumption 1. $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ < \pi_A^{++}$.

To abstract from integer problems we also make the following genericity assumption throughout the paper.

Assumption 2. $\forall n \in \mathbb{N} \left(1 + \frac{\lambda}{\rho}n\right)\pi_A^+ \neq \pi_A^{++}$.

3.2. Roadmap. We proceed in two steps. In section 4 we introduce intermediaries into our simple model above and show that a necessary and sufficient condition to guarantee existence of a cooperative equilibrium for a wide range of parameters is the existence of a single intermediary who is able to intermediate all of the interactions in the economy. Subsequently, in section 5, we derive richer predictions on the patterns of the intermediation by developing a model that takes more seriously the idea of trust relationships.

¹⁴The specifics of the result relies on the assumption that there is only one agent—having more agents introduces the possibility of contagious strategies which increase the range of parameters for which cooperative equilibria exist. However, we show in Proposition 2 and Theorem 1 below that to motivate the role of intermediaries as enabling cooperation, it is sufficient to note that in this simple setup with many agents there exists a cooperative equilibrium *only if* $\left(1 + \frac{\lambda}{\rho}|C|\right)\pi_A^+ > \pi_A^{++}$ (rather than $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ \geq \pi_A^{++}$). This is true regardless of the number of agents in the model.

4. FULLY EXCLUSIVE INTERMEDIATION: AN ILLUSTRATION

Now suppose that in addition to the agent and the set of clients, there is also a finite set of intermediaries denoted by I , with an individual intermediary denoted by i . An intermediary is simply an individual who is able to participate in the interaction on behalf of clients. If an intermediary is used in an interaction, the intermediary incurs a cost (e.g., effort) of participation, and shares the benefits of an interaction if the agent cooperates. For example, an intermediary can be an individual hired by the client to witness or supervise the direct interaction, a financial entity which executes the interaction on behalf of the client, or a manager that is paid a percentage of the client's payoffs. Formally, the payoffs in an intermediated interaction are as follows: if the agent cooperates then the payoffs to the client, agent, and intermediary (respectively) are $(\pi_C^+, \pi_A^+, \pi_I^{++})$, whereas if the agent defects then the corresponding payoffs are $(\pi_C^-, \pi_A^{++}, \pi_I^-)$ —compared with $(\pi_C^{++}, \pi_A^+, 0)$ and $(\pi_C^-, \pi_A^{++}, 0)$ respectively in a direct interaction (recall that for any individual $J \in \{C, I, A\}$, $\pi_J^{++} > \pi_J^+ > 0 > \pi_J^-$). As before, an interaction and its outcome are observable only to the participants in the interaction (now including the intermediary if any).¹⁵

Patterns of interaction can be affected by exogenous factors as well as by strategic behavior. In the remainder of this section, we propose and study a simple model of networked markets. Informally, there is an exogenously given network connecting some of the individuals (client-agent, client-intermediary, or intermediary-agent). If two individuals are connected, it means that they are *able* to interact with each other if the opportunity arises at any given time t . It is then up to the individuals to decide whether and how to interact. We make the role of the network more precise when reviewing the timeline of the market below.

4.1. The interaction network: definitions. The network of connections between individuals is described by a graph $G = \langle V, E \rangle$, where $V = \langle C, I, a \rangle$ is the set of all individuals and $E \subseteq (C \times \{a\}) \cup (C \times I) \cup (I \times \{a\})$ is the set of connections (or links/edges). That is, a link always connects two individuals of different types. For ease of notation we denote by jk the link (j, k) and by jkl the path including the two links jk and kl .

Let $C_i^G = \{c \in C \mid ci \in G\}$ be the set of clients who are connected to intermediary i in network G , and define similarly C_a^G , I_a^G , and I_c^G . We drop the superscript G when it is clear from the context. We say that $ca^G = \text{true}$ if and only if c and a are connected in G , and that $cia^G = \text{true}$ if and only if both c and a are connected to i in G (ci^G and ia^G). Similarly, $cIa^G = \text{true}$ if and only if c and a are connected or there exists at least one intermediary i such that both c and a are connected to i ($\exists i \in I \mid cia^G$). If cIa^G (that is, if $cIa^G = \text{true}$) we say that there is an *interaction path* between c and a and that they are *network related*. We denote by $S_a^G = \{c' \mid c'Ia^G\}$ the *network support* of the agent, i.e., the set of clients who are network related to a .

¹⁵To emphasize that the role of intermediation in our model is not due to cost advantages, we note that all of our results hold regardless of whether intermediaries have a cost advantage ($\pi_C^+ + \pi_I^{++} > \pi_C^{++}$) or a cost disadvantage ($\pi_C^+ + \pi_I^{++} < \pi_C^{++}$) in executing an interaction.

4.2. **Timeline.** Consider an economy with an exogenously given network G that is commonly known to all of the individuals. At any time $t \in [0, \infty)$, if client c has an interaction opportunity with the agent, then the following happens (instantaneously in order):

- (1) Client c and all individuals connected to c observe the opportunity.¹⁶
- (2) If client c is directly connected to the agent (that is, if ca^G), then c chooses whether to interact directly or not.
- (3) If an intermediary i is connected to the agent and to client c , she chooses whether to offer (to c and a) that she intermediate the interaction. If multiple intermediaries make intermediation offers, one offer (selected exogenously and u.a.r.) is revealed to c and a .¹⁷ Then, if client c didn't interact in step 2., he chooses whether to interact using the intermediation offer (if any), or not interact.
- (4) If c decides to interact, interaction takes place.

4.3. **Exclusive intermediation.** It turns out that networks that facilitate cooperative equilibria for the largest range of parameters' values have a specific structure.

Definition 2. We say that a network G enables *exclusive intermediation* if there exists an intermediary i who is connected to the agent and to all of the clients in C .

Proposition 2. *The following statements are equivalent:*

- [1] *The network G enables exclusive intermediation.*
- [2] *If $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$, then there exists a cooperative equilibrium.*

Proposition 2 reveals that only networks that enable exclusive intermediation admit a cooperative equilibrium for the entire range of parameters that satisfy $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$. We do not prove Proposition 2 directly. Instead, in the appendix we prove a stronger result consisting of two lemmas: First, Lemma 1 shows that in any network that enables exclusive intermediation there exists a cooperative equilibrium if and only if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$. Second, Lemma 2 shows that in any network that does not enable exclusive intermediation there exists a cooperative equilibrium only if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ > \pi_A^{++}$. The intuition behind these results is as follows: if a network enables exclusive intermediation, then as long as $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$, there exists an equilibrium in which only one intermediary ever makes offers to intermediate, and in which all of the interactions are intermediated by that intermediary. Such an equilibrium can be supported by a threat that if the agent ever defects, no further transaction take place. On the other

¹⁶The assumption that any individual connected to c observes of the opportunity is made for simplicity only. In a previous version of the paper that is available on the author's website, we present a model in which a decentralized process of information diffusion, which is necessary for parties to find interaction partners, leads to the transmission of information about interaction opportunities.

¹⁷The assumption that one offer is revealed captures unmodeled stochastic elements in information diffusion and in the order of making offers, which allows us to abstract from the process of choice of an intermediary. In section 7.5 when we introduce price competition between intermediaries, we relax this assumption and allow an intermediary to lower her price to guarantee that her offer is revealed.

hand, if a network does not enable exclusive intermediation, then information on the agent's defection must propagate in the network (via individuals' actions) in order to reach all of the active intermediaries and direct interaction channels. For any level of impatience (discount rate $\rho > 0$), this implies that a cooperative equilibrium does not exist for any $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \leq \pi_A^{++}$.

In fact, if a network enables exclusive intermediation, there is no cooperative equilibrium that does better than the one in which one intermediary intermediates all of the interactions in the economy. That is, if the aforementioned equilibrium does not exist, then there is no cooperative equilibrium.

5. NETWORKS OF TRUST RELATIONSHIPS

In the simple model above, community enforcement is hindered because monitoring is private: an interaction and its outcome are observed only by the interacting parties. Contagious and ostracizing strategies may help, but they require time for information about a defection to propagate. If the agent is not infinitely patient, then the delay in punishment constrains cooperation. Exclusive intermediation improves the ability to enforce cooperation by cutting down on this delay—an intermediary functions as a central information and enforcement agency that acts immediately to punish defection.

To motivate the analysis in the remainder of the paper we make the following observations: First, from a practical perspective, Proposition 2 provides appealing intuition, but limited predictions. It does not tell us much about the patterns of interactions that enforce cooperation, or about the set of networks that facilitate cooperation when the constraints on cooperation are not maximally demanding (i.e., when $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ > \pi_A^{++}$). This leaves open several questions that are crucial for our understanding of intermediation, and for empirical work. For example, we would like to know whether intermediation, and the exclusivity of intermediation, are important when the constraints on cooperation are not maximally demanding. If exclusivity remains important, is cooperation achieved only through market fragmentation into several smaller and separate markets, or can more integrated structures achieve cooperation?

The model above is not rich enough to answer these questions because it captures only a physical network in which two individuals are connected if they are able to interact, and does not tell us which links are used. Taking more seriously the idea that individuals have long term relationships that are captured by an interaction network requires rethinking the meaning of a link. For example, if a link represents a long term trust based relationship, then individuals may sever links. Taking the perspective that a link is a relationship also reopens the question of what do individuals know about the structure of the network in which they are embedded, and whether changes in the network can be used to communicate between individuals. On the one hand, individuals may not observe the entire network structure. On the other hand, when a link is never used, it is effectively eliminated. The elimination of a link may affect some

individuals indirectly; such individuals may be able to make inference on the change in the network structure and therefore on deviations from equilibrium path.

We now consider a richer model in which individuals may affect the network structure, and in which an individual's knowledge of the network structure is directly related to the role of the individual and to what she can observe through her interactions. Thus, the network is determined by exogenous factors (i.e. initial state) as well as by individuals' decisions, and can evolve over time, with a connection between two individuals at time t capturing that they are *able and willing* to interact with each other if the opportunity arises at time t .

To highlight the result that some links may not exist only because a client or intermediary expects that the agent will defect, we assume throughout that there is no cost associated with maintaining a link (our results continue to hold if maintaining links is costly).

5.1. The observability operator. The following notation accommodates different assumptions with respect to the observability of the network structure.

Definition 3. Consider an individual j and let K_j map (deterministically) any network G to a set of networks $K_j(G)$. We interpret $K_j(G)$ as the set of networks that are not ruled out by j 's observations with respect to the network structure. For example, if for every network G , $K_j(G) = \{G\}$ then we say that individual j perfectly observes the network structure. We let $K = \{K_j\}_{j \in V}$ and refer to K as *the observability operator*.

Given j 's prior, K_j can be thought of as a mapping from any subset of networks to a signal received by individual j and conveying the subset of networks in the support of j 's posterior. The mapping K is deterministic and commonly known. Thus, if the network is drawn from some distribution, then any differences in beliefs between individuals must take the form of having different posteriors on the structure of the network G (due to different observations). On the other hand, individuals agree on what others observe given a network structure.

5.2. Timeline. The timeline of this model follows closely the timeline above (section 4.2) with small modifications to allow for the deletion of links and for uncertainty over the network structure. We highlight all such modifications in bold.

At time $t = 0$ the game begins with an initial network $G^0 = \langle V, E^0 \rangle$ being generated by an arbitrary process, which individuals believe to be summarized by some distribution \mathcal{G} over some set of networks.¹⁸ Note that the set of individuals $V = \langle C, I, a \rangle$ is also generated at time $t = 0$.

At any time $t \in [0, \infty)$ the following happens (instantaneously in order):

- (1) **Any individual j observes $K_j(G^t)$ and decide which of her links to eliminate and which to maintain.**

¹⁸Whether the process generating the networks is, in fact, summarized by \mathcal{G} does not affect our results, but affects their interpretation.

