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Comparative Performance of Cryptocurrencies through the Aumann and Serrano Economic Index of Riskiness

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Toshiyuki Yamawake*
Faculty of Economics, Nagoya University of Commerce and Business

Joseph Sheely†
Department of Economics, Brown University

Roberto Serrano‡
Department of Economics, Brown University

Jiro Hodoshima§
Faculty of Economics, Nagoya University of Commerce and Business

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Abstract

We present comparative performance of cryptocurrencies by the performance index based on the Aumann-Serrano –henceforth, AS– economic index of riskiness. We consider three cryptocurrencies: Bitcoin, Ethereum and Binance-coin, which have the largest market capitalizations among all cryptocurrencies. Ethereum is about twice as good as Binance-coin and fourteen times as good as Bitcoin when we evaluate them by the AS performance index. Therefore, Bitcoin is rated quite poorly –much riskier– compared to other cryptocurrencies although it is the most popular and has the largest market capitalization among all the cryptocurrencies. On the other hand, SPY is rated the best –least risky– and much better than the three cryptocurrencies by the AS performance index. The evaluation by the AS

*Toshiyuki Yamawake, E-mail: yamawake@nucba.ac.jp
†Joseph Sheely, E-mail: joseph sheely@alumni.brown.edu
‡Roberto Serrano, E-mail:Roberto serrano@brown.edu
§Corresponding author. Address correspondence: Jiro Hodoshima, Faculty of Economics, Nagoya University of Commerce and Business, 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi 470-0193, Japan E-mail:hodoshima@nucba.ac.jp; Tel: +81 561-73-2111 Fax: +81 561-73-1202

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JEL codes: G11; C22; C46

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*Toshiyuki Yamawake, E-mail: yamawake@nucba.ac.jp

†Joseph Sheely, E-mail: joseph_sheely@alumni.brown.edu

‡Roberto Serrano, E-mail:Roberto_serrano@brown.edu

§Corresponding author. Address correspondence: Jiro Hodoshima, Faculty of Economics, Nagoya University of Commerce and Business, 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi 470-0193, Japan E-mail:hodoshima@nucba.ac.jp; Tel: +81 561-73-2111 Fax: +81 561-73-1202

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1 Introduction

Cryptocurrencies have received considerable attention recently. For example, the price of Bitcoin, which has the largest market capitalization among all the cryptocurrencies, is now reported in major financial news and cited every day around the globe. Cryptocurrencies have increased their market capitalizations tremendously in the last decade so that they now play important roles as financial products and targets in financial and risk management and investing. On the other hand, cryptocurrencies also have a great deal of variation compared to traditional financial assets, such as stocks, commodities, derivatives, etc. It is therefore not surprising to find many studies that try to capture the volatility of cryptocurrencies. Cryptocurrencies are now one of major investment targets, so that assessment of cryptocurrencies is quite important in investment and financial risk and management. In this paper, we present evaluation of cryptocurrencies by an axiomatic performance measure of the Aumann-Serrano –AS hereafter– performance index due to Kadan and Liu (2014), which is based on the economic index of riskiness proposed in Aumann and Serrano (2008).

Studies of the properties, modeling, and forecasting of cryptocurrencies abound. A partial list follows. Their market efficiency has been studied in Urquhart (2016); Nadarajah and Chu (2017); Bariviera et al. (2017); Khuntia and Pattanayak (2018); Tiwari et al. (2018). On the other hand, Lo (2004) proposed an alternative of the market efficiency hypothesis, i.e., the adaptive market hypothesis where the efficiency evolves over time. There are some studies of the adaptive market hypothesis for cryptocurrencies (e.g., Chu et al. (2019); Noda (2020)).

Cryptocurrencies have extremely large volatility. One of the well-known methods of capturing volatility is the autoregressive conditional heteroskedasticity (ARCH) model by Engle (1982), which is generalized to the general autoregressive conditional heteroskedasticity (GARCH) model by Bollerslev (1986). Glaser et al. (2014) estimated a standard GARCH(1,1) model for Bitcoin. Gronwald (2014) found an autoregressive jump-intensity GARCH model fits better than a standard GARCH(1,1) model for Bitcoin. Dyhrberg (2016) estimated an asymmetric GARCH model for Bitcoin. Katsiampa (2017) found that an autoregressive model with a component GARCH model fits best for Bitcoin.

Chu et al. (2017) tried 12 GARCH specifications with different distributions for the error term.

There are also studies of long memory and volatility for cryptocurrencies, such as Bariviera et al. (2017), Caporale et al. (2018), Cheah et al. (2018), Charfeddine and Maouchi (2019), and Yaya et al. (2019).

Bauwens et al. (2010,2014) found structural change results in biased estimation and poor forecasting of GARCH models. To deal with this issue, Markov switching models have been proposed. Ardia et al. (2018) estimated a number of Markov switching GARCH models and found Markov switching GARCH models outperform single regime models in forecasting value at risk and expected shortfall.

On the other hand, not many studies exist that obtain the comparative assessment of cryptocurrencies, with the possible exception of popular performance measures such as the Sharpe ratio. We intend to present the assessment of cryptocurrencies by performance measures in this paper. Although several performance measures have been proposed in the literature, most of them are somewhat ad-hoc (e.g., Eling et al. (2007); Farinelli et al. (2008)). In this paper, we choose to employ the AS performance index, which is based on axiomatic principles and applies to any population of risk-averse investors (Aumann and Serrano (2008); Kadan and Liu (2014)).

Hodoshima and Otsuki (2019) presented the AS performance index of Bitcoin as compared to traditional assets of U.S. stocks and gold. That paper assumed that!! the underlying gamble of Bitcoin is a static gamble, i.e., its observations are sample data of a static random variable and assumed the underlying distribution follows the class of normal mixture distributions. However, it may be more appropriate to treat observations of cryptocurrencies as realizations of stochastic processes, since cryptocurrencies vary tremendously both in the short-run and in the long-run, so that treating them as static random variables may not be as reasonable. Therefore, in this paper, we assume observations of cryptocurrencies are realizations of stochastic processes, instead of static random variables. In other words, we assume cryptocurrencies are multi-period gambles, which are treated as stochastic processes. Under this assumption, it is appropriate to use the AS performance index for multi-period gambles instead of one-period gambles, as formulated by Kadan and Liu (2014) (cf. Section 3.2 of Kadan and Liu (2014)). Furthermore, Hodoshima and Otsuki (2019) did not model volatility clustering for the assets they investigated, unlike what is done in the current study.

In order to obtain the AS performance index for multi-period gambles instead of one-period gambles, we must model and estimate the underlying process of the multi-period gambles to compute the index. We employ a class of normal mixture distributions with GARCH volatility models. The class of normal mixture distributions is a flexible class that can reproduce symmetric distributions as well as asymmetric and/ or heavy tail distributions often found in financial products, such as stocks, commodities, foreign currencies, etc. The class of normal mixture distributions can be considered to be one of Markov switching models where the transition matrix is time independent. Since GARCH volatility models are also well-known to capture volatility clustering, we incorporate them into the class of normal mixture distributions. In particular, we examine up to four-components normal mixture distributions to examine whether such higher order normal mixture models are useful to capture the enormous variation of cryptocurrencies.

