# Monetary equilibria with monopolistic competition and sticky prices

Tomoyuki Nakajima[*] Herakles Polemarchakis[†]

Working Paper No. 02-25

Department of Economics, Brown University

October, 2002

**Abstract**

We consider a cash-in-advance economy under uncertainty in which monetary policy sets either short-term nominal interest rates or money supplies. We show that both the initial price level and the distribution of the inflation rate up to its expectation are indeterminate, regardless of the degree of competition or the flexibility of prices in commodity markets. This indeterminacy is not related to the stability of a deterministic steady state.

Key words: sticky prices; monopolistic competition; monetary policy; uncertainty; indeterminacy.

JEL classification numbers: D50; E31; E40; E50.

[*]Department of Economics, Brown University; tomoyuki_nakajima@brown.edu
[†]Department of Economics, Brown University; herakles_polemarchakis@brown.edu

# 1 Introduction

We consider a stochastic cash-in-advance economy with monopolistic competition and sticky prices, and we examine the extent to which monetary policy can control the path of the equilibrium inflation rate. The controlability of inflation is of practical importance. If the indeterminacy of the inflation rate is associated with indeterminacy of the real allocation, it is the source of undesirable fluctuations. In the presence of indeterminacy, the definition of optimal monetary-fiscal policy as one that supports an optimal allocation [1] is inadequate. The possible role of fiscal policy in price-level determination, as emphasized, for example, by Woodford (1996), depends on the ability of monetary policy to control inflation.

Indeterminacy of non-monetary economies is often related to the dynamic properties of a deterministic steady state [2]. However, there is a different source of indeterminacy in monetary economies, which is closely related to the well known fact that only relative prices are determined in equilibrium. This type of indeterminacy, which is the one we examine here, does not derive from the stability of a deterministic steady state, and it does not rely on an infinite horizon. To stress this point, we consider a two-period economy, although the extension of our results to the infinite-horizon case would be straightforward.

We consider two forms of monetary policy: interest-rate policy and money-supply policy. The latter sets an exogenous path of money supplies, and the former an exogenous path of one-period nominal interest rates [3]. Fiscal policy satisfies an intertemporal budget constraint, and it pays off the public debt at the end of the last period, for all possible, equilibrium or out-of-equilibrium values of price levels, money supplies, and interest rates [4]. Monopolistically competitive producers supply differentiated commodities. Their prices are either flexible or sticky over time.

We show that both the initial price level and the distribution of the inflation rate up to its expectation are indeterminate, regardless of the degree of competition or the flexibility of prices in commodity markets; more formally, a "nominal

[1] A useful survey of this literature is given by Chari and Kehoe (1999).

[2] A useful survey of this literature is Benhabib and Farmer (1999).

[3] The monetary policy we examine here is not Taylor rule. However, given our specification of fiscal policy, considering Taylor rule does not affects the results.

[4] In the terminology of Woodford (1996), Benhabib, Schmitt-Grohé and Uribe (2001, 2002), fiscal policy is Ricardian.

equivalent martingale measure," and the initial price level index the indeterminacy at equilibrium. In particular, the degree of indeterminacy is exactly the same as in the economy with perfect competition in Nakajima and Polemarchakis (2001). In the flexible-price case, under interest-rate policy, the real allocation is unique, and hence, indeterminacy is nominal: different nominal equivalent martingale measures only affect inflation rates. In the sticky-price case, even under interest-rate policy, indeterminacy is real: different nominal equivalent martingale measures are associated with different real allocations. Under money-supply policy, indeterminacy is real both in the flexible-price and sticky-price cases. Also, under money-supply policy, there exists a zero-interest-rate equilibrium, unless money supply decreases too fast.

Our argument explains why fiscal policy may matter for the price-level determination. Monetary policy leaves the initial price level and the nominal equivalent martingale measure undetermined. This means that the degree of indeterminacy is exactly equal to the number of the terminal nodes of the date-event tree. The fiscal policy we consider leaves no public debt at any terminal node, whether at equilibrium or out-of equilibrium price levels, money supplies or interest rates. If, however, the fiscal policy is not constrained to satisfy an intertemporal budget constraint[5], it may impose additional restrictions on equilibrium price levels, since, at equilibrium, public debt is a fortiori paid off at each terminal node.

Our result is closely related to the indeterminacy result in Cass (1985), Balasko and Cass (1989), and Geanakoplos and Mas-Colell (1989). They considered an incomplete-market economy where money serves only as a unit of account, and they showed that the indeterminacy of inflation has real effects; since money is only an abstract unit of account, there is no room for monetary policy there. Bloise, Drèze and Polemarchakis (2000a,b) and Nakajima and Polemarchakis (2001) extended the results to a cash-in-advance economy with monetary and fiscal policy. Here, we extend those results to an economy with monopolistic competition and sticky prices.

The rest of the paper is organized as follows: In Section 2, we analyze equilibria in a monopolistic-competition model with flexible prices. In Section 3, we consider the case in which prices must be set in advance. In Section 4, we examine the case in which prices are set in a staggered manner.