- (2) If client c has an interaction opportunity with the agent, then
- (a) Client c and all individuals connected to c observe the opportunity.
 - (b) If client c is directly connected to the agent (that is, if ca^G), then c chooses whether to interact directly or not.
 - (c) If an intermediary i is connected to the agent and to client c , she chooses whether to offer (to c and a) that she intermediate the interaction. If multiple intermediaries make intermediation offers, one offer (selected exogenously and u.a.r.) is revealed to c and a . Then, if client c did not interact in step (b), he chooses whether to interact using the intermediation offer (if any), or not interact.
 - (d) If c decides to interact, interaction takes place. **A link that has been defected on is automatically eliminated.**

5.3. Robust interaction networks. We are interested in characterizing the patterns of interactions that we expect to observe in markets. To this end, we need to fill in the following missing parts: first, the game we study has a rich strategy space and there is no theoretical guarantee that links are used (or used in a way that we expect a relationship to be used). Thus, we are required to define an equilibrium criterion that respects the notion of trust relationships. Second, we are yet to define an appropriate observability structure, i.e., put economic content into the observability operator (K) in a way that is consistent with knowledge that individuals can acquire during repeated interactions. Finally, given incomplete observability of the network structure, it is prohibitively permissive to require only the existence of an equilibrium with a certain network structure, or even with particular patterns of interaction. The cause is the induced freedom in the selection of equilibrium beliefs.¹⁹

In this section we define a general notion of network robustness, which accepts as parameters an equilibrium selection criterion and a set of beliefs. A network is robust if given the observability operator, for any belief profile from the defined set, there exists an equilibrium satisfying the selection criterion in which the network is sustained indefinitely with probability 1. We now make this more precise.

Let Ω be the set of all possible states of the world. Each state $\omega \in \Omega$ specifies a set of individuals V , an initial network G^0 , a time t , and the entire history of play up to time t . Incorporating the time index and history into the state is done for notational convenience only – when it plays a role in the analysis we add a superscript t (ω^t).

Denote by $\Omega(G)$ the set of states of the world in which the underlying network is G and denote by $\omega(G)$ a member of $\Omega(G)$. We also denote by $G(\omega)$ the underlying network in state ω . An

¹⁹This is clear for PBE, because of the continuous time modeling. However, the problem is not less severe for PEBE (see Fudenberg and Tirole 1991b and Battigalli 1996) and other strong equilibrium concepts. The reason lies in the high dimensionality of networks, and the different information on the network structure that each individual may be exposed to. Below, we show that the problem goes away when individuals have complete observability of the networks structure, but at the cost of losing predictive power.

information set of individual j is denoted $h_j(\omega)$ (or h_j when ω is clear from the context). A state ω' belongs to $h_j(\omega)$ if j cannot distinguish between ω and ω' (and then $h_j(\omega) = h_j(\omega')$). We denote by $H_j = \{h_j(\omega)\}_{\omega \in \Omega}$ the knowledge partition of individual j , and $H = \times_{j \in V} H_j$. A belief is captured by a mapping $\mu: H \rightarrow \Delta(\Omega)^{|V|}$ (also written as a vector of mappings $\mu_j: \Omega \rightarrow \Delta(\Omega)$ such that $\mu_j(\omega) = \mu(h_j(\omega))$ when convenient) where $\Delta(\Omega)$ is the set of probability distributions over Ω .

Definition 4. Given an equilibrium selection rule \mathcal{S} and a set of beliefs \mathcal{M} , we say that a network G is $\langle \mathcal{S}, \mathcal{M} \rangle$ -robust (or simply *robust* when clear from the context) if for any belief $\mu \in \mathcal{M}$ there exist strategies σ such that (σ, μ) is a PBE that satisfies equilibrium selection rule \mathcal{S} and such that if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1.

The definition of a robust networks is quite general and can be applied to a large variety of games in networks in which individuals have a choice of whether to sustain or eliminate links.²⁰

It is easy to see why we are required to restrict the set of beliefs: not all beliefs are consistent with any equilibrium play of any PBE—e.g., beliefs that are inconsistent with Bayes rule cannot be part of any PBE, regardless of whether the network is sustained indefinitely or not. An equilibrium selection rule is also necessary: with no restrictions that require individuals to interact over existing links, any network is trivially robust—if the strategies σ are such that clients never interact and the agent always defects then for any network G and for any belief μ which is consistent with any PBE, it is also the case that (σ, μ) is a PBE in which if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1.

5.4. Equilibrium selection rule—Network Equilibrium (NetE). We focus attention on a family of equilibria in which relationships are implicit long term commitments.

Definition 5. A PBE is a *Network equilibrium* (NetE) if in any continuation game the following three requirements hold.

- (1) Interaction network: Any client c always chooses to interact upon having the choice of whether to interact or not.
- (2) Trust network: The agent does not eliminate links. Intermediaries do not eliminate links to clients.
- (3) No arbitrary shutdowns: An individual eliminates a link at time $t > 0$ only immediately after observing a defection or a change to the network.

The first part of the definition governs the connection between the network structure and the pattern of interactions. That is, in a NetE, clients who have direct connections to the agent interact directly with her as long as they keep these links. Similarly, clients with only indirect

²⁰Examples of such games include the network *favor exchange game* in Jackson, Rodriguez-Barraquer, and Tan (2011), and the network client-agent *trust game* in Fainmesser (2012) and Fainmesser and Goldberg (2012).

connection to the agent agree to interact through any intermediary with whom they keep the connection. In equilibrium, this reduces the strategy space (however, the complete strategy space is still considered for evaluating deviations). The second and third parts formalize the idea that connections capture trust—agents do not eliminate connections in which they are the trustees rather than the trusting parties, and a client who trusts the agent does not cease trusting the agent without observing any deviation or change in the environment that alerts him to reasons not to trust the agent. In particular, the third part rules out “bad” continuation games in which a client and the agent are stuck: the client eliminates the link to the agent because the agent would always defect (although she never did) and the agent is willing to defect in any interaction with the client because the client is expected to eliminate the link regardless of what happens. Notably, a NetE is not necessarily cooperative in the sense of definition 1. However, a NetE in which a network $G^0 = G$ is sustained with probability 1, is a cooperative equilibrium by definition.

Formally, we note that (1) and (2) allow us to fully describe any pure strategy NetE strategies as follows (our analysis includes also mixed strategies NetE, yet formalizing pure strategies is useful to illustrate the notion of a NetE, and is also useful in our proofs). A candidate NetE pure strategy for the agent can be described as a mapping $\sigma_a : H_a \rightarrow \mathcal{P}(C \cup (I \times C))$ from a 's information set to the set of clients and intermediary-client pairs, such that the agent will cooperate in a direct interaction with a client / interaction with a client via its paired intermediary—i.e., if $c \in \sigma_a(h_a)$ ($ci \in \sigma_a(h_a)$) then the agent will cooperate when interacting directly with c (via i) if a 's information set is h_a . Similarly, a candidate NetE pure strategy for a client c is a mapping $\sigma_c : H_c \rightarrow \mathcal{P}(I \cup \{a\})$ from c 's information set to the set of intermediaries and agent to whom c maintains a connection, and a candidate NetE pure strategy for an intermediary i is a mapping $\sigma_i : H_i \rightarrow \mathcal{P}(\{a\})$. We denote by $\sigma = \{\sigma_j\}_{j \in V}$ a candidate NetE strategy profile, and by $\Sigma(G)$ the set of NetE such that if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1.²¹

5.5. Beliefs. We now define two sets of beliefs that we consider in the analysis of robust networks.

Definition 6. Belief μ is an *Equilibrium Enabling Belief* (EB) if there exists a strategy profile σ such that (σ, μ) is a NetE. Belief μ is a *Initial Trust Enabling Belief* (ITB) for network G if there exists a strategy profile σ such that (σ, μ) is a NetE, and if $G^0 = G$ then according to σ all clients keep all of their links to intermediaries in G at time 0 ($\forall_c \sigma_c(\omega^0(G)) \supseteq I_c^G$). Denote by *EB* the set of all EBs and by *ITB*(G) the set of all ITBs for network G .

²¹Clearly, to prove that a candidate NetE strategy profile is a NetE strategy profile, we are required to rule out deviations that cannot be described by the above formulation. For example, in a NetE an agent should not find it profitable to eliminate a link to a client, and a client should not find it profitable to refuse an interaction over a link that he maintains.

Thus, for example, a network G is $\langle \text{NetE}, EB \rangle$ -robust if for any belief profile $\mu \in EB$ there exists a NetE such that starting with $G^0 = G$, the network G is sustained indefinitely with probability 1 ($\forall \mu \in EB \exists \sigma (\sigma, \mu) \in \Sigma(G)$).

Notably, the sets EB and $ITB(G)$ depend on what individuals observe. However, for any observability operator K , our definition of EB captures a natural restriction on the set of beliefs—beliefs should be consistent with Bayes rule. We defer the discussion of the notion of ITB to section 5.7 in which we analyze the corresponding game with incomplete network observability—we do that because with complete network observability, the sets EB and $ITB(G)$ coincide regardless of G . Thus, the set of $\langle \text{NetE}, EB \rangle$ -robust networks and the set of $\langle \text{NetE}, ITB \rangle$ -robust networks are identical (see Proposition 3 below).

5.6. Network observability. Much of the economic literature on networks makes the assumption that agents have complete knowledge of the network structure, and that they perfectly observe any change made to the network structure throughout the game. This assumption is a reasonable approximation of reality in small markets, and when links capture a physical connection—both requirements do not fit the markets motivating this paper. In other works, the perfect observability approach is often justified as a good approximation for setups in which observability may be imperfect and knowledge of the network structure incomplete. This is not true in our setup. To show that, we first characterize the set of robust networks in the perfect network observability case.

5.6.1. Benchmark: perfectly observable networks. With complete observability of the network structure (for all j and G , $K_j(G) = G$), the robustness criterion does not restrict the set of networks that our model predicts. That is, a network is robust if and only if there exists a NetE in which the network is sustained indefinitely with probability 1. We now make this claim formally.

The following definition will be useful for the statement of many of our results.

Definition 7. Let $m^* \triangleq \min \left\{ n \in \mathbb{N} \mid \left(1 + \frac{1}{\rho} n \right) \pi_A^+ \geq \pi_A^{++} \right\}$.

Note that $\left(1 + \frac{1}{\rho} n \right) \pi_A^+$ is the expected present and future payoffs from cooperating to an agent who is network related to exactly n clients and who plans to always cooperate. Thus, m^* captures the minimal number of clients such that the agent (perhaps weakly) prefers to always cooperate rather than defect now and lose her network relations with m^* clients immediately. Also, by assumption 1, $m^* > 1$.

Proposition 3. *Suppose that for any individual j and network G , $K_j(G) = G$. Then, the following statements are equivalent:*

- (1) *There exists a NetE such that if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1 (i.e., $\Sigma(G) \neq \emptyset$);*
- (2) *The network G is $\langle \text{NetE}, EB \rangle$ -robust;*

- (3) The network G is $\langle \text{NetE}, \text{ITB} \rangle$ -robust;
 (4) The agent is network-related to at least m^* clients in G .

The proof relies on simple bang-bang strategies—if the agent ever defects, at least one link is eliminated immediately. Since the elimination of a link is immediately observed by everyone, all clients can coordinate on disconnecting all of their links simultaneously and the entire market shuts down. So for instance, if there are more than m clients in the economy, the complete network could be sustained. In the complete network all clients are connected to the agent and to all of the intermediaries in the economy, and all interactions are direct—with no use of intermediation.

While providing a benchmark, as well as assuring us that our assumptions so far did not lead to deviations from the classic prediction on community enforcement, there are issues with the assumption of perfectly observable connections and the corresponding analysis. First, even if individuals observe the exogenous (potentially physical) restrictions imposed on the network as captured by G^0 , it is unlikely that a client c observes immediately when some other client c' is no longer willing to interact with the agent, or with some intermediary. This is especially so if c and c' are in a remote parts of the network (we make this claim more precise below). The use of bang-bang strategies further highlights the unrealistic nature of the analysis assuming perfect and immediate observability in this market.²²

For example, consider Figure 5.1 and assume that the parameters of the model (in particular λ, ρ, π_A^+ , and π_A^{++}) are such that $m^* = 4$. Now suppose that a defects in a direct interaction with c_6 : how can c_6 be certain that i_1 (or c_5) will punish a ? How would c_1 know the exact time that he is supposed to punish? Note that if $c_1 - c_5$ cannot learn about the defection and c_6 's punishment, then i_1 prefers to continue to intermediate interactions with a . Assuming instantaneous complete knowledge of the network structure as well as relying on bang-bang strategies provides a theoretical underpinning, but one that relies heavily on potentially unreasonable assumptions.

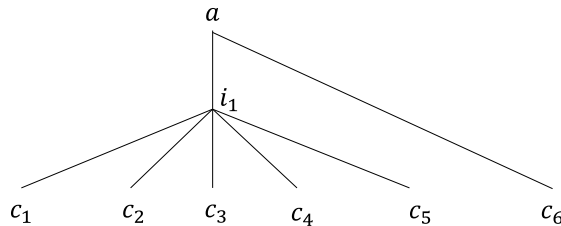


FIGURE 5.1.