Summarizing our main findings, We consider three cryptocurrencies: Bitcoin, Ethereum and Binance-coin, which have the largest market capitalizations among all cryptocurrencies. Ethereum is about twice as good as Binance-coin and fourteen times as good as Bitcoin when we evaluate them by the AS performance index.[1] Therefore, Bitcoin is rated quite poorly –much riskier– compared to other cryptocurrencies although it is the most popular and has the largest market capitalization among all the cryptocurrencies. On the other hand, SPY is rated the best –least risky– and much better than the three cryptocurrencies by the AS performance index. The evaluation by the AS performance index contrasts sharply to that made by the Sharpe ratio, according to which the difference among the three cryptocurrencies and SPY is fairly small.

The rest of the article is organized as follows. Section 2 presents the setup of our models. Section 3 explains data. Section 4 provides the empirical examples. Section 5 presents concluding comments.

[1] As two benchmark comparisons –see Aumann and Serrano (2008)–, (i) a gamble that results in a loss of l with probability $1/e$, and a "very large" gain with the rest of probability has AS-riskiness l, for any $l > 0$; and (ii) given any base binary gamble with gain g and loss l, for any $\lambda > 0$, its AS riskiness is multiplied by λ when both gain and loss are also multiplied by λ.

2 Models

In this section, we present the models that are used in this paper. Modeling volatility is key in capturing the variation of cryptocurrencies, and to that end, we use the GARCH(1,1) family of volatility models. In particular, we use the standard GARCH(1,1) model as well as the asymmetric GARCH(1,1) (AGARCH(1,1)) model (cf. Engle (1990)) and the GJR(1,1) model (the model based on Glosten et al. (1993)). In the latter two asymmetric GARCH(1,1) models, the response is asymmetric with respect to the sign of the return of one-period behind.

We employ the class of normal mixture distributions to capture the underlying distribution of the return of cryptocurrencies. As the underlying model, we use a class of normal mixture distributions with three volatility specifications of GARCH(1,1) families. In other words, we assume the return X_t of the three cryptocurrencies follows a mixture of K normal distributions with a time-varying volatility process

$$X_t|I_{t-1} \sim \pi_k N(\mu_k, \sigma_{k,t}^2) \tag{1}$$

for $t = 1, \cdots, T$ and $k = 1, \cdots, K$, where $N(\mu_k, \sigma_{k,t}^2)$ denotes normal distribution with mean μ_k and variance $\sigma_{k,t}^2$, I_{t-1} is the information set up to time $t-1$, $0 \leq \pi_k \leq 1$, and $\sum_{k=1}^{K} \pi_k = 1$. We further assume that the conditional variance of the k-th component follows three possible processes:

(1)GARCH(1,1) process

$$\sigma_{k,t}^2 = \omega_k + \alpha_k X_{t-1}^2 + \beta_k \sigma_{k,t-1}^2 \tag{2}$$

(2)AGARCH(1,1) process

$$\sigma_{k,t}^2 = \omega_k + \alpha_k (X_{t-1} - \lambda_k)^2 + \beta_k \sigma_{k,t-1}^2 \tag{3}$$

(3) GJR(1,1) process

$$\sigma_{k,t}^2 = \omega_k + \alpha_k X_{t-1}^2 + \lambda_k d_{t-1}^- X_{t-1}^2 + \beta_k \sigma_{k,t-1}^2 \tag{4}$$

where $d_t^- = 1$ if $X_t < 0$ and 0 otherwise, the component conditional variance depends on the previous return X_{t-1} as well as its own previous conditional variance. Under these assumptions, the conditional mean, variance, skewness, and kurtosis of X_t given the information set up to time $t-1$ are given respectively by:

$$\begin{aligned}
\mu &= \sum_{k=1}^{K} \pi_k \mu_k \\
\sigma_t^2 &= \sum_{k=1}^{K} \pi_k (\sigma_{k,t}^2 + \mu_k^2) - \mu^2 \\
\tau_t &= \frac{1}{\sigma_t^3} \sum_{k=1}^{K} \pi_k (\mu_k - \mu) \left[3\sigma_{k,t}^2 + (\mu_k - \mu)^2\right] \\
\kappa_t &= \frac{1}{\sigma_t^4} \sum_{k=1}^{K} \pi_k \left[3\sigma_{k,t}^4 + 6(\mu_k - \mu)^2 \sigma_{k,t}^2 + (\mu_k - \mu)^4\right].
\end{aligned} \tag{5}$$

The AS performance index for multi-period gamble $\boldsymbol{g} = (g_1, g_2, \cdots, g_T)$ is given by the unique positive solution $P^{AS}(\boldsymbol{g})$ of the implicit equation

$$\sum_{t=1}^{T} \rho^{t-1} E[\exp(-P^{AS}(\boldsymbol{g}) \cdot g_t)] = \sum_{t=1}^{T} \rho^{t-1} \tag{6}$$

where $\rho \in (0,1)$ is a discount factor (cf. Section 3.2 of Kadan and Liu (2014)). Equation (6) is actually equation (8) of Kadan and Liu (2014). In this paper, we use conditional expectation instead of unconditional expectation in equation (6), since expectation for X_t is conditional, as per its description above.

When the return $\boldsymbol{X} = (X_1, X_2, \cdots, X_T)$ follows the above normal mixture process with time-varying volatility of GARCH(1,1) families, the following equality holds for $E[\exp(-P^{AS}(\boldsymbol{X}) \cdot X_t)]$ in the implicit equation (6) of the AS performance index:

$$E[\exp(-P^{AS}(\boldsymbol{X}) \cdot X_t)] = \sum_{k=1}^{K} \pi_k \exp(-\mu_k P^{AS}(\boldsymbol{X}) + \sigma_{k,t}^2 P^{AS}(\boldsymbol{X})^2/2) \tag{7}$$

where $\boldsymbol{X} = (X_1, X_2, \cdots, X_T)$ is treated as a multi-period gamble since the moment-generating function (MGF) $E[\exp(sY)]$ of a random variable Y is given by

$$\exp(\mu s + \sigma^2 s^2/2) \tag{8}$$

when Y follows normal distribution $N(\mu, \sigma^2)$. Notice $E[\exp(-P^{AS}(\boldsymbol{g}) \cdot g_t)]$ is, besides the minus sign, nothing but the MGF of g_t as a function of $P^{AS}(\boldsymbol{g})$.