[5]Such fiscal policy is non-Ricardian in Woodford (1996), Benhabib, Schmitt-Grohé and Uribe (2001, 2002); this is, also, the case for the specification in Dubey and Geanakoplos (1992).

## 2 Monopolistic Competition with Flexible Prices

We first describe the baseline economy with monopolistic competition. Money is valued through the cash-in-advance constraint as in Lucas and Stokey (1987). We shall see that the nominal equivalent martingale measure, as well as the initial price level, are indeterminate, and the degree of indeterminacy is equal to the number of terminal nodes of the date-event tree. This exactly parallels the case of perfect competition.[6]

### 2.1 Households

There are two dates: 0 and 1. States of the world at date 1 are indexed by $s \in \mathcal{S} = \{1, \dots, S\}$. Each state occurs with a probability $f(s) > 0$, $s \in \mathcal{S}$.

There is a continuum of households, distributed uniformly over $[0, 1]$. At each date-event, household $j \in [0, 1]$ produces a differentiated product $j$. Let $y_0(j)$ and $y_1(s, j)$ denote the amount of output produced by household $j$ at date 0 and at state $s \in \mathcal{S}$ at date 1, respectively. The amount of commodity $i \in [0, 1]$ consumed by household $j$ is denoted by $c_0^j(i)$ and $c_1^j(s, i)$, $s \in \mathcal{S}$.

The preferences of household $j$ are described by the lifetime expected utility

$$u\big[c_0^j, \overline{y}_0 - y_0(j)\big] + \beta \sum_{s=1}^{S} u\big[c_1^j(s), \overline{y}_1(s) - y_1(s, j)\big] f(s), \tag{1}$$

where $c_0^j$ and $c_1^j(s)$ are consumption of the "composite" goods defined by

$$c_0^j = \left\{ \int_0^1 \big[c_0^j(i)\big]^{\frac{\theta-1}{\theta}} di \right\}^{\frac{\theta}{\theta-1}},$$

$$c_1^j(s) = \left\{ \int_0^1 \big[c_1^j(s, i)\big]^{\frac{\theta-1}{\theta}} di \right\}^{\frac{\theta}{\theta-1}}, \qquad s \in \mathcal{S};$$

we interpret $\overline{y}$ as the endowment of time, and $(\overline{y} - y)$ as the consumption of leisure, $l$ [7].

**Assumption 1.** *The flow utility function, $u : \mathbb{R}^2_{++} \to \mathbb{R}$, is continuously differentiable, strictly increasing, and strictly concave. It also satisfies satisfies $u_{11}u_2 - u_{12}u_1 < 0$, $u_{22}u_1 - u_{12}u_2 < 0$, $\lim_{c \to 0} u_1 = \lim_{l \to 0} u_2 = \infty$.*

[6] For example, in Nakajima and Polemarchakis (2001).

[7] In the terminology of Lucas and Stokey (1987), $\overline{y}$ and $(\overline{y} - y)$ are the endowment and consumption of "credit goods," and $c$ is consumption of "cash goods."

Note that this assumption guarantees that $u_1(c, y-c)/u_2(c, y-c)$ is strictly decreasing in $c$; it guarantees the existence of an equilibrium, but not its determinacy.

Let $p_0(i)$ and $p_1(s, i)$ be the spot prices of good $i$ at date 0 and at state $s$ at date 1, respectively. Then, the prices of the composite goods, $P_0$ and $P_1(s)$, $s \in \mathcal{S}$, are given by

$$P_0 = \left\{ \int_0^1 \left[p_0(i)\right]^{1-\theta} di \right\}^{\frac{1}{1-\theta}},$$
$$P_1(s) = \left\{ \int_0^1 \left[p_1(s,i)\right]^{1-\theta} di \right\}^{\frac{1}{1-\theta}}, \qquad s \in \mathcal{S}.$$

Cost minimization by households leads to

$$P_0 c_0^j = \int_0^1 p_0(i) c_0^j(i)\, di,$$
$$P_1(s) c_1^j(s) = \int_0^1 p_1(s,i) c_1^j(s,i)\, di, \qquad s \in \mathcal{S}.$$

Let $c_0$ and $c_1(s)$, $s \in \mathcal{S}$, be the aggregate consumption at date 0 and state $s$, that is,

$$c_0 = \int_0^1 c_0^j \, dj, \quad \text{and} \quad c_1(s) = \int_0^1 c_1^j(s)\, dj.$$

The demand for product $j$, then, is

$$y_0(j) = \left( \frac{p_0(j)}{P_0} \right)^{-\theta} c_0, \tag{2}$$
$$y_1(s,j) = \left( \frac{p_1(s,j)}{P_1(s)} \right)^{-\theta} c_1(s), \qquad s \in \mathcal{S}. \tag{3}$$

Given these demand functions, household $j$ chooses $p_0(j)$ and $p_1(s,j)$, $s \in \mathcal{S}$, to maximize its lifetime utility.

As in Lucas and Stokey (1987), we assume that a household cannot consume what it produces; instead