²²E.g. in the context of favor exchange in small rural villages, Jackson, Rodriguez-Barraquer, and Tan (2011) assume that all agents publicly announce at the beginning of every period who they are willing to provide favors to. This captures the idea that information diffusion in a small village is fast relative to the frequency of transactions. However, Jackson, Rodriguez-Barraquer, and Tan (2011) agree that even in their setup, enforcement using market-wide bang-bang strategies does not provide the correct criteria for predictive purposes.

5.6.2. *Partly observed networks.* A good alternative to the assumption of publicly observable trust relationships is hard to come by. In the language of networks, this amounts to assuming that individuals have incomplete knowledge of the network that they are embedded in. To-date, the few papers that explore games with incomplete knowledge of the network structure make assumptions directly on what agents know about the network, rather than make a connection between interactions and the acquisition of knowledge.²³ This approach provides a reasonable starting point for the analysis. However, a more satisfactory (and challenging) approach requires an analysis of what individuals *can learn* based on their interactions and the information available to them in the market. For example, if over a long period of time an intermediary i intermediates very few interactions between a client c and the agent, then i should be able to infer that (with high probability) there are additional intermediaries connecting c to the agent. As long as the network stays fixed, this inference becomes more accurate the longer the period of time used for the inference. In fact, if the time frame considered is very long, then i may have a pretty accurate idea of the number of other intermediaries connecting c to the agent. More generally, consider a network G and assume that it is sustained for a long period of time. Then, in a NetE, each individual can use the frequencies of her own interactions with different other individuals to learn some attributes of the underlying network structure.

Learning of the network structure may depend on an individual's prior and on the realizations of the stochastic elements in the market (e.g. arrival of interaction opportunities). As a result, analyzing individuals' learning continuously may be intractable. Instead, we next analyze the patterns of interactions in a NetE, and propose an *upper bound* on individuals' knowledge of the network structure based on their observations—we show that as long as an individual starts with a rich enough prior and learns about the network structure only based on her observations in the game, there is some information that over an asymptotically long period of time an individual receives with probability 1. Similarly, there is some information that is never revealed to the individual. We next explain with some more details our notion of observability, which we term *steady-state observability*.

Consider the following hypothetical exercise: assume that beyond their immediate connections, individuals learn about the network structure only from their histories of play. For an individual $j \in V$, let $\mu_j \in \mathcal{M}_j$ if belief μ_j is consistent with Bayes rule, puts strictly positive probability on the true state at time t , and puts probability 1 on the network not changing from time t onwards. Suppose that starting at time t the network does not change and let $K_j^\infty(G)$ be the set of networks (including sets of individuals V) such that for any network $G' \in K_j^\infty(G)$ there exists a belief $\mu_j \in \mathcal{M}_j$ that assigns G' a strictly positive probability after an asymptotically long period of play. We find that all of the networks in the set $K_j^\infty(G)$ have several common characteristics. We call $K_j^\infty(G)$ the set of *steady state believable networks* of individual j (see also Definition 9).

²³See section 1.1 for details.

Being able to characterize what j eventually knows suggests an upper bound on the knowledge that individuals can acquire based only on their observations. Any additional “knowledge” of j must be the product of j ’s beliefs. Example 1 demonstrates how one can derive as a result what individuals can and cannot infer from their observations in a long term play in a fixed network.

Example 1. Consider the network G in figures 5.2 and 5.3, and assume that G is sustained indefinitely. Over a long enough time interval, the event that c_4 has an interaction opportunity with a will occur many times. In a NetE, all three intermediaries will repeatedly offer to intermeditate, and only one offer will be revealed to c_4 each time. Because the revealed offer is chosen at random, after a sufficiently long time interval, c_4 learns that all three intermediaries connect her to the agent. On the other hand, c_4 cannot use her frequencies of interactions and offers received to make inferences about connections between the intermediaries and other clients, or between the agent and other clients. Similarly, client c_1 can infer that he is connected to the agent both directly and via intermediary i_1 . The same reasoning can be applied to derive that the agent eventually learns who she is connected to, and the sets of clients connected to each of the intermediaries who are connected to her.²⁴

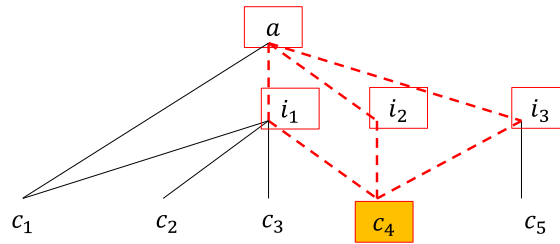


FIGURE 5.2. The knowledge of the network—client c_4 . Suppose that the network in the figure is sustained indefinitely. Rectangles mark the individuals whose identities must eventually be revealed to client c_4 , and dashed lines represent the links that client c_4 must eventually know to exist.

Interestingly, intermediaries must eventually learn about the existence of some interaction paths that do not pass through them. For example, the event that c_4 has an interaction opportunity with the agent occurs an indefinite number of times. Each time, the probability that i_1 gets to intermeditate the transaction is $\frac{1}{3}$. Thus, i_1 eventually learns that there are three distinct interaction paths between c_4 and the agent, and that the three paths involve three distinct intermediaries (one of whom is i_1 herself). Similarly, i_1 is able to learn that c_1 and the agent are directly connected to each other.

The formal result requires additional notation and definitions, which are not used in the remainder of the paper and are therefore deferred to the Appendix together with the result itself (see Definition 9 and Proposition 6). Notably, the intuition from example 1 generalizes and the

²⁴Since we focus on the one agent case, this implies that the agent can learn the entire network structure. For the multi-agent case, see an older draft of this paper—available on the author’s webpage.

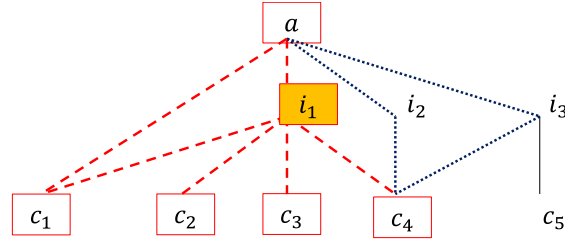


FIGURE 5.3. The knowledge of the network—intermediary i_1 . Suppose that the network in the figure is sustained indefinitely. Rectangles mark the individuals whose identities must eventually be revealed to intermediary i_1 , and dashed lines represent the links that intermediary i_1 must eventually know to exist. Interestingly, i_1 can learn that additional interaction paths connecting c_4 to the agent exist, but cannot learn the identity of the intermediaries along those paths.

following is true for any believable network $G' = \langle V', E' \rangle \in K_j^\infty(G)$: if j is a client then the set of intermediaries connected to j , as well as the links between these intermediaries and the agent, are the same in G' and G ; if j is the agent then the set of intermediaries and clients connected to j , as well as the set of clients connected to any intermediary who is connected to j are the same in G' and G ; and finally, if j is an intermediary then the set of clients (and agent) connected to j are the same in G' and G , and additionally, if j is connected to the agent, then for each client c who is connected to j , c is connected directly to the agent in G if and only if they are connected directly to each other in G' , and if c is not directly connected to the agent in G , then the number of distinct intermediaries who connect c to the agent is the same in G and G' .

For the remainder of the paper, we analyze the case in which individuals observe the network structure as captured by K_j^∞ , that is $\{K_j\}_{j \in V} = \{K_j^\infty\}_{j \in V}$.

5.7. Main result. In this section we characterize the set of $\langle \text{NetE}, \text{ITB} \rangle$ -robust networks (henceforth robust networks). That is, the set of all networks G such that for any belief $\mu \in \text{ITB}(G)$ there exist strategies σ such that (σ, μ) is a NetE and such that if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1.²⁵

²⁵A perhaps more natural set of interest is the set of $\langle \text{NetE}, \text{EB} \rangle$ -robust networks. However, we cannot rule out the existence of belief profiles that are consistent with Bayes rule and make it impossible to sustain any network. For example, consider a client c who is connected to one intermediary i who is in turn connected to the agent. Client c observes very little of the network structure and we cannot rule out the possibility of a belief profile that facilitates only NetE in which c disconnects from i at time 0. Notably, such a belief profile must be very specific and exhibit a particular combination of beliefs of c and i ; otherwise c would trust i to decide based on i 's knowledge. In addition, the fact that this reasoning can eliminate any network precludes any discrimination between networks that are 'more' and 'less' robust. Moreover, we note that by definition, if a belief profile μ is not an ITB of network G , then there is no strategy profile σ such that $(\sigma, \mu) \in \Sigma(G)$. In that sense, the set $\text{ITB}(G)$ is the largest set that can be considered—there is no set of belief profiles that contains a belief profile $\mu' \notin \text{ITB}(G)$ such that network G can be sustained in equilibrium given any belief profile from the set. Finally, Lemma 3 in the appendix teaches us that the set $\text{ITB}(G)$ is never empty. Thus, we do not per-se rule out any network by focusing on ITBs.

5.7.1. *Locally monopolistic networks.* It turns out that robust networks have a particular structure.

Definition 8. An intermediary i is a k -local monopoly in G if for any interaction path cia in G there exist at least k clients whose only interaction path with a passes via i . A network G is k -locally monopolistic if no client is connected to the agent and if all the intermediaries are k -local monopolies in G .

From the definition, if any client is connected directly to the agent, then the network is not k -local monopolistic for any k . Therefore, the network in Figure 5.1 on page 19 is not k -local monopolistic for any k , even though the intermediary in the network is a 5-local monopoly. Figure 5.4 provides an additional example.

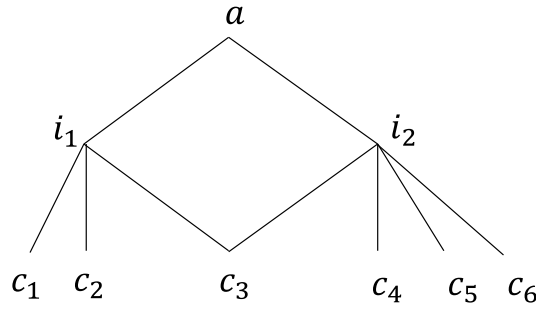


FIGURE 5.4. A 2-locally monopolistic network. Intermediary i_1 is a 2-local monopoly whereas i_2 is a 3-local monopoly. Consequently, the network as a whole is 2-locally monopolistic (but not 3-locally monopolistic).

Recall that $m^* = \min \left\{ n \in \mathbb{N} \mid \left(1 + \frac{1}{\rho} n \right) \pi_A^+ \geq \pi_A^{++} \right\}$. The following result provides a complete characterization of robust networks.

Theorem 1. *A network is robust if and only if it is m^* -locally monopolistic.*

Theorem 1 implies that there are no direct links between clients and the agent in any robust network. That is, all interactions must be intermediated. In fact, Theorem 1 provides a more complete picture. In order for an intermediary i to successfully incentivize the agent to cooperate, i must provide the agent with unique access to at least m^* clients.

Theorem 1 provides new insights that are related to the discussion of the optimal size of intermediaries in financial and other markets: in our model, an intermediary is effective (and needed) if she provides the agent with a sufficient stream of interactions that would not have been accessible to the agent otherwise. Thus, it is not the absolute size of an intermediary, or the overall number of interactions that she intermediates. It is rather the exclusivity over a sufficient number of interaction paths. Such exclusivity can obviously be achieved by a large intermediary who controls a large fraction of the market—such as Airbnb.com in the short term

apartment rentals market. However, there are several other ways of achieving sufficient exclusivity over interaction paths. For example, an intermediary can require that any agent who she works with does not work with additional intermediaries, or that her clients are exclusive to her—this is the case for real estate agents in many states. Alternatively, an intermediary who cannot require exclusivity contracts can specialize on a particular niche market, which is not served by bigger intermediaries in the market—small financial institutions often specialize in serving particular sectors, and the same segmentation is observed in the short-term apartment rentals, where HomeAway Inc. owns several separate websites including HomeAway.com and VRBO.com. The discussion boards of HomeAway Inc. reveals that it is agreed that these websites serve different communities:

“The overwhelming majority of travelers use either VRBO or HomeAway, not both.” (Laura, a HomeAway Inc. representative on 'Community HomeAway')²⁶

5.7.2. *Implications of Theorem 1.* A straightforward implication of Theorem 1 is an upper bound on the number of active intermediaries (that is, intermediaries who are connected to the agent and at least one client) that is related to the primitives of the market through m^* .