In order to obtain the AS performance index for multi-period gambles, we must first estimate μ_k and $\sigma_{k,t}$ under the parametric assumption of the normal mixture process of

$\boldsymbol{X} = (X_1, X_2, \cdots, X_T)$. To estimate these parameters, we follow the continuous empirical characteristic function (CECF) approach of Xu and Wirjanto (2010), which facilitates the estimation of a class of normal mixture distributions with three volatility models of GARCH(1,1) families containing three or four normal mixture components.[2] The CECF approach of Xu and Wirjanto (2010) has several advantages as a method of estimating the parametric model: a closed-form objective distance function is available, the estimator is strongly consistent and asymptotically normal, and the characteristic function is, unlike the likelihood function, always uniformly bounded (cf. Xu and Wirjanto (2010)).

The characteristic function of X_t associated with equations (1)-(4) is defined by

$$C_t(r, \theta) = E[e^{irX_t}] = \sum_{k=1}^{K} \pi_k \exp\left(i\mu_k r - \frac{1}{2}\sigma_{k,t}^2 r^2\right) \tag{9}$$

where $i = \sqrt{-1}$, r is a real number, and θ denotes the set of parameters in the model.

The empirical characteristic function of the above equation is given by

$$C_t(r, X_t) = \exp(irX_t). \tag{10}$$

Then, we consider the following distance measure defined by

$$D_t(\theta; X_t) = \int \mid C_t(r, X_t) - C_t(r, \theta) \mid^2 \exp(-br^2) dr. \tag{11}$$

where b is a parameter to be specified.

We have the following result for the closed-form expression of the above distance function $D_t(\theta; \boldsymbol{X}_t)$.

Theorem 1 (Proposition 1 of Xu and Wirjanto (2010)) *If the return X_t is generated by equations (1)-(4) and the distance measure under the CECF is given by Equation (11), then the closed-form-expression for the distance measure $D_t(\theta; X_t)$ is given*

[2]It was mentioned in Alexander and Lazar (2009) that estimation of normal mixture distributions with three normal mixture components is difficult by the traditional maximum likelihood method.

by

$$\begin{aligned} D_t(\theta; X_t) &= \sqrt{\frac{\pi}{b}} + \sum_{k=1}^{K} \pi_k^2 \sqrt{\frac{\pi}{b+\sigma_{k,t}^2}} \\ &\quad -2\sum_{k=1}^{K}\left(\pi_k \sqrt{\frac{\pi}{\frac{1}{2}\sigma_{k,t}^2 + b}} \exp\left(-\frac{(X_t-\mu_k)^2}{4b+2\sigma_{k,t}^2}\right)\right) \\ &\quad +2\sum_{k\neq h} \pi_k \pi_h \sqrt{\frac{\pi}{b+\frac{1}{2}(\sigma_{k,t}^2+\sigma_{h,t}^2)}} \\ &\quad \times \exp\left(-\frac{(\mu_k-\mu_h)^2}{4b+2(\sigma_{k,t}^2+\sigma_{h,t}^2)}\right). \end{aligned} \tag{12}$$

The conditional variance $\sigma_{k,t}^2$ of the k-th component in the closed-form-expression given above can be any of the three possible processes of GARCH(1,1) families given above. In other words, the closed-form-expression (12), originally for the standard GARCH models, continues to hold for other forms of GARCH families such as AGARCH and GJR models. We employ $b = 1$ when we implement estimation by minimizing the closed-form expression as in Xu and Wirjanto (2010).

The CECF estimation of the model is to minimize $D(\theta) = \sum_{t=1}^{T} D_t(\theta; \boldsymbol{X}_t)$ with respect to the set of unknown parameters in the model. The following result states the asymptotic normality:

Theorem 2

$$\sqrt{T}(\hat{\theta} - \theta) \Longrightarrow N(0, \Lambda^{-1}\Omega\Lambda^{-1}) \tag{13}$$

where $\hat{\theta}$ denotes the estimator by the CECF approach, $\Longrightarrow$ denotes convergence in distribution, $\Lambda = E\left[\frac{\partial^2 D(\theta)}{\partial\theta\partial\theta'}\right]$, and $\Omega = E\left[\frac{\partial D(\theta)}{\partial\theta}\frac{\partial D(\theta)}{\partial\theta'}\right]$.

See Heathcote (1977) for the proof of the above theorem. The CECF estimation of the model was carried out by Hodoshima and Yamawake (2020) to estimate the AS performance index for multi-period gambles using U.S. stock data.

3 Data

In order to study cryptocurrencies, we focus on Bitcoin, Ethereum, and Binance-coin, which have the largest market capitalizations among all the cryptocurrencies except stable coins such as Tether as of July 31, 2022. In addition, we use a stock ETF with

S&P500 as the benchmark, i.e., SPY,[3] as a representative traditional asset to compare with the three cryptocurrencies. We employ daily return data of the three cryptocurrencies from January 1, 2018 to July 31,2022[4] for our analysis. We use daily return r_t at time t defined by

$$r_t = 100 \times (P_t/P_{t-1} - 1) \tag{14}$$

where P_t and P_{t-1} denote the price at time t and t-1, respectively. There are 1673 daily return data for the three cryptocurrencies. On the other hand, there are 1152 daily return data for SPY since there are no obserbations for weekends and holidays for SPY. The data are downloaded from Yahoo Finance.

We provide figures of the three cryptocurrencies and SPY at Figures 1-4 at the end of the paper. We present summary statistics of daily return data of the three cryptocurrencies and SPY at Table 1. Summary statistics of the three cryptocurrencies show different characteristics in the three cryptocurrencies. Binance-coin is the one with the highest standard deviation and highest mean, while Bitcoin is the one with the lowest standard deviation and lowest mean among the three cryptocurrencies. Bitcoin and Ethereum are negatively skewed, while Binance-coin is positively skewed. The three cryptocurrencies have heavy tails compared to the normal distribution. Bitcoin has the smallest extreme values of the maximum and minimum among the three cryptocurrencies. On the other hand, the stock ETF SPY has significantly lower mean and standard deviation compared to the cryptocurrencies. SPY is negatively skewed and has heavy tails compared to the normal distribution. SPY also has smaller maximum and minimum values compared to the cryptocurrencies.

[3]SPY is also analyzed as an asset to compare with cryptocurrencies in Sheely (2022).

[4]This sample period excludes the period when the three cryptocurrencies do not receive a lot of attention and their prices do not vary a great deal jointly.

Table 1: Summary Statistics of Daily Return Data for the Assets

name	mean	s.d.	skew	kurt	max	min
BTC	0.108	3.925	-0.382	10.057	18.746	-37.170
ETH	0.179	5.070	-0.309	8.183	25.949	-42.346
BNB	0.383	6.039	1.925	26.903	69.765	-41.889
SPY	0.053	1.344	-0.643	14.733	9.060	-10.942

In the table, s.d., skew, kurt, max, and min denote respectively standard deviation, skewness, kurtosis, maximum, and minimum. BTC, ETH, and BNB stand for Bitcoin, Ethereum, and Binance-coin respectively. SPY denotes the SPY ETF.