Corollary 1. *The number of (active) intermediaries in any robust network is at most $\lfloor \frac{C}{m^*} \rfloor$.*

Comparative statics on the maximal number of intermediaries as well as the predicted levels of exclusivity in the market are then directly implied by the comparative statics on $m^* = \min \left\{ n \in \mathbb{N} \mid \left(1 + \frac{\lambda}{\rho} n \right) \pi_A^+ \geq \pi_A^{++} \right\}$. Therefore, the required level of exclusivity is higher, and the maximal number of intermediaries is smaller when: (1) the frequency of arrival of interaction opportunities (λ) is lower; (2) the agent is more impatient (high ρ); (3) the agent's payoff from a cooperative interaction (π_A^+) is lower; and (4) the agent's payoff from defecting (π_A^{++}) is higher. Notably, m^* does not depend on the size of the market, as captured by $|C|$ and $|I|$.

An additional implication of Theorem 1 is that even a reputation system which records and publicly announces any default of the agent cannot replace an intermediary, who is required to know that she has a sufficient level of exclusivity. We make this point formal in section 7.4 when we discuss the role of credit rating agencies in financial markets. For now we just note that to replace the role of an active intermediary, a reputation system must incorporate into the information revealed to consumers also information on the patterns and frequencies of interactions (i.e., the network structure).

In the example of Airbnb.com and the market for short-term apartment rentals (see section 2), clients are guests, the agent is a host, and Airbnb.com, Craigslist.org, HomeAway Inc., etc. are intermediaries. Our model assigns a special value to the intermediary observing the transactions and their outcomes (e.g., whether the security deposit was claimed by the host). This

²⁶<https://community.homeaway.com/message/3241>

may account for the success of Airbnb.com in attracting guests and hosts that could not overcome the incentive issues on their own in order to transact via Craigslist.org.

The security deposit is captured in our model by $\pi_A^{++} - \pi_A^+$ which is the additional profit a host can gain by defecting and charging the security deposit even if no damage was done to the property. The larger the security deposit, the higher level of exclusivity Airbnb.com is required to have to deter hosts from defecting. The maximal security deposit sustainable with no defections is also increasing in the frequency with which guests wish to rent the property of a host, and decreasing in the outside option of the host (e.g., rent in the long-term apartment rentals market). The opposite is true for the minimal requirement on the level of exclusivity. In addition, higher rental fees (captured by π_A^+) allow for higher security deposits for the same level of intermediary's exclusivity. This is good news, because properties requiring larger security deposits are often rented for higher fees.

In section 7 we apply our analysis to financial markets with lenders/investors and borrowers/entrepreneurs and derive a range of testable implications.

5.7.3. Sketch of the proof of Theorem 1. A subtle observation is at the heart of the proof of Theorem 1: if at any time $t > 0$ an intermediary i is an m^* -local monopoly, then in NetE, i must keep the link with the agent indefinitely, and the agent never defects in an interaction intermediated by i . This is true because [1] if an intermediary is an m^* -local monopoly, this is common knowledge to the intermediary and the agent; and [2] given that links are eliminated only by individuals who observe a defection or a change in the network, an intermediary can effectively signal to the agent that she is willing to intermediate interactions with the agent as long as the agent does not defect in an interaction intermediated by her. The formal claim is available in Lemma 4 in the Appendix.

Returning to Theorem 1, it is now relatively simple to show that m^* -locally monopolistic networks are robust. More challenging is to show that *only* m^* -locally monopolistic networks are robust. The reason that this is true lies in the knowledge of the network structure that intermediaries (do not) have. This is demonstrated in Example 2.

Example 2. *Consider the two networks in Figure 5.5 and assume that the parameters of the model are such that $m^* = 3$. Note that the two networks are observationally identical from the point of view of intermediary i_1 , i.e. they induce the same $K_{i_1}(\bullet)$. If all individuals would have known that they were in the leftmost network, there would have existed a NetE in which the leftmost network is sustained indefinitely. In such a NetE, if i_1 defected in an interaction intermediated by i_1 then i_2 would have observed the elimination of the link i_1a and known that in the remaining network a has access to liquidity only from two lenders. Thus i_2 would have eliminated the link i_2a in response.*

However, this is not the case for all beliefs. Consider a belief such that i_1 and all of the clients ($c_1 - c_3$, as well as the hypothetical c_4 and c_5) assign probability 1 to the network being the rightmost network (in which i_2 is a 3-local monopoly). Now, if the network was indeed the rightmost network, then if c_3 , c_4 , and c_5 would keep their links at time $t = 0$, and if i_2 would keep her link to a at any time t , then the best strategy of the agent is to defect in an interaction intermediated by i_1 . To verify that the leftmost network is not robust, it is then sufficient to show that in any NetE in which the leftmost network is sustained, it is also the case that if the network is the rightmost one then all of the clients connected to i_2 keep their links at $t = 0$, and that i_2 keeps her link to a at any time t .

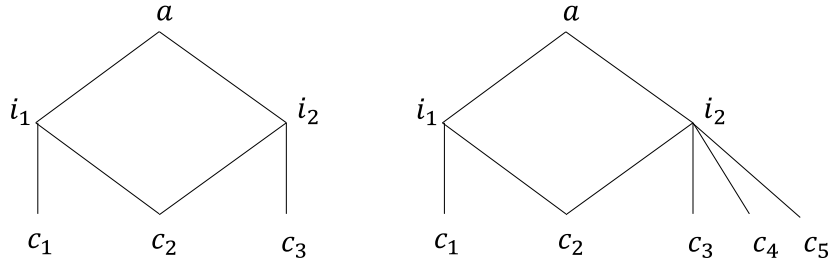


FIGURE 5.5.

6. WELFARE

As demonstrated in section 3.1, a market with no intermediaries allows for successful interactions only if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ \geq \pi_A^{++}$. Therefore, intermediaries enhance welfare in any economy in which interactions provide the agent with larger temptations to defect, that is, if $\left(1 + \frac{\lambda}{\rho}\right)\pi_A^+ < \pi_A^{++}$. Moreover, intermediaries with higher levels of exclusivity allow for a larger range of interactions in terms of the agent's temptation to defect.

The latter also implies that a network with fewer and more exclusive intermediaries is better able to handle economy wide shocks that impact m^* . For example, in an economic downturn, the frequency of interactions (λ) may go down. As a result, intermediaries with low levels of exclusivity will no longer be able to serve the market, leading to a multiplier effect through the disconnect between clients and agents who were able to interact only via these intermediaries.

However, in a richer environment this welfare boost has also a cost in terms of the resilience of the market to the collapse of any single intermediary. To see how, suppose that for any intermediary there is a separate Poisson arrival process with a small parameter $\underline{\lambda}$ such that an arrival means that the intermediary goes out of business for an exogenous reason (with a small enough $\underline{\lambda}$ our analysis above carries through with only minor changes). The collapse of an intermediary in a k -locally monopolistic network inevitably leads to a disconnection between at least k clients and the agent and to the loss of all transactions between them. In other words, high levels of exclusivity imply that a collapse of an intermediary shuts down a large segment of

the market, making the market less resilient to exogenous shocks. This tradeoff is related to the discussion of *too-big-to-fail* financial institutions which we get back to in section 7 below.

Another relevant comparison is how close does intermediation bring us towards the first best—a market in which all transactions take place, regardless of the temptation to defect and with no intermediaries. Theorem 1 shows that as long as $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$ then a single intermediary is able to guarantee that all interactions take place. Thus, abstracting from the issue of resilience, intermediation replicates the first best as long as $\pi_C^+ + \pi_I^{++} \geq \pi_C^{++}$. That is, as long as intermediation is not costly. This will be the case if there are additional cost advantages to intermediation. Otherwise, it is more plausible that an intermediary incurs some cost, at least in terms of effort or opportunity costs.

7. FINANCIAL INTERMEDIATION

We now apply our model to a simple market with investors and entrepreneurs (or lenders and borrowers). We show that the level of exclusivity required from an active intermediary is a function of the frequency of arrival of investment opportunities, their riskiness and their expected returns, as well as of the risk free interest rate through its effect on the division of surplus between investors and entrepreneurs. We also derive predictions on how the exclusivity requirement for an intermediary is affected by self-financing requirements and the availability of collateral. For example, we are able to show that the availability of full collateral is likely to undermine the role of intermediaries for the **riskiest and safest investments**, and intermediaries are likely to continue trading the **intermediately risky assets** without collateral. We follow up with an analysis of the role of central credit information agencies (a generalized version of credit rating agencies), and conclude the section with an extension of our model that allows for price competition between intermediaries and bargaining between investors and entrepreneurs over the division of surplus.

7.1. A model of investments in risky assets. An entrepreneur comes up stochastically over time with investment opportunities, which require one unit of liquidity and have stochastic returns: with probability q the return is $\frac{1+r}{q}$ (for some $r > 0$) and with probability $1 - q$ it is zero (the parameter q affects both the probability of a successful investment and the return on a successful investment and $\frac{1}{q}$ be thought of as the volatility of the risky asset/investment. An investor comes up stochastically over time with liquidity (e.g., has funds released from previous investments). The event that an entrepreneur has an investment opportunity and at the same time an investor has unit liquidity occurs according to a Poisson arrival process with parameter λ . Intermediaries are agents who may obtain funds from investors and invest them with the entrepreneur, possibly at some cost.

We follow much of the literature on contract theory and on financial intermediation and assume limited liability and limited availability of collateral.²⁷ Therefore, to satisfy liquidity constraints repayment must be at least partially conditioned on investment outcome.²⁸

For simplicity, assume for now *fixed-rate equity contracts* that take the following form. Suppose that at time t investor c invests one unit of liquidity with the entrepreneur, then:

- (1) If c invests directly with the entrepreneur, and if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then the entrepreneur is required to pay $\frac{1+\phi r}{q}$ to c .
- (2) If c invests via an intermediary i , and if the investment opportunity has a positive outcome, then the entrepreneur is required to pay $\frac{1+\phi r}{q}$ to i who upon receiving the payment is required to pay $\frac{1+\phi r - r_I}{q}$ to c , and keep to herself an intermediation fee of $\frac{r_I}{q}$.²⁹
- (3) If the investment opportunity has a negative outcome, then the entrepreneur is not required to pay to c or i .

In section 7.5 we extend the model to allow for price competition and bargaining in determining contract rates.

Outcomes of investment opportunities are not verifiable. Therefore, the aforementioned contracts are not enforceable by court, and (as in Hart and Moore 1998 and Bolton and Scharfstein 1990) there is room for *strategic default*—e.g., a manager may divert funds from the investment to herself, preventing the firm from repaying a loan. Money transfers are verifiable and enforceable by court. Thus, intermediaries cannot strategically default.³⁰

Letting π_A^+ and π_A^{++} be random variables that capture the payoffs of the agent given a successful investment and under cooperation and defection respectively, we get that Theorem 1 applies, with

$$(7.1) \quad m^* = \min \left\{ n \in \mathbb{N} \mid \frac{\lambda}{\rho} n(1-\phi)r \geq \frac{1+\phi r}{q} \right\}.$$

7.2. Exclusive intermediation and the economy. Theorem 1 provides sharp predictions with respect to the structure of the financial network in terms of $m^* = m^*(\rho, \lambda, \phi, r, q)$. A larger m^* implies that intermediaries are required to have higher levels of exclusivity in order to enforce repayment.

²⁷The important assumption is that full collateral is costly. In section 7.3 we generalize our model and study the roles of self-financing and of collateralized debt. Our results remain qualitatively intact.

²⁸Debt contracts are also possible. However, in this case there is a difference between a liquidity default that occurs when an entrepreneur does not have the funds to repay a loan, and a strategic default that occurs when an entrepreneur refuses to pay a loan despite having the funds (see also Bolton and Scharfstein 1990). We focus on strategic default.

²⁹Liquidity constraints imply that $\phi \in [-\frac{1}{r}, 1]$, and $r_I \in [0, 1 + \phi r]$.

³⁰It is often assumed in the finance literature that strategic default is observable to the manager and the investors. However, it is acknowledged that default may not be observable to outside parties. Thus, it may be impossible to prove that strategic default took place. On the other hand, money transfer across firms leave a paper / electronic communication trail. See also Bolton and Scharfstein (1996), and Babus (2010) for similar assumptions.

Notably, m^* is decreasing in the frequency of the arrival of investment opportunities and liquidity (λ), the expected return on a successful investment (r), and in the entrepreneur's share of the profit from investment ($1 - \phi$). On the other hand, m^* is increasing in the volatility of investments ($\frac{1}{q}$) and the entrepreneur's discount rate (ρ).

These comparative statics have a wide range of implications. For example, older or less established entrepreneurs, who may have shorter expected horizons as entrepreneurs (and thus larger effective ρ) are trusted only by intermediaries with high levels of exclusivity, whereas established young to mid-career entrepreneurs are trusted also by intermediaries with lower levels of exclusivity as captured by m^* . Similarly, entrepreneurs who are considered to have higher expected rates of arrival of investment opportunities, or investment opportunities with higher expected returns or lower volatilities (higher λ , r , and q respectively) can be trusted by intermediaries with a wider range of levels of exclusivity.

From a macroeconomic perspective, the model predicts that in times of economic booms (high λ and r) there is room for a large number of intermediaries with lower levels of exclusivity. The same is true for highly liquid markets (e.g., low interest rates) in which the entrepreneur expects to receive a larger share of the profit ($1 - \phi$), as well as for markets with lower volatility (higher q). On the other hand, in times of economic downturns, the model predicts that a small number of intermediaries may serve to mitigate the crisis by preventing a multiplier effect driven by the loss of trust. This prediction may be intensified if the economic downturn is triggered by a liquidity crunch and increases the bargaining power of the investors (as captured by ϕ).

7.3. Self-financing, collateralized debt, and bankruptcy. So far, we focused on the stylized case in which investors assume the entire risk of the investment opportunity. However, in some financial markets, investors may require that an entrepreneur (or other borrowers) pledge full or partial collateral, and bankruptcy rules make investors the residual claimants of a firm's assets. It is also common that investors require that an entrepreneur self-finance a part of the investment opportunity from his personal funds or through a previous investment of family and friends. These measures often come with substantial efficiency costs. For example, liquidating collateral may be costly,³¹ and there is often a gap between the value of a firm's assets to the entrepreneur and the corresponding value to investors (which is often simply the resale value of the assets).³²

In this section, we evaluate the effect of the availability of self-financing, full and partial collateral, and bankruptcy laws on the set of robust networks. We find that the aforementioned

³¹Pledging collateral may also block relatively liquid funds that can otherwise be invested at a positive return elsewhere.

³²See also Bolton and Scharfstein (1996) and Kiyotaki and Moore (1997).

instruments do not change our results qualitatively. However, they do affect the level of exclusivity that an intermediary is required to have in order to enforce repayment. More specifically, allowing an entrepreneur to self-finance a positive fraction of the investment opportunity reduces the level of exclusivity required from intermediaries in order to enforce repayment. The same is true for the availability of partial collateral and the presence of bankruptcy laws. On the other hand, introducing the possibility of pledging full collateral increases the level of exclusivity that intermediaries need in order to enforce repayment of uncollateralized investments. Therefore, if the cost of pledging full collateral is sufficiently low, the market may revert to simple debt contracts even when equity-like contracts are more efficient. Our analysis in this section requires extending Assumption 2 in the obvious way.

7.3.1. Self-financing. In this section, we extend our model to allow for an industry norm, or a policy mandate, requiring a level of self-financing. We show that partial self-financing allows for the existence of intermediaries with lower levels of exclusivity, and that all of our comparative statics extend immediately to this setup. Extending the model to allow for heterogeneous self-investment clauses is also discussed.

Consider the following contract with self-investment: suppose that investor c has liquidity and the entrepreneur has an investment opportunity. Then, c invests a fraction $\psi \in (0, 1)$ of liquidity under the condition that the entrepreneur invests the remainder $(1 - \psi)$ from his personal funds. The details of the contract follow the ones suggested in section 7.1, scaled down by the factor ψ . I.e., if c invests the money directly, and if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then the entrepreneur is required to pay $\psi \left(\frac{1+\phi r}{q}\right)$ to c . Similarly, if c invests the money via an intermediary i who invests on his behalf, and if the investment opportunity has a positive outcome then the entrepreneur is required to pay $\psi \left(\frac{1+\phi r}{q}\right)$ to i who upon receiving the payment is required to pay $\psi \left(\frac{1+\phi r - r_I}{q}\right)$ to c , and keep to herself an intermediation fee of $\frac{\psi r_I}{q}$.³³ Let χ be such that $1 > \chi > \phi$ and assume that the entrepreneur has a cost of $1 + \chi r$ per one unit of self-financing (e.g., cost of liquidating illiquid assets), and that it is not feasible for the entrepreneur to take the investment opportunity with 100 percent self-financing—e.g., due to limited personal wealth or need for diversification (allowing the entrepreneur to fully self-finance at some cost is qualitatively identical to the case in which the entrepreneur has an illiquid asset that can be used as full collateral with some cost of liquidation—see section 7.3 below).

We say that a network is $\text{robust}_{|\psi, \chi}$ if it is robust given that contracts have $1 - \psi$ self-financing at cost $(1 - \psi)(1 + \chi r)$. The following Corollary is a direct implication of Theorem 1.

Corollary 2. *Let $m^{\psi, \chi} = \min \left\{ n \in \mathbb{N} \mid \frac{1}{\rho} \cdot n \cdot \lambda \cdot [\psi(1 - \phi) + (1 - \chi)(1 - \psi)] r \geq \psi \left(\frac{1 + \phi r}{q}\right) \right\}$. Then, a network is $\text{robust}_{|\psi, \chi}$ if and only if it is $m^{\psi, \chi}$ -locally monopolistic.*

³³Liquidity constraints and investors' incentive constraints imply that $\phi \in [-\frac{1}{r}, 1]$, and $r_I \in [0, \phi r]$.

Corollary 2 suggests that self-financing allows for the existence of smaller intermediaries with lower levels of exclusivity, and that all of our results and comparative statics extend immediately substituting $m^{\psi, \chi}$ for m^* .

Extending our analysis to endogenously determined fractions of self-financing would require additional structure. This is because one needs to understand the entrepreneur's reasons to seek external funding to begin with. If the reason is limited liquid balance then an entrepreneur always prefers to self-finance the largest fraction possible given her constraint. On the other hand, if there is an efficiency cost for using self-financing, e.g., cost of liquidating illiquid assets, then under some specifications an entrepreneur prefers the lowest fraction of self-financing.

7.3.2. Collateralized debt and bankruptcy. An additional widely used instrument for preventing strategic default is the use of a firm's (potentially illiquid) assets as collateral. This can be done explicitly by assigning collateral to the loan itself, or implicitly by endowing investors with the power to compel the firm to announce bankruptcy and distribute its assets between stake holders.

Assume that at any point in time, the entrepreneur has one illiquid asset, with a discounted present value of V_A to the entrepreneur and V_C to the investor. Following Kiyotaki and Moore (1997), we assume that $V_A > V_C$. Thus, using a collateral is costly and generates an efficiency loss of $V_A - V_C$ whenever the collateral is transferred to the investor.

Consider the following contract: suppose that investor c has liquidity and the entrepreneur has an investment opportunity. Then, if c invests directly, and if the investment opportunity has a negative outcome, then the entrepreneur is required to give the collateral to c , whereas if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then the entrepreneur is required to pay $\frac{1+\phi r - (1-q)V_C}{q}$ to c . If c invests via an intermediary i , and if the investment opportunity has a negative outcome, then the entrepreneur is required to give the collateral to c , whereas if the investment opportunity has a positive outcome, then the entrepreneur is required to pay $\frac{1+\phi r - (1-q)V_C}{q}$ to i who upon receiving the payment keeps to herself an intermediation fee of $\frac{\Gamma}{q}$ and transfers the remainder to c .³⁴

We say that a network is robust $_{|V_A, V_C}$ if it is robust with respect to contracts parametrized by V_A and V_C . The following Corollary is a direct implications of Theorem 1.

Corollary 3. *Let $m^{V_A, V_C} = \min \left\{ n \in \mathbb{N} \mid \frac{1}{\rho} \cdot n \cdot \lambda \cdot ((1-\phi)r + (1-q)(V_C - V_A)) \geq \frac{1+\phi r - (1-q)V_C}{q} \right\}$. Then, a network is robust $_{|V_A, V_C}$ if and only if it is m^{V_A, V_C} -locally monopolistic.*

³⁴To see how the repayment sum upon successful investment $\left(\frac{1+\phi r - (1-q)V_C}{q}\right)$ is calculated, note that $q \frac{1+\phi r - (1-q)V_C}{q} + (1-q)V_C = 1 + \phi r$. Thus, conditional on not strategically defaulting, the expected payment by an entrepreneur did not change under this specification relative to the main analysis of this paper. Allowing investors and intermediaries to demand lower shares due to the reduction in uncertainty does not change our analysis as long as it does not eliminate completely the expected loss to the entrepreneur from needing to liquidate an asset worth V_A in order to pay V_C .

Using collateral enables a larger set of networks to be robust if and only if $m^* > m^{V_A, V_C}$. On the other hand, if $m^* \leq m^{V_A, V_C}$ then using a collateral has efficiency cost with no additional benefits in terms of enforcement. Whether $m^* > m^{V_A, V_C}$ or $m^* \leq m^{V_A, V_C}$ depends on two countervailing effects of the use of collateral. First, collateral decreases the immediate benefit of an entrepreneur from strategically defaulting. Second, the need to pledge collateral in the future reduces the expected future payoffs of the same entrepreneur. Evaluating the expressions for m^* and m^{V_A, V_C} reveals that collateral enhances investment when the collateral is valuable and induces little expected efficiency loss in any given transaction (that is, when V_C is large and $V_A - V_C$ is small). The following simple cases provide additional intuition:

- (1) If $V_A = V_C$ then collateral always allows for intermediaries with (weakly) lower levels of exclusivity.
- (2) If $V_C \geq 1 + \phi r$ and $V_A - V_C < \frac{(1-\phi)r}{1-q}$, then collateral allows for direct investment, with no role for intermediation.

In the latter case, fully collateralized debt contracts are available. However, as long as $V_A > V_C$, collateralized debt contracts come with an efficiency loss. This raises new questions which we study in the next section.

7.3.3. Fully collateralized debt. In this section we focus on the case where $V_C \geq 1 + \phi r$ and $0 < V_A - V_C < \frac{(1-\phi)r}{1-q}$, so that fully collateralized debt contracts are available but carry an efficiency cost. We answer the following questions: what networks can sustain repayment in a robust and efficient (i.e. no collateral) manner? Or more generally, how does the availability of fully collateralized debt contracts affect the market's ability to enforce efficient investment contracts that require no collateral? This question is related to another question that has a long history in the financial literature, namely: which markets do we expect to transact mostly on collateral, and which markets do we expect to transact based on long term relationships (potentially using intermediaries for enforcement).³⁵

In the presence of fully collateralized contracts, it is a dominant strategy for any investor to agree to invest using collateral. Thus, an entrepreneur who is refused an uncollateralized investment via the network will nevertheless be able to get funding in the form of fully collateralized debt. This increases the outside option of the entrepreneur and makes it more difficult to enforce the repayment of investments made with no collateral.

Corollary 4. Let $\bar{m} = \min \left\{ n \in \mathbb{N} \mid \frac{1}{\rho} \cdot n \cdot \lambda \cdot (1-q)(V_A - V_C) \geq \frac{1+\phi r}{q} \right\}$. Then, a network is robust if and only if it is \bar{m} -locally monopolistic.³⁶

³⁵See also Hall and Lerner (2009), Babus (2010), and references therein.

³⁶The expression for \bar{m} is an algebraically simplified representation of $\bar{m} = \min \left\{ n \in \mathbb{N} \mid \frac{1}{\rho} \cdot n \cdot \lambda \cdot (1-\phi)r \geq \frac{1+\phi r}{q} + \frac{1}{\rho} \cdot n \cdot \lambda \cdot \left(q \frac{1+r}{q} + (1-q)(V_A - V_C) - (1+\phi r) \right) \right\}$.

Reducing the cost of pledging collateral ($V_A - V_C$) has two effects: on the one hand, it makes fully collateralized investments more efficient. On the other hand, Corollary 4 implies that it also makes it more difficult to enforce efficient uncollateralized investments.

In addition, Corollary 4 implies that the ability to transact without collateral can be traced back to the riskiness of the underlying investment opportunity. In particular, holding fixed the expected return on investment ($1 + r$), a higher probability that the investment succeeds implies that the expected cost of pledging collateral $(1 - q)(V_A - V_C)$ is lower, which makes it more difficult to enforce repayment without collateral. At the same time, a higher probability that the investment succeeds implies that the expected temptation to default $\left(\frac{1+\phi r}{q}\right)$ is lower, which makes it easier to enforce repayment without collateral. Summing over these two effects produces the following non-monotonic result.