4 Empirical Results

In this section, we present empirical results for the three cryptocurrencies and stock ETF SPY.

We first adjust the autocorrelation of returns of the assets in question by autoregressive models. In other words, we first fit the autoregressive model by the Bayesian information criterion (BIC) for the cryptocurrencies and SPY. We present the best autoregressive model in equations (15)-(18) for the assets.

$$r_t = 0.10825 + u_t \qquad \text{Bitcoin} \tag{15}$$

$$r_t = 0.16708 + 0.05921 r_{t-1} - 0.04841 r_{t-2} + u_t \qquad \text{Ethereum} \tag{16}$$

$$\begin{aligned} r_t &= 0.29995 - 0.00044 r_{t-1} + 0.05821 r_{t-2} - 0.01507 r_{t-3} - 0.00689 r_{t-4} - 0.01545 r_{t-5} \\ &\quad + 0.08157 r_{t-6} - 0.04221 r_{t-7} + u_t \qquad \text{Binance-coin} \end{aligned} \tag{17}$$

$$\begin{aligned} r_t &= 0.04974 + 0.13127 r_{t-1} - 0.09490 r_{t-2} + 0.14974 r_{t-3} - 0.09777 r_{t-4} - 0.00262 r_{t-5} \\ &\quad - 0.06639 r_{t-6} + 0.03770 r_{t-7} + 0.06223 r_{t-8} - 0.10064 r_{t-9} + u_t \qquad \text{SPY} \end{aligned} \tag{18}$$

where u_t in equations (15)-(18) denote unpredictable returns.

We first provide summary statistics of unpredictable returns at Table 2. Mean at Table 2 is 0, which results from the fact that unpredictable returns are residuals in the autocorrelation adjustment. Standard deviation at Table 2 is similar to that for the original returns at Table 1. Other summary statistics at Table 2 are also similar to those at Table 1. Therefore, unpredictable returns have similar characteristics to those of the original return data except for mean.

Table 2: Summary Statistics of Unpredictable Return Data for the Assets

name	mean	s.d.	skew	kurt	max	min
BTC	0	3.925	-0.382	10.057	18.638	-37.278
ETH	0	5.046	-0.327	8.267	26.134	-42.620
BNB	0	5.655	1.093	21.314	68.253	-42.791
SPY	0	1.266	-0.683	9.763	6.915	-8.948

In the table, s.d., skew, kurt, max, and min denote respectively standard deviation, skewness, kurtosis, maximum, and minimum. BTC, ETH, and BNB stand for Bitcoin, Ethereum, and Binance-coin respectively. SPY denotes the SPY ETF.

Table 3: Diagnostic Test Results for Unpredictable Returns for the Assets

name	BTC	ETH	BNB	SPY
Ljung-Box(12) in levels	11.012	17.061	5.010	1.061
Ljung-Box(12) in squares	45.537**	35.844**	39.050**	72.267**
Sign Bias	1.270	1.252	-0.177	2.876**
Negative Size Bias	−3.055**	−3.624**	−1.827†	−9.631**
Positive Size Bias	-0.329	-0.367	6.556**	5.237**
Joint Test	9.698*	14.068**	60.312**	160.724**

Table 3 provides the results of Ljung-Box test statistic, modified by Diebold (1988), of the first twelve autocorrelations in levels and squares of the unpredictable return data as well as the results of the sign bias test statistic, negative size bias test statistic, positive size bias test statistic, and joint test statistic. One and two asterisks indicate significance at the five and one percent levels respectively and † denotes significance at the ten percent level.

We then present diagnostic test results for the unpredictable returns at Table 3. We follow Engle and Ng (1993) to carry out the Ljung-Box test of the first twelve autocorrelations in levels and squares, sign bias test, negative size bias test, positive size bias test, and joint test described in Engle and Ng (1993). The sign bias test, negative size bias test, and positive size bias test are respectively defined as the t-test for the coefficient associated with an explanatory variable of a dummy variable of taking one when the unpredictable return at one day behind is negative and zero otherwise, an explanatory variable of the dummy variable times the unpredictable return at one day behind, and an explanatory variable of the dummy variable, which takes one when the unpredictable return at one day behind is positive and zero otherwise, times the unpredictable return at one day behind in the regression model for the square of the normalized residual under the volatility model hypothesized (cf. Engle and Ng (1993)). The joint test is defined as a Lagrange multiplier (LM) test of adding the three explanatory variables of the sign bias test, negative size bias test, and positive sign bias test in the regression model for the square of the normalized residual under the volatility model hypothesized.

The Ljung-Box tests of the existence of the first twelve autocorrelations in the level of unpredictable returns show they are all insignificant, which is natural since unpredictable returns are residuals after the autocorrelation adjustment. On the other hand, the Ljung-Box tests of the existence of the first twelve autocorrelations in the square of unpredictable returns show they are highly significant, indicating the phenomenon of volatility clustering in unpredictable returns. The sign bias tests are all insignificant for cryptocurrencies but highly significant for SPY. This indicates that the cryptocurrencies and stock ETF SPY are different with respect to the response of the sign of the return at one day behind. The negative sign bias tests are significant at the one percent significance level except for Binance-coin with the ten percent significance, which is similar to the result in Engle and Ng (1993). The positive size bias tests are only significant at the one percent significance level for Binance-coin and SPY. The joint tests are significant at the five and one percent significance levels.

We then model unpredictable returns as follows. We intend to find the best model for the unpredictable returns of the assets at this stage. We first estimate a class of normal mixture distributions with the three volatility models of GARCH(1,1) families and employ the best model to be used to estimate the AS performance index for multi-period gambles.

Below we provide the best model by the BIC for the assets in question. We present the best model for Bitcoin at Table 4. The best model for Bitcoin among the three classes of GARCH(1,1) families is a three-components normal mixture model of GARCH(1,1) volatility model. The best GARCH(1,1) model is, unlike the AGARCH model and GJR model, symmetric with respect to the response of the sign of the unpredictable return at one day behind. The first component is a stable component with probability of 0.6111, the second component is another stable component with probability of 0.319, and the third component is a crash component with a probability of 0.070. The volatility reaction (α_i) is low and the volatility persistence (β_i) is high in the first and second components. On the other hand, the volatility reaction is extremely high but the volatility persistence is zero in the crash component. Mean is near zero in the first and second components. On the other hand, mean is -0.624 in the third component although it is insignificant.