Corollary 5. *Consider two economies $\mathcal{H}_1 = (\rho, \lambda, \phi, r, q_1, V_C, V_A)$ and $\mathcal{H}_2 = (\rho, \lambda, \phi, r, q_2, V_C, V_A)$ such that $q_1 > q_2$. Then, there exists a threshold $\bar{q}(\rho, \lambda, \phi, r, V_C, V_A) \in (0, 1)$ such that:*

[1] *if $q_1, q_2 < \bar{q}$ then if a network G is robust in economy \mathcal{H}_2 , then G is also robust in economy \mathcal{H}_1 ; whereas*

[2] *if $q_1, q_2 > \bar{q}$ then if a network G is robust in economy \mathcal{H}_1 , then G is also robust in economy \mathcal{H}_2 .*

That is, given a fixed network, Corollary 5 implies that **the riskiest and safest investments are made using collateral, whereas intermediately risky investments are made by intermediaries and without collateral.**

7.4. Credit rating agencies and generalized central information entities. Standard credit rating agencies and bureaus have no explicit role in the setting considered in this paper. Credit rating agencies generally rate the *ability of a debtor to repay*, rather than his *incentives to repay*. That is, credit rating agencies focus on enabling creditors to avoid suffering from *liquidity defaults*, rather than from *strategic defaults*. Similarly, credit bureaus do not distinguish between liquidity and strategic defaults.

Therefore, in this section we go beyond the standard credit bureaus and credit rating agencies. We consider (hypothetical) Generalized Central Information Entities (GCIEs) and their effect on the set of robust networks. We show that given plausible capabilities, a GCIE can relax the constraints imposed by the m^* -local monopolism requirement. However, as long as the frequencies of arrival of liquidity and investment opportunities are sufficiently low (so that $m^* > 1$), it is still the case that in all robust networks any investor and entrepreneur who are network related must also be connected to at least one intermediary in common. Also, any intermediary must be connected to at least m^* investors or to none ($\forall_{ci \in G} |C_i^G| \geq m^*$).

Clearly, if a GCIE can observe the entire network of financial connections instantaneously at any moment in time, it can facilitate ostracism and the result captured by Proposition 3 is

recovered: *a network G is robust if and only if the entrepreneur is network related to at least m^* investors or to none ($\forall_{c \in a^G} |S_a^G| \geq m^*$).* To enforce repayment, the perfectly informed GCIE gives the highest rating to an entrepreneur who is financially related to at least m^* investors and never strategically defaulted in the past, and the lowest rating otherwise. This facilitates the bang-bang strategy profile used for proving Proposition 3—a single strategic default by an entrepreneur leads to the elimination of all of the links in the network.

On the other extreme, if a GCIE cannot observe the network structure and cannot verify that a lack of repayment is indeed due to strategic default, then our main result (Theorem 1) goes through without change: *the set of robust networks and the set of m^* -locally monopolistic networks are identical.*

The more interesting case is a GCIE that can perfectly observe a strategic default whenever such a default occurs, yet does not have the necessary information to map the financial network reliably. Consider a GCIE such that whenever an entrepreneur strategically defaults, it is announced by the GCIE. Proposition 4 offers partial characterization of the set of robust networks in the presence of such a GCIE.

Proposition 4. *Consider the model with a GCIE that publicly announces any default. Then, a network G is robust only if*

[1] *If an investor c is connected directly to the entrepreneur (ca^G), then there exists an intermediary i who is connected to both c and the entrepreneur; and*

[2] *Every intermediary i , who is connected to at least one investor and to the entrepreneur, is connected to at least m^* distinct investors in G ($\forall_{c \in a^G} |C_i^G| \geq m^*$).*

Proposition 4 establishes that even in a model with a GCIE who publicly announces any default, robustness requires that intermediaries exist and be well connected.

7.5. Pricing and market structure. In this section we explore the connection between the details of the investment contracts with respect to prices (i.e., shares of returns) and the set of robust financial networks. First, we verify that our results are *not* artifacts of the fixed price contracts; they carry over to an environment in which intermediaries compete in prices à la Bertrand for each investment. Second, we show that when the division of surplus between investors and the entrepreneur follows the Nash bargaining solution, the set of robust financial networks may include also networks in which intermediaries are local duopolies, not only local monopolies.

7.5.1. Price competition among intermediaries. In this section we verify that Theorem 1 extends to an environment in which intermediaries compete in prices à la Bertrand. Formally, suppose that when an intermediary i who is connected to client c informs him that she is able to intermediate the transaction, i also chooses a fee $\frac{r_i(h_i)}{q}$ for the transaction, where $r_i(h_i) \in [r_{min}, r_{max}]$ where r_{min} is the break-even point for the intermediary (zero profit) and $r_{max} <$

ϕr .³⁷ To accommodate price competition, we modify the game such that the intermediation offer revealed to an investor must be from the set of offers with the lowest intermediation fee.

A *Bertrand NetE* is a NetE in which intermediaries choose prices in the following way. Suppose that at time t investor c has liquidity and the entrepreneur has an investment opportunity. Suppose further that intermediary i is connected to both c and the entrepreneur. Then, i sets her fee as follows:

$$(7.2) \quad r_i(h_i) = \begin{cases} r_{min} & \text{if there exists an intermediary } i' \neq i \text{ such that } ci' a^G \\ r_{max} & \text{otherwise} \end{cases}.$$

A network G is *Bertrand-robust* if for any belief profile $\mu \in ITB(G)$ there exists a Bertrand NetE such that starting with $G^0 = G$, the network G is sustained indefinitely with probability 1. The proof of Corollary 6 follows the same argument as the proof of Theorem 1 and is omitted.

Corollary 6. *A network is Bertrand-robust if and only if it is m^* -locally monopolistic.*

7.5.2. *Bargaining and local duopolies.* We now show that considering richer bargaining outcomes between investors and the entrepreneur has the potential to enrich our results in an interesting way: if we allow an investor and an entrepreneur to divide their surplus according to a generalized Nash bargaining solution, the set of robust networks may include networks in which some intermediaries have sufficient *duopolistic* exclusivity, even if they lack any full exclusivity.

Formally, suppose that when an interaction opportunity is revealed (with or without an intermediation offer with a fee $\frac{r_i(h_i)}{q}$), the investor and entrepreneur bargain à la Nash on the terms of the investment: $\phi_{ca}(h_c, h_a)$ for a direct investment, and $\phi_{cia}(h_c, h_a, r_i)$ for an intermediated investment.

A generalized Nash bargaining solution requires that there is some $\bar{\phi}$ such that, upon successful investment, the entrepreneur pays to the intermediary

$$(7.3) \quad \frac{1 + \bar{\phi}(r - r_i) + r_i}{q},$$

and the intermediary transfers to the investor

$$(7.4) \quad \frac{1 + \bar{\phi}(r - r_i)}{q}.$$

If the investment is made directly, then the same applies with the fee set to $r_i = 0$.

³⁷We define $r_i(h_i)$ to belong to a closed set to allow for the existence of an equilibrium in which intermediaries compete in prices à la Bertrand. The additional restriction that $r_{max} < \phi r$ allows us to abstract from indifferences on the investors' side of the market. An alternative approach that yields identical results without restricting $r_i(h_i)$ and r_{max} is to model money using a discrete variable.

A *Bertrand NetE with Nash bargaining* is a NetE in which any intermediary determines her fee according to (7.2), and repayment is done according to (7.3) and (7.4) for some $\bar{\phi}$ such that $\bar{\phi}(r - r_{max}) + r_{max} \leq r$. A network G is then *Bertrand-robust with Nash bargaining* if for any belief profile $\mu \in ITB(G)$ there exists a Bertrand NetE with Nash bargaining such that starting with $G^0 = G$, the network G is sustained indefinitely.

With a slight modification of m^* to incorporate $\bar{\phi}$ and r_{max} instead of ϕ in equation (7.1) it is still true that m^* -locally monopolistic networks are Bertrand-robust with Nash bargaining. In addition, since the entrepreneur pays lower intermediation fees when he has more than one investment path to a given investor, the entrepreneur also values a relationship with an intermediary who has sufficient duopolistic exclusivity—an intermediary who is a part of a large number of investment paths; each being one of exactly two intermediated paths connecting an investor to the entrepreneur. Formally, let

$$\begin{aligned}
(7.5) \quad M^D &= M^D(\rho, \lambda, r, p, c, \bar{\phi}, r_{max}) \\
&= \left\{ (n^M, n^D) \in \mathbb{N}^+ \times \mathbb{N} \mid \frac{1}{\rho} \cdot n^M \cdot \lambda \cdot (1 - \bar{\phi})(r - r_{max}) + \right. \\
&\quad \left. + \frac{1}{\rho} \cdot n^D \cdot \lambda \cdot (1 - \bar{\phi})(r_{max} - c) \geq \frac{1 + \bar{\phi}(r - r_{max}) + r_{max}}{q} \right\} \cup \\
&\quad \cup \left\{ (0, n^D) \in \{0\} \times \mathbb{N} \mid \frac{1}{\rho} \cdot n^D \cdot \lambda \cdot (1 - \bar{\phi})(r_{max} - c) \geq \frac{1 + \bar{\phi}(r - c) + c}{q} \right\}.
\end{aligned}$$

We say that an intermediary i is a (k^M, k^D) -local duopoly in G if for any investment path cia in G there exist at least k^M investors whose unique investment path to the entrepreneur is via i , and at least k^D investors for whom one of exactly two intermediated investment paths to the entrepreneur passes via i . A network G is M^D -locally monopolistic if any intermediary in G is a (k^M, k^D) -local duopoly in G for some $(k^M, k^D) \in M^D$. The proof of the following partial characterization result follows the same argument as Theorem 1 and is omitted.³⁸

Proposition 5. *A network is Bertrand-robust with Nash bargaining only if it is M^D -locally duopolistic and has no direct links between investors and the entrepreneur.*

8. CONCLUSION

This paper proposes a theory of intermediation: an intermediary who exclusively represents a large pool of clients in their transactions with an agent can enforce good behavior by the agent even if the frequency of interactions of the agent with each individual client is low. In a market with many clients and agents such exclusivity can be achieved by exclusivity of the intermediary over an agent's interactions, over the interactions of a sufficiently large number

³⁸For the proof one needs to update Assumptions 1 and 2 with r_{max} and $\bar{\phi}$ in the obvious way.

of clients, or over a sufficient number of interaction paths, with no exclusivity over all of the interactions of any client or agent.

Our analysis shows that incomplete knowledge of the network of the patterns of interaction in a market is key for understanding the role of intermediation in the informal enforcement of contracts. Imperfect observability of network connections prevents clients from relying on community enforcement via ostracism or contagion. Therefore, only intermediaries with sufficient exclusivity (as captured by the notion of m^* -local monopoly) can be certain that they are able to execute cooperative interactions successfully. As a result, intermediation can lead to a welfare improvement by allowing for the execution of transactions that would not have been possible otherwise. This is true even if intermediation is costly.

By emphasizing exclusivity, our analysis suggests that the size of an intermediary (as measured by her connectivity to clients and agents) matters. However, it is not a sufficient statistic to determine whether an intermediary can enforce cooperation. The important measure of the effectiveness of an intermediary is the number of clients whom the intermediary exclusively represents when dealing with a particular agent. In that sense our model of intermediation is global—rather than focusing on a single transaction and a single intermediary, we characterize networks that are robust and show that in any such network all intermediaries have a sufficiently high level of exclusivity. The minimal level of exclusivity required of an intermediary depends on the fundamentals of the market. If the frequency of arrival of interaction opportunities and the expected benefits are high, intermediaries may enforce repayment without holding high levels of exclusivity. However, if interaction opportunities are scarce and carry low expected returns, only intermediaries who have exclusivity over a large number of interaction paths can effectively participate in the market.

This seemingly simple set of observations carries significant implications. For example, we show that our theory sheds light on the rise of Airbnb.com in the market for short-term apartment rentals. Similarly, when applied to financial markets, our theory provides an array of testable implications, including which assets are expected to be traded using collateral and which using incomplete contracts enforced informally by intermediaries.

APPENDIX

Proofs and additional results for section 4

Lemma 1. *Let the underlying network G be any network which enables exclusive intermediation. Then, there exists a cooperative equilibrium if and only if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$.*

Proof of Lemma 1. Let let G be a network which enables exclusive intermediation, and let intermediary i be connected to the agent and to all of the the clients in C .

We first show that there exists a cooperative equilibrium if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$. Consider the following strategies. For clients: interact if and only if receive an intermediation offer from intermediary i and only using that offer. For intermediary i : offer to intermediate whenever an interaction opportunity rises and if and only if the agent never defected in an interaction intermediated by i before. For any intermediary $i' \neq i$: never offer to intermediate. For the agent: cooperate in an interaction if and only if the interaction is intermediated by i . It is straightforward to verify that these strategies are cooperative equilibrium strategies for $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$.

To show that there exists a cooperative equilibrium only if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \geq \pi_A^{++}$, it is sufficient to note that in any cooperative equilibrium, at any time t , an agent who has an interaction has an expected payoff of at most $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+$ from following the equilibrium strategies, and at the same time the agent can guarantee herself a payoff of π_A^{++} if she defects. ■

Lemma 2. *Let the underlying network G be any network which **does not** enable exclusive intermediation. Then, there exists a cooperative equilibrium only if $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ > \pi_A^{++}$.*

Proof of Lemma 2. Let G be a network which **does not** enable exclusive intermediation, and suppose that $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ \leq \pi_A^{++}$. Assume by contradiction that there exists a cooperative equilibrium. Now consider an agent who is required to decide whether to cooperate or defect in an interaction at time t . If $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ < \pi_A^{++}$ then the agent is better off defecting regardless of the network structure and equilibrium strategies, hence we now focus on the case that $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+ = \pi_A^{++}$. If the agent cooperates, she receives an expected payoff of $\left(1 + \frac{\lambda}{\rho} |C|\right) \pi_A^+$ (unless she plans to increase her payoff by defecting in the future, in which case this is not an equilibrium and we are done). We now show that if the agent defects in all of her interactions from time t onward, she receives an expected payoff higher than π_A^{++} , which contradicts the assumption that this is an equilibrium. The agent's immediate payoff at time t from defecting is π_A^{++} . In addition, because the network does not enable exclusive intermediation, there is a positive probability that the subsequent interaction opportunity is with a client c' who is not connected to the intermediary who intermediated in the aforementioned interaction (if there was such an intermediary—if the interaction was direct, it is sufficient that a different client has an interaction opportunity). Thus, c' and any intermediary connected to her will not be informed of the agent's deviation in the previous interaction. Since this is a cooperative equilibrium, the client will interact with the agent either directly or via an intermediary. Thus, the agent receives a positive payoff in addition to the one in the previous interaction. ■

Proofs and additional results for section 5

Proof of Proposition 3. The following strategy profile $\hat{\sigma}$ will be useful in several stages of the proof. For any client c (intermediary i): If at time t the agent is network-related to at least m^* clients in G^t then maintain all of your links and choose to participate (offer to intermediate). Otherwise, disconnect all of your links. For the agent: If at time t the agent is network-related to at least m^* clients in G^t then cooperate in any interaction at time t (perhaps unless the agent has multiple simultaneous interactions, which happens with probability 0). Otherwise, defect in any interaction at time t .

(1) \Rightarrow (4) Consider a network G in which the agent is network-related to fewer than m^* clients. Since the network is perfectly observable, so that for any belief that is consistent with any PBE, the agent knows that and strictly prefer to deviate in any of her interactions.

(3) \Rightarrow (1) This is true because, by definition, if a network G is $\langle \text{NetE}, \text{ITB} \rangle$ -robust then there exists at least one NetE such that if the initial network is $G^0 = G$ then G is sustained indefinitely with probability 1.

(2) \Rightarrow (3) This is true because, by definition, for any network G it is the case that $\text{ITB}(G) \subseteq \text{EB}$.

(4) \Rightarrow (2) We note that any belief $\mu \in \text{EB}$ assigns probability zero to any state of the world that is inconsistent with G^t , thus for any belief $\mu \in \text{EB}$, it is also the case that $(\hat{\sigma}, \mu)$ is a NetE in which if the agent is network-related to at least m^* clients in G , then if the initial network is $G^0 = G$ it is guaranteed that G is sustained indefinitely with probability 1. ■

Definition 9. Suppose that the state is ω^t , that play follows some NetE play, and that the network $G(\omega^t)$ is sustained indefinitely from time t onwards. Consider an individual $j \in V$ and let $\mu_j \in \mathcal{M}_j$ if belief μ_j satisfies the following conditions:

[1] μ_j puts positive probability only on networks in which j is connected to the same other individuals as in $G(\omega)$;

[2] $Pr_{\mu_j^t}(\omega = \omega^t) > 0$ – i.e. at time t , individual j puts positive probability on the true state;

[3] $\forall t' > t Pr_{\mu_j^t}(G^{t'} = G(\omega^t)) = 1 - j$ puts probability 1 on the network not changing anytime after time t ; and

[4] μ_j is consistent with Bayes rule.

Denote by $K_j^{T,\epsilon}(G(\omega^t, \mu_j))$ the set of networks such that $G' \in K_j^{T,\epsilon}(G(\omega^t, \mu_j))$ if at time $T > t$ the network G' is assigned a probability larger than ϵ by μ_j ($K_j^{T,\epsilon}(G(\omega^t, \mu_j)) = \{G' | Pr_{\mu_j}(G^T = G') > \epsilon\}$), and let $K_j^\infty(G(\omega^t)) = \{G' | \exists \mu_j \in \mathcal{M}_j, \exists \epsilon > 0 G' \in \lim_{T \rightarrow \infty} K_j^{T,\epsilon}(G(\omega^t, \mu_j))\}$ be the set of *steady state believable networks* for individual j .

Proposition 6. *The following two statements are equivalent.*

[1] $G' = \langle \langle C', I', a \rangle, E' \rangle$ is a steady state believable network for individual $j \in V$.

[2] All of the following hold:

2a. If $j \in C$, then $ja^{G'}$ if and only if $ja^{G(\omega^t)}$, $I'_j = I_j(\omega^t)$, and for all $i \in I'_j$ it holds that $ia^{G'}$ if and only if $ia^{G(\omega^t)}$;

2b. If $j \in I$, then $C'_j = C_j(\omega^t)$, and $ja^{G'}$ if and only if $ja^{G(\omega^t)}$, plus for each $c \in C_j$ such that $cja^{G(\omega^t)}$ it holds that $ca^{G'}$ if and only if $ca^{G(\omega^t)}$ and if $\neg ca^{G(\omega^t)}$ then $\left| \left\{ i' \in I | ci' a^{G(\omega^t)} \right\} \right| = \left| \left\{ i' \in I | ci' a^{G'} \right\} \right|$;

2c. If $j = a$, then $C'_j = C_j(\omega^t)$, $I'_j = I_j(\omega^t)$, and $C'_i = C_i(\omega^t)$ for any $i \in I_j$.

Proof of Proposition 6. [1] \Rightarrow [2]: Consider a network $G' = \langle \langle C', I', a \rangle, E' \rangle$ such that one of the conditions 2a-2c does not hold. Then it is left to show that for any belief μ_j that is consistent with the requirements of Definition 9, $G' \notin K_j^\infty(G(\omega^t))$. Namely, for any $\epsilon > 0$ there exists $t' > t$ such that $Pr_{\mu_j}(G^{t'} = G') < \epsilon$. For 2a, and 2c, as demonstrated in Example 1, as $t' \rightarrow \infty$ the probability of an event after which $Pr_{\mu_j}(G^{t'} = G') = 0$ goes to one. Finally, as demonstrated

in Example 1, an application of the law of large numbers suggests that the probability of the network being a network that violates 2b goes asymptotically to zero as t' goes to infinity.

[2] \Rightarrow [1]: Consider a belief μ_j that is consistent with the requirements of Definition 9 such that $\mu_j(\omega^t)$ puts probability p on G' and probability $1-p$ on the real network $G(\omega^t)$, i.e. $\forall t' \geq t \Pr_{\mu_j}(G = G') = p$ and $\forall t' \geq t \Pr_{\mu_j}(G = G(\omega^{t'})) = 1 - p$. It is left to verify that for any $t' > t$ and any history of play that is consistent with NetE play (and given that the network is sustained during $[t, t']$), the probability that the history is observed by j given G' equals the probability that the same history is observed by j given $G(\omega^t)$ during the same time period. The verification follows the same steps as in Example 1 and is omitted. ■

Lemma 3. *For any G , there exists an ITB. Moreover, there exists a belief profile μ which is an ITB for all networks ($\exists \mu \forall G \mu \in ITB(G)$).*

Proof of Lemma 3. Let $\bar{\mu}$ be any belief profile that is consistent with the following in any state of the world: For any $j \in C \cup I$, after observing $K_j(G^t)$, $\bar{\mu}_j^t$ puts probability 1 on some network G such that $\forall i' a \in G | i' \in I$ and $i' \neq j | S_a^G | - | S_a^{G-i'a} | > m^*$ (recall definition 7 for m^*).

Let $\bar{\sigma}$ be a strategy profile that is consistent with the following conditions for all G and $c i a \in G$:

- (1) The agent never cooperates in a direct interaction with c (even if $c a^G$) and cooperates in an interaction intermediated by i if $|S_a^G| - |S_a^{G-ib}| \geq m^*$.
- (2) The agent defects in an interaction intermediated by i if $|S_a^G| - |S_a^{G-ia}| < m^*$ and $\forall i' a \in G | i' \neq i | S_a^G | - | S_a^{G-i'a} | > m^*$.
- (3) i maintains her link to a the agent if and only if $|S_a^G| - |S_a^{G-ib}| \geq m^*$.
- (4) c maintains all of her links to intermediaries.
- (5) c does not maintain her link to the agent (if any).

Note that for any network G' and two paths $c i a^{G'}$ and $c' i' a^{G'}$ such that $i \neq i'$ and for a network $G'' = G' - i a$, $|S_a^{G'}| - |S_a^{G'-i'a}| \geq m^*$ only if $|S_a^{G''}| - |S_a^{G''-i'a}| \geq m^*$. Thus, given $\bar{\sigma}$ an intermediary i who trusts the agent at $t = 0$, continues trusting her as long as she does not defect in an interaction intermediated by i and as long as no client c disconnected from i . Given that, the agent's strategy is a best response. Similarly, intermediaries' and clients' strategies are best responses given their beliefs and the strategies of all other individuals. ■

Fact 1. $\forall k \in \mathbb{N}, G, c i a^G, \omega \in \Omega(G), \mu \in ITB(G)$,

$$\Pr_{\mu_i}(|S_a^G| - |S_a^{G-ia}| \geq k) = \Pr_{\mu_a}(|S_a^G| - |S_a^{G-ia}| \geq k) = \Pr(|S_a^G| - |S_a^{G-ia}| \geq k | G) \in \{0, 1\}.$$

Following Fact 1 and for $j \in \{i, a\}$ we often substitute

$$\Pr_{\mu_j}(|S_a^G| - |S_a^{G-ia}| \geq k) = 1 \left(\Pr_{\mu_j}(|S_a^G| - |S_a^{G-ia}| < k) = 1 \right) \text{ with the shorter } |S_a^G| - |S_a^{G-ia}| \geq k \text{ (} |S_a^G| - |S_a^{G-ia}| < k \text{)}.$$

Let $\bar{C}_i^G \equiv \{c \in C_i^G | c i' a^G \Rightarrow i' = i\}$ be the set of clients whose unique interaction path with the agent in network G is via i .

Lemma 4. *Consider any network G , intermediary i , and state ω' such that $G^0 = G$, $\bar{C}_i^G \subseteq C_i^{G(\omega')}$, and $c' i a^{G(\omega')}$ for at least one client c' . Consider any NetE (σ, μ) such that according to σ all clients*

keep all of their links to intermediaries in G at time 0 ($\forall_c \sigma_c(\omega^0(G)) \supseteq I_c^G$). Then, σ satisfies the following. If i is an m^* -local monopoly in G then in state ω' :

- [1] i keeps her link to the agent ($a \in \sigma_i(h_i(\omega'))$); and
- [2] the agent does not defect in an interaction intermediated by i ($i \in \sigma_a(h_a(\omega'))$).

Proof of Lemma 4. For any $i \in I$ and state ω' that satisfies the conditions of the lemma, let $O_i^I(\omega') = \left| \left\{ (c, i') \mid i' \in I - i, cia^{G(\omega')}, ci'a^{G(\omega')} \right\} \right|$ and let $O_i^C(\omega') = \left| \left\{ c \mid cia^{G(\omega')}, ca^{G(\omega')} \right\} \right|$. Note that by definition $O_i^I(\omega')$ and $O_i^C(\omega')$ can be perfectly inferred from $K_a(\omega')$ as well as from $K_i(\omega')$. Thus, $O_i^I(\omega')$ and $O_i^C(\omega')$ are common knowledge for i and a . We first prove Lemma 4 for the case that $O_i^C(\omega') = 0$ by induction on $O_i^I(\omega')$. We later extend the proof to include any $O_i^C(\omega') > 0$.