Table 4: Best Estimation of the Three Classes of GARCH(1,1) Families in Bitcoin

GARCH-3component							
μ_1	μ_2	μ_3	ω_1	α_1	β_1	ω_2	α_2
-0.000 (0.006)	-0.005 (0.027)	-0.624 (0.583)	1.913E-09 (0.160)	0.069 (0.031)	0.933 (0.018)	4.116E-10 (0.019)	0.011 (0.006)
β_2	ω_3	α_3	β_3	π_1	π_2	π_3	BIC
0.930 (0.017)	4.788E-08 (1.750)	75.245 (2.393)	0 (1.009E-05)	0.611 (0.180)	0.319	0.070 (0.019)	9086.315

Table 4 shows the best estimation result for Bitcoin in the three classes of GARCH(1,1) families, i.e., GARCH(1,1) model, AGARCH(1,1) model, and GJR(1,1) model. Standard errors, shown in parentheses, are obtained using asymptotic variance given in equation (13) in Theorem 2. In models with more than one component, probability of one component is determined automatically by the restriction of the sum of probabilities of all the components being equal to one so that its standard error is not given.

We present the best model for Ethereum at Table 5. The best model is again a symmetric three-component normal mixture model with the GARCH(1,1) volatility model. The first component has positive mean with probability of 0.429, the second component has negative mean with probability 0.407, and the third component has large positive mean 1.238 with probability of 0.164. The volatility reaction is small but the volatility persistence is high in the first component. The mean return (ω_i) in the volatility equation is large with 1.657 in the first component. The volatility reaction is small and the volatility persistence is high in the second component too. The same is true in the third component. The third component with the highest mean 1.238 has the smallest reaction and the largest volatility persistence. Therefore, the best model for Ethereum does not contain a crash component as in the third component for Bitcoin. Instead, all the components for Ethereum are stable components with positive mean in the first and third components and negative mean in the second component.

Table 5: Best Estimation of the Three Classes of GARCH(1,1) Families in Ethereum

GARCH-3component							
μ_1	μ_2	μ_3	ω_1	α_1	β_1	ω_2	α_2
0.345 (0.005)	-0.681 (0.025)	1.238 (0.292)	1.657 (0.001)	0.099 (0.009)	0.906 (0.002)	0.000 (0.118)	0.081 (0.007)
β_2	ω_3	α_3	β_3	π_1	π_2	π_3	BIC
0.772 (0.003)	0.063 (0.008)	0.001 (0.000)	0.984 (0.002)	0.429 (0.026)	0.407	0.164 (0.016)	9915.745

Table 5 shows the best estimation results for Ethereum in the three classes of GARCH(1,1) families, i.e., GARCH(1,1) model, AGARCH(1,1) model, and GJR(1,1) model. Standard errors, shown in parentheses, are obtained using asymptotic variance given in equation (13) in Theorem 2. In models with more than one component, probability of one component is determined automatically by the restriction of the sum of probabilities of all the components being equal to one so that its standard error is not given.

We present the best model for Binance-coin at Table 6. The best model for Binance-coin is again a symmetric three-component normal mixture model with the GARCH(1,1)

Table 6: Best Estimation of the Three Classes of GARCH(1,1) Families in Binance-coin

GARCH-3component							
μ_1	μ_2	μ_3	ω_1	α_1	β_1	ω_2	α_2
-1.297 (0.140)	-0.475 (0.047)	1.559 (0.560)	0.000 (0.342)	0.258 (0.088)	0.845 (0.071)	0.331 (0.157)	0.065 (0.010)
β_2	ω_3	α_3	β_3	π_1	π_2	π_3	BIC
0.674 (0.008)	0.000 (0.024)	0.002 (0.001)	0.995 (0.002)	0.373 (0.011)	0.321 (0.071)	0.306	9981.776

Table 6 shows the best estimation result for Binance-coin in the three classes of GARCH(1,1) families, i.e., GARCH(1,1) model, AGARCH(1,1) model, and GJR(1,1) model. Standard errors, shown in parentheses, are obtained using asymptotic variance given in equation (13) in Theorem 2. In models with more than one component, probability of one component is determined automatically by the restriction of the sum of probabilities of all the components being equal to one so that its standard error is not given.

Table 7: Best Estimation of the Three Classes of GARCH(1,1) Families in SPY

AGARCH-1component					
μ_1	ω_1	α_1	λ_1	β_1	BIC
0.049 (0.003)	0 (0.069)	0.134 (0.042)	0.619 (0.324)	0.823 (0.067)	3257.185

Table 7 shows the best estimation result for SPY in the three classes of GARCH(1,1) families, i.e., GARCH(1,1) model, AGARCH(1,1) model, and GJR(1,1) model. Standard errors, shown in parentheses, are obtained using asymptotic variance given in equation (13) in Theorem 2.

volatility model. The first component has large negative mean with probability 0.373. The second component has negative mean with probability 0.321. The third component has large positive mean with probability 0.306. The volatility reaction 0.258 is rather high in the first component. However, the volatility reaction is small in the second and third components. On the other hand, the volatility persistence is much higher than the volatility reaction in all the components. The third component for Binance-coin is similar to that for Ethereum. As in Ethereum, there is no crash component for Binance-coin.

We present the best model for SPY at Table 7. The best model for SPY is an asymmetric one component normal mixture model with the AGARCH(1,1) volatility model. It has positive mean 0.049. The volatility reaction is small and the volatility persistence is larger, which is not particularly different from those in the non-crash components in the best model for the cryptocurrencies. The effect (λ_1) of the sign of the unpredictable return at one day behind is highly significant in SPY. This implies that volatility increases when the sign of the unpredictable return at one day behind is negative.

Overall, the best model for the three cryptocurrencies is a three-component normal mixture model with the GARCH(1,1) volatility model. Hence, it is symmetric with respect to the sign of the unpredictable return at one day behind. The best model contains a crash component for Bitcoin but does not for the remaining cryptocurrencies. The three components are stable components with positive as well as negative means in Ethereum and Binance-coin. On the other hand, the best for the stock ETF SPY is a one-component normal mixture model with the AGARCH(1,1) volatility model. Therefore, the asymmetric one-component model fits better than higher-components models for the stock index. We emphasize the BIC 3257.185 for SPY is much smaller, about one-third, than that for the cryptocurrencies. Therefore, modelling the variation of the underlying asset is much more successful in the stock index than in the cryptocurrencies.

We remark that in the case of one-regime or one-component normal mixture model with three classes of GARCH(1,1) families, the AGARCH(1,1) model, which is asymmetric with respect to the response of the sign of the unpredictable return at one day behind, is the best model for the three cryptocurrencies. The BIC of the AGARCH(1,1) model is about twice as small as that of the symmetric GARCH(1,1) model in one-component (regime) model for the three cryptocurrencies. On the other hand, in more than one-component (regime) models, the symmetric GARCH(1,1) model always has the smallest BIC for the three cryptocurrencies. Therefore, the asymmetry with respect to the response of the sign of the unpredictable return at one day behind is captured by the AGRCH(1,1) or GJR(1,1) model in one-component (regime) GARCH(1,1) families. However, the asymmetry appears to be assimilated into the normal mixture model with the symmetric GARCH(1,1) model in multiple components (regimes).