Consider a link ia such that ia^G and $|S_a^{G(\omega')}| - |S_a^{G(\omega')-ia}| \geq m^*$. By Fact 1 this is common knowledge for i and a . If $O_i^C(\omega') = 0$, $O_i^I(\omega') = 0$, and $a \in \sigma_i(h_i(\omega'))$ then a 's unique best response in any NetE is $i \in \sigma_a(h_a(\omega'))$. This is true because in a NetE links that were not severed at a certain time t are not severed at any time $t' > t$ unless a defection or a severing of another link are observed, and since $O_i^C(\omega') = 0$ and $O_i^I(\omega') = 0$, the agent knows that no defection will be observed by i unless the agent defects in an interaction intermediated by i . Subsequently, i 's unique best response in any NetE is $a \in \sigma_i(h_i(\omega'))$. Now, let $O_i^C(\omega') = 0$ and let k be a non negative integer such that for all $O_i^I(\omega') \leq k$, i 's unique best response is $a \in \sigma_i(h_i(\omega'))$ and a 's unique best response is $i \in \sigma_a(h_a(\omega'))$. We now prove that this is also true for $O_i^C(\omega') = 0$ and $O_i^I(\omega') = k + 1$. Note that since $\bar{C}_i^G \subseteq C_i^{G(\omega')}$ then $|S_a^{G(\omega')}| - |S_a^{G(\omega')-ia}| \geq m^*$ and this is the case as long as a does not default on i and ia is not eliminated. Thus, feasible network changes that are observable to i according to K_i include a disconnection of ia due to a 's defection in an interaction intermediated with i , or a decrease in $O_i^I(\omega')$ without a defaulting on i . Assume the latter, then by the induction assumption, in the new state ω'' , i 's unique best response is $a \in \sigma_i(h_i(\omega''))$ and a 's unique best response is $i \in \sigma_a(h_a(\omega''))$. Therefore in state ω' (and in any state in which K_i is identical to $K_i(G(\omega'))$) if $a \in \sigma_i(h_i(\omega'))$ then a 's unique best response is $i \in \sigma_a(h_a(\omega'))$, hence i 's unique best response is $a \in \sigma_i(h_i(\omega'))$. This concludes the proof that for $O_i^C(\omega') = 0$ and any $O_i^I(\omega')$, i 's unique best response is $a \in \sigma_i(h_i(\omega'))$ and a 's unique best response is $i \in \sigma_a(h_a(\omega'))$.

We now prove Lemma 4 for any $O_i^I(\omega')$ and $O_i^C(\omega')$. As a first step, assume that for some non negative integer k , if $O_i^C(\omega') \leq k$ then the claim is true for any $O_i^I(\omega')$. We now prove that if $O_i^C(\omega') = k + 1$ then the claim is true for $O_i^I(\omega') = 0$. Note that since $\bar{C}_i^G \subseteq C_i^{\omega'}$ then $|S_a^{G(\omega')}| - |S_a^{G(\omega')-ia}| \geq m^*$ and this is the case as long as a does not default on i and ia is not eliminated. In addition, this is common knowledge to i and a . If $O_i^I(\omega') = 0$ and $O_i^C(\omega') > 0$ then feasible network changes that are observable to i according to K_i include a disconnection of ia due to a 's defection in an interaction intermediated by i , or a decrease in $O_i^C(\omega')$ without a defecting in an interaction intermediated by i . Assume the latter, then the new state ω'' is such that $O_i^C(\omega'') = k$. So by the induction assumption, i 's unique best response is $a \in \sigma_i(h_i(\omega''))$ and a 's unique best response is $i \in \sigma_a(h_a(\omega''))$. Therefore, in state ω' (and in any state in which K_i is identical to $K_i(\omega')$) if $a \in \sigma_i(h_i(\omega'))$ then a 's unique best response is $i \in \sigma_a(h_a(\omega'))$. Consequently, i 's unique best response is $a \in \sigma_i(h_i(\omega'))$.

Finally, assume that for some non negative integers k_C and k_I the claim is true for any $O_i^C(\omega') \leq k_C$ regardless of $O_i^I(\omega')$, as well as for $O_i^C(\omega') = k_C + 1$ if $O_i^I(\omega') \leq k_I$. We now prove that it is true for $O_i^C(\omega') = k_C + 1$ and $O_i^I(\omega') = k_I + 1$. As before, since $\bar{C}_i^G \subseteq C_i^{G(\omega')}$ then $|S_a^{G(\omega')}| - |S_a^{G(\omega')-ia}| \geq m^*$ and this is the case as long as a does not defect in an interaction intermediated by i and ia is not eliminated. Thus, feasible network changes that are observable to i according to K_i include a disconnection of ia due to a 's defection in an interaction intermediated by i , a decrease in $O_i^I(\omega')$ without a defecting in an interaction intermediated by i , and a decrease in $O_i^C(\omega')$ without a defecting in an interaction intermediated by i . For either case, we can employ our induction assumptions as above. ■

Proof of Theorem 1. Part 1 - locally monopolistic \Rightarrow robust: By definition for any $ITB(G)$ there exists a NetE such that at time $t = 0$, $\forall_c \sigma_c(\omega^0(G)) \supseteq I_c^G$. From Lemmas 4 and 3, and the definition of ITB it is sufficient to note that for any σ that satisfies conditions (1) and (2) below, no direct client-agent links and $\forall_{cia^G} |S_a^G| - |S_a^{G-ia}| \geq m^*$ imply that G is sustained indefinitely with probability 1.

$$(1) \quad \forall_c \sigma_c(\omega^0(G)) \supseteq I_c^G.$$

$$(2) \quad \text{For any intermediary } i, \text{ network } G, \text{ and state } \omega' \text{ such that } G^0 = G, \bar{C}_i^G \subseteq C_i^{G(\omega')} \text{ and } ia^{G(\omega')}, \text{ if } i \text{ is an } m^* \text{-local monopoly in } G \text{ then } i \in \sigma_a(h_a(\omega')) \text{ and } a \in \sigma_i(h_i(\omega')).$$

Part 2 - robust \Rightarrow locally monopolistic: We prove that if $\forall_{\mu \in ITB(G)} \exists_\sigma (\sigma, \mu) \in \Sigma(G)$, then there are no client-agent links in G and $\forall_{cia^G} |S_a^G| - |S_a^{G-ia}| \geq m^*$.

Consider a network G such that $\exists_{cia^G} |S_a^G| - |S_a^{G-ia}| < m^*$. We proceed by constructing a belief profile $\bar{\mu} \in ITB(G)$ such that $\forall_\sigma (\sigma, \bar{\mu}) \notin \Sigma(G)$ which provides the necessary contradiction.

Consider the following network G' : Starting from the network G , for each intermediary in G add links to m^* distinct clients that are not connected to any other intermediary or to a (add clients to C if necessary). Note that G' is m^* -locally monopolistic. Let $\bar{\mu}$ be any belief profile that induces the following posterior once individuals observe $K(G^0)$: for each individual $j \in V$ the belief $\bar{\mu}_j$ puts probability 1 on the network being G' minus the links that they observe that do not exist (i.e. links that belong $K_j(G')$ but not to $K_j(G)$). We have shown in the proof of Lemma 3 that $\forall_G \bar{\mu} \in ITB(G)$.

Consider first a strategy profile σ that dictates that if the initial network is G' then all of the clients in G' keep all of their connections to intermediaries at $t = 0$ ($\forall_c \sigma_c(\omega^0(G')) \supseteq I_c^{G'}$). Now consider cia^G such that $|S_a^G| - |S_a^{G-ia}| < m^*$, and assume by contradiction that $(\sigma, \bar{\mu}) \in \Sigma(G)$. Suppose that at some point in time, a defects in an interaction intermediated by i and denote by ω' the new state immediately after the elimination of ia . From Lemma 4 and according to the beliefs of i and all of the clients connected to i , $\forall_{i'a^G \neq ia} i' \in \sigma_a(h_a(\omega'))$ and $a \in \sigma_{i'}(h_{i'}(\omega'))$. Therefore, given $\bar{\mu}$ and for every $ci'^G \neq ci$, c 's unique best response is $i' \in \sigma_c(h_c(\omega'))$. Now consider again the network G . Both i and c know that $|S_c^G| - |S_c^{G-ic}| < m^*$. Hence, according to $\bar{\mu}_i$ it must be the case that $i \notin \sigma_a(h_a(\omega^0(G)))$. Subsequently, $|S_a^G| - |S_a^{G-ia}| < m^* \Rightarrow a \notin \sigma_i(h_i(\omega^0(G)))$.

Note that we have implicitly assumed that all clients maintain their links to intermediaries in G' (rather than G) at $t = 0$. This does not require further attention for links in G , because we focus on a belief that belongs to $ITB(G)$. We now show that if all individuals play any equilibrium

such that if the network is G it would be sustained indefinitely, then if the network is G' it would also be sustained indefinitely. Therefore, our implicit assumption is not restrictive.

Now consider a client c for which a connection to some intermediary i exists in G' and does not exist in G . Consider any σ such that c disconnects her link to i at $t = 0$ (this possibility needs to be considered because ci is not in G and thus we cannot take for granted that an equilibrium with c keeping his link to i at $t = 0$). Remember that i is the (unique) intermediary that c is connected to. Assume by contradiction that $(\sigma, \bar{\mu}) \in \Sigma(G)$. Since $(\sigma, \bar{\mu}) \in \Sigma(G)$ it must be that $\forall_{\omega \in \{\omega' | G^0 = G\} \cap \Omega(G), c' \in G} \sigma_{c'}(\omega) \supseteq I_{c'}^G$ and $\forall_{\omega \in \{\omega' | G^0 = G\} \cap \Omega(G), i' \in G} \sigma_{i'}(\omega) = \{a\}$. Consider a strategy $\hat{\sigma} = (\hat{\sigma}_c, \sigma_{-c})$ where $\hat{\sigma}_c$ is identical to σ_c with the only exception that $i \in \hat{\sigma}_c(\omega^0(G'))$. This changes only the information sets of a , i , and c . Thus, the actions of all other individuals are not affected as long as ω is such that $ia^{G(\omega)}$. We now show that given $\hat{\sigma}$ and given that $(\sigma, \bar{\mu}) \in \Sigma(G)$, a 's unique best response is not to defect in an interaction intermediated by i . Therefore, σ such that $i \notin \sigma_c(\omega^0(G))$ is not an equilibrium strategy given σ_{-c} and $\bar{\mu}$ – contradicting out assumption that $(\sigma, \bar{\mu}) \in \Sigma(G)$.

Finally, suppose that given $\hat{\sigma}$ there exists a sequence of actions that makes it strictly profitable for the agent to defect in an interaction intermediated by i , then the same sequence would have made it strictly profitable for the agent to defect in an interaction intermediated by i if c was not connected to i . Moreover if given $\hat{\sigma}$ there exists a sequence of actions that makes it weakly profitable for the agent to defect in an interaction intermediated by i , then the same sequence would have made it strictly profitable for the agent to defect in an interaction intermediated by i if c was not connected to i . This is true because $i \in \hat{\sigma}_c(\omega^0(G))$ implies that cia^G as long as ia^G . Thus, if c is connected to i , then a losses strictly more from the elimination of the link ia .

We are left to prove that if there are any direct client-agent links then there exists a belief profile $\mu \in ITB(G)$ such that $\forall_{\sigma} (\sigma, \mu) \notin \Sigma(G)$. This part of the proof involves an almost exact repetition of the argument above and is omitted. ■

Proof of Proposition 4. For any network G , consider any belief profile μ^G that is consistent with the following:

- (1) $\mu^G \in ITB(G)$.
- (2) The posterior of each intermediary i given her observations in G puts probability 1 on her being connected to all of the investors who are network related to the entrepreneur (i.e. no other investor is network related to the entrepreneur).
- (3) If an investor c is connected directly to the entrepreneur and if there is no intermediary i who is connected to both c and the entrepreneur, then μ_c^G puts probability 1 on the entrepreneur not being network related to any investor besides c .
- (4) If an investor c is connected to an intermediary i who is connected to the entrepreneur, then μ_c^G puts probability 1 on i being connected to all of the investors who are network related to the entrepreneur.

One can verify that for any network G there exists at least one such belief profile, and that for any network G and belief profile μ^G , if G does not correspond with conditions [1] and [2] in Proposition 4 then there is no strategy profile σ' such that $(\sigma', \mu^G) \in \Sigma(G)$. ■

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