We then present diagnostic test results at Table 8 in the best model for the assets to follow Engle and Ng (1993). The tests are applied to the normalized residuals of the best model in the three t-tests related to the three coefficients associated with the sign of the unpredictable return at one day behind and the joint test of the three coefficients to examine how the best model performs in these tests for the normalized residuals. The Ljung-Box statistics of serial correlation of the first twelve autocorrelations in the level are all insignificant. We omit the Ljung-Box statistics of serial correlation of the first twelve autocorrelations in the squares. The sign bias test is also all insignificant for the cryptocurrencies but highly significant for SPY. On the other hand, the negative size bias test is significant with the ten and one percent significance level respectively for Bitcoin and Ethereum but insignificant for Binance-coin and SPY. The positive size bias test is significant with the five and one percent significance level for the cryptocurrencies but insignificant for SPY. The joint test is significant with the five and one percent significance level for the cryptocurrencies but significant with the ten percent significance level for SPY. Overall, the residual of the best volatility model does not either contain serial correlation in the level or indicate different characteristics as compared to the residual of the original unpredictable return given at Table 3 with respect to the sign bias test. However, the residual of the best volatility model shows somewhat different characteristics compared to the residual of the original unpredictable return given at Table 3 with respect to other tests.

We then compare summary statistics of the estimated conditional variance of the return in the best model at Table 9. In the table, u^2 denotes the squared unpredictable return after the autocorrelation adjustment. Therefore, Table 9 compares the squared unpredictable return and estimated conditional variance of the unpredictable return in the best model in their summary statistics. In Bitcoin, mean, standard deviation, maximum, and minimum largely differ although skewness and kurtosis are quite similar between u^2 and estimated conditional variance of the best model. Therefore, low moments are largely different but higher moments such as skewness and kurtosis are close between u^2 and estimated conditional variance of the best model for Bitcoin. In Ethereum, Binance-coin, and SPY, mean is similar but other summary statistics are strikingly dif-

Table 8: Diagnostic Test Results for the Best Model of the Assets

name	Model	LB (12)	Sign Bias	- Size Bias	+ Size Bias	Joint
BTC	GARCH-3	16.005	1.575	$-1.744^{\dagger}$	-2.297^{*}	11.541^{**}
ETH	GARCH-3	9.112	1.544	-3.079^{**}	-2.776^{**}	8.958^{*}
BNB	GARCH-3	10.870	0.242	1.373	2.418^{*}	13.826^{**}
SPY	AGARCH-1	5.627	3.257^{**}	-0.296	0.962	$6.884^{\dagger}$

Table 8 provides diagnostic test results for the best model in the three classes of GARCH(1,1) families with respect to the BIC in the three cryptocurrencies and SPY. LB (12) denotes Ljung-Box statistics, modified by Diebold (1988), of serial correlation of the the first twelve autocorrelations in levels of the normalized residuals. Sign Bias, - Size Bias, + Size Bias, and Joint denote respectively the sign bias test statistic, negative size bias test statistic, positive sign bias test statistic, and joint test statistic. These tests are applied to the normalized residuals, i.e., the residuals divided by the conditional standard deviation estimate. In the table, GARCH-3 denotes a GARCH(1,1) three components model and AGARCH-1 denotes an AGARCH(1,1) one component model. One and two asterisks indicate significance at the five and one percent levels respectively and † denotes significance at the ten percent level.

ferent between u^2 and estimated conditional variance of the best model. Therefore, the best model works similarly to capture the variation of the squared unpredictable return in Ethereum, Binance-coin, and SPY, which is different from Bitcoin. Overall, the best model does not reproduce well the variation of the original squared unpredictable return although the best model is the most successful in specification of the assets. However, the outcome of modelling the variation of the underlying asset is similar in the cryptocurrencies of Ethereum and Binance-coin, but quite different in Bitcoin. Therefore, the variation of the underlying asset in Bitcoin appears to be different from that in the other two cryptocurrencies and the stock index ETF SPY.

Finally, we present the AS performance index for multi-period gambles as well as the Sharpe ratio for the three cryptocurrencies and SPY at Table 10. We provide the AS performance index for multi-period gambles when the discount rate ρ is $0.01, 0.05,$

Table 9: Summary Statistics of the Conditional Variance Estimate

name	Model	mean	s.d.	max	min	skewness	kurtosis
BTC	u^2	15.398	46.355	1389.631	0.000	17.092	469.556
	GARCH-3	91.409	246.923	7372.323	0.839	16.850	459.744
ETH	u^2	25.443	68.609	1816.444	0.000	13.201	296.527
	GARCH-3	24.048	13.459	162.457	10.383	3.778	26.797
BNB	u^2	31.964	144.107	4658.537	0.000	22.286	660.997
	GARCH-3	29.115	40.893	697.844	6.815	8.231	101.701
SPY	u^2	1.601	4.741	80.072	0.000	9.126	116.025
	AGARCH-1	1.499	2.380	28.679	0.153	6.208	52.433

Table 9 provides summary statistics of the conditional variance estimate of the best model of the three classes of GARCH(1,1) families in each asset with respect to the BIC. In the table, u^2 denotes the squared unpredictable return after the autocorrelation adjustment. In the table, s.d., max, and min stand for respectively standard deviation, maximum, and minimum. In the table, GARCH-3 denotes a GARCH(1,1) three components model and AGARCH-1 denotes an AGARCH(1,1) one component model.

and 0.1. The results do not differ much when we use different discount rates. The AS performance index increases as the discount rate increases in SPY, which is different from the previous result in stock data (cf. Hodoshima and Yamawake, 2020). Ethereum is rated the best, Binance-coin is rated the second best, and Bitcoin is rated the third best by the AS performance index. In particular, Ethereum is about twice as good as Binance-coin and fourteen times as good as Bitcoin. Therefore, the difference between the three cryptocurrencies by the AS performance index is quite large. On the other hand, Binance-coin is about 1.8 times as good as Ethereum and 2.4 times as good as Bitcoin by the Sharpe ratio. Therefore, the evaluation of the three cryptocurrencies by the Sharpe ratio is different from that by the AS performance index, and strikingly so for Bitcoin. Bitcoin is rated the worst by the two performance measures. However, the performance of Bitcoin is much worse by the AS performance index than by the Sharpe ratio.

On the other hand, SPY is rated the best and much better than the three cryptocurrencies by the AS performance index. However, SPY is not rated highly by the Sharpe ratio: it is rated the second best by the Sharpe ratio among the four assets. The AS performance index is known to be a performance index sensitive to losses but insensitive

Table 10: The AS Performance Index for Multi-Period Gambles and Sharpe ratio for the Assets

name	ρ=0.01	ρ=0.05	ρ=0.1	Sharpe ratio
BTC	0.0014	0.0014	0.0013	0.0266
ETH	0.0201	0.0201	0.0200	0.0345
BNB	0.0109	0.0109	0.0108	0.0628
SPY	0.1140	0.1146	0.1154	0.0369

Table 10 presents the AS performance index for multi-period gambles in the best model with respect to the BIC in the class of normal mixture models with the three GARCH(1,1) volatility families. In the table, the best model with respect to the BIC is always the three-components model with the GARCH(1,1) volatility model in the three cryptocurrencies while it is a one-component model with the AGARCH(1,1) volatility model in the stock ETF SPY.

to gains of the underlying asset, and hence appropriate to investors afraid of the riskiness of the underlying target. This suggests that the evaluation of the AS performance index should be more valuable to assess the assets, instead of that of the Sharpe ratio, for a population of risk-sensitive investors.

Overall, the stock index ETF is much better than the three cryptocurrencies by the AS performance index. Among the three cryptocurrencies, Bitcoin is rated the worst and Ethereum is about twice as good as Binance-coin and fourteen times as good as Bitcoin by the AS performance index.

5 Conclusion

We have computed the AS performance index for multi-period gambles in the three cryptocurrencies, which have the largest market capitalizations, and in the stock index ETF SPY. In order to obtain the AS performance index for multi-period gambles, we have estimated parametric models of the class of normal mixture models with three GARCH(1,1) volatility families. Unlike preceding studies in the literature, we have tried to fit the class of normal mixture models up to four-component normal mixture models with the GARCH volatility families by using the continuous empirical characteristic function (CECF) approach of Xu and Wirjanto (2010).

The best model is found to be always the three-component normal mixture model with the GARCH(1,1) volatility model in the three cryptocurrencies of Bitcoin, Ethereum, and Binance-coin, while it is the AGARCH(1,1) model in the stock index ETF SPY. The best volatility model we have found is symmetric with respect to the sign of the unpredictable return at one day behind in the three cryptocurrencies. In other words, asymmetric models of the AGARCH model and GJR model are not chosen as the best model. Ethereum is found to be rated the best, Binance-coin to be rated the second best, and Bitcoin to be rated the worst by the AS performance index for multi-period gambles. In particular, Ethereum is twice as good as Binance-coin and more than fourteen times as good as Bitcoin by the AS performance index for multi-period gambles. Therefore, Bitcoin is rated quite poorly by the AS performance index, although it is the most popular and has the largest market capitalization among all cryptocurrencies. The evaluation of the three cryptocurrencies by the AS performance index is different from that made by the Sharpe ratio. The results for Bitcoin are particularly striking: its evaluation by the Sharpe ratio is not too bad, while it is very poor when one uses the AS performance index. The AS performance index is a measure much more sensitive to losses than to gains of the underlying asset (cf. Aumann and Serrano (2008); Kadan and Liu (2014)). On the other hand, the AS performance index of the stock index ETF SPY is much better than that of the cryptocurrencies. Indeed, the cryptocurrencies are evaluated poorly compared to the traditional stock index by the AS performance index. One cannot reach this conclusion if one uses the Sharpe ratio, perhaps a red flag about its use for our purpose. The three cryptocurrencies perform quite poorly and, in particular, Bitcoin is the worst performer by the risk-sensitive AS performance index. Therefore, we should be careful about recommending the three cryptocurrencies to risk-sensitive investors. Since cryptocurrencies vary a great deal as compared to traditional financial assets such as stocks, bonds, commodities, etc., assessing their risk is crucial in investment and financial management. Our study should be viewed as an attempt to do just that. We have provided an assessment of the three most popular cryptocurrencies by the risk-sensitive measure of the AS performance index. Our findings could be used as guidance for risk-sensitive investors who might be interested in cryptocurrencies, and our final recommendation is probably "buyer, beware!."

Figure 1: Bitcoin Price

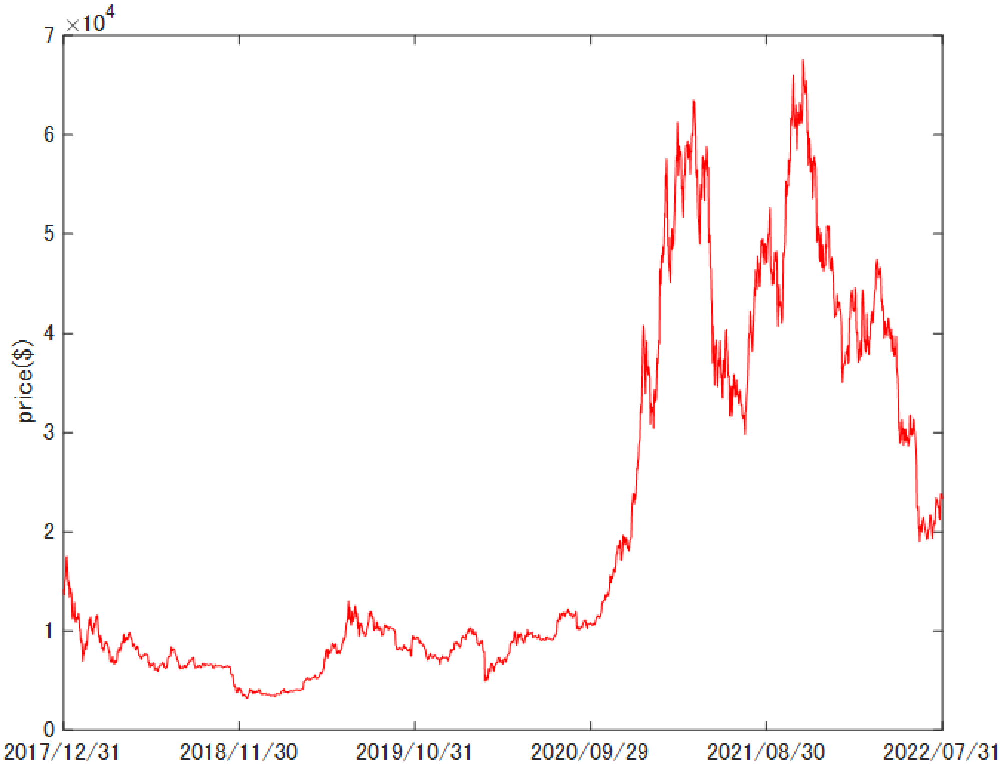


Figure 2: Ethereum Price

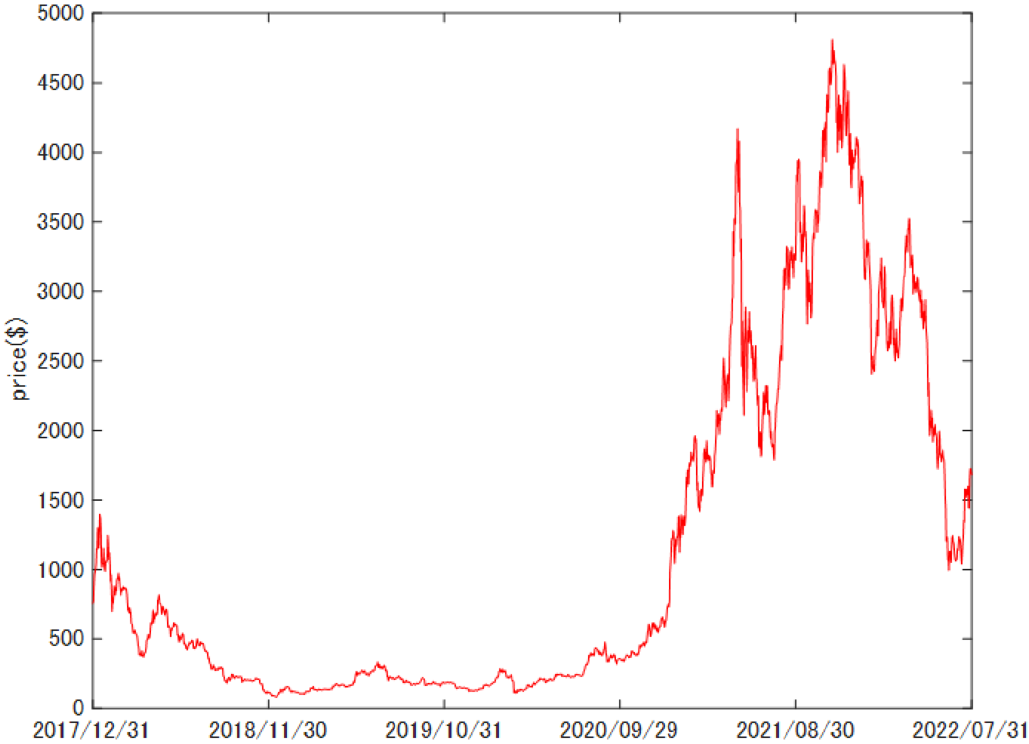


Figure 3: Binance-coin Price

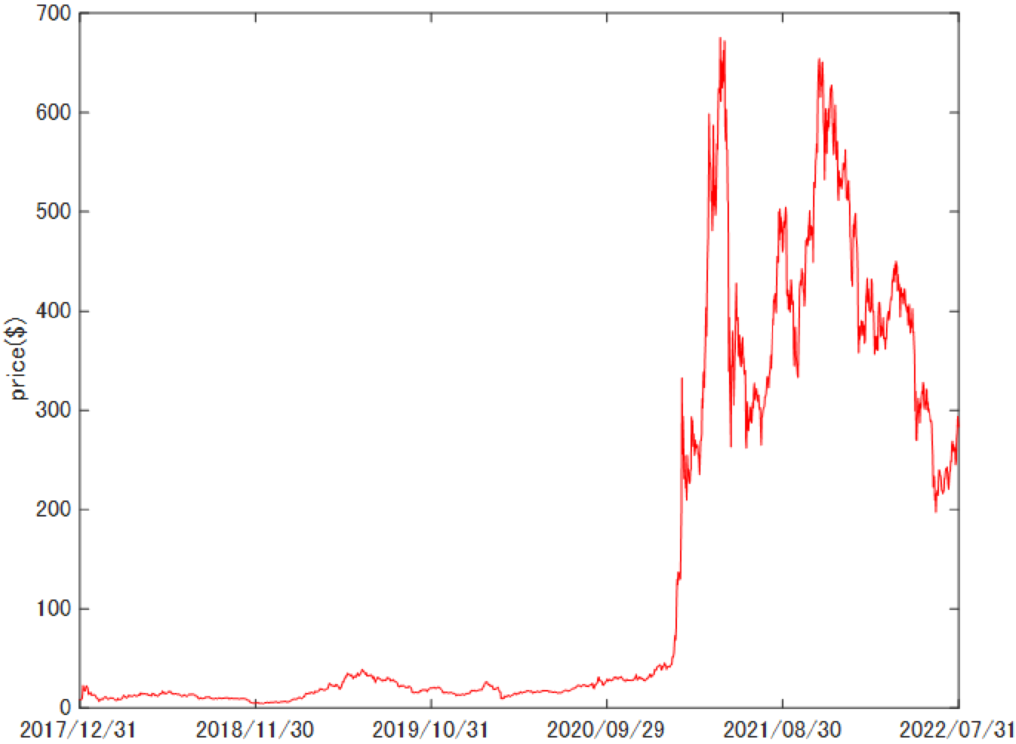
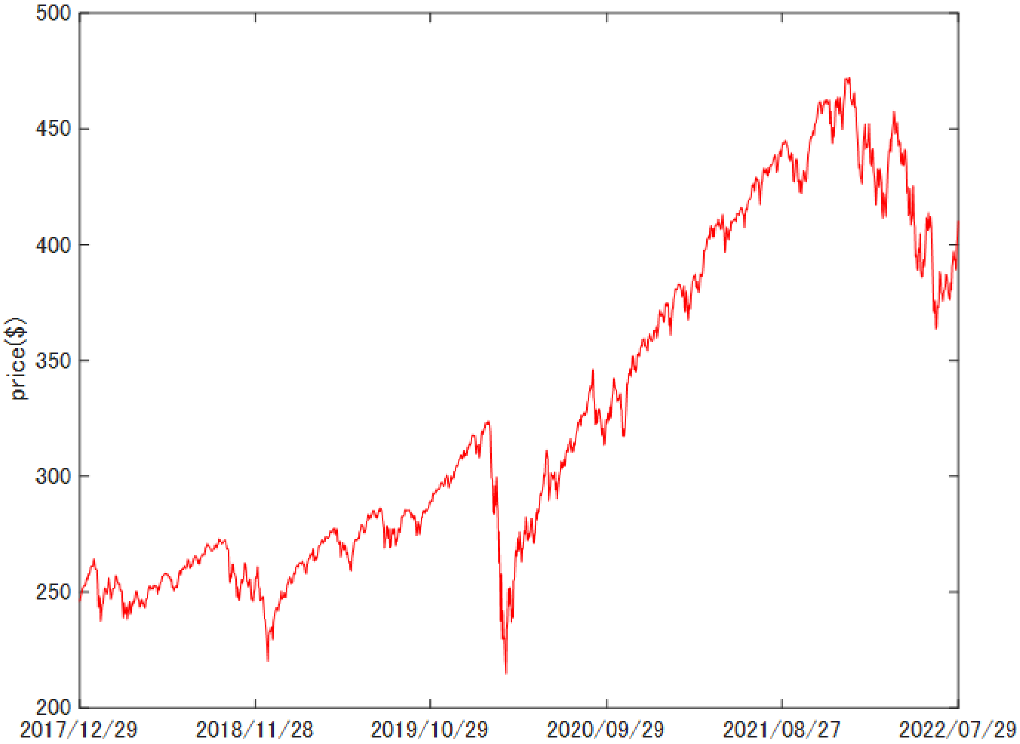


Figure 4: SPY Price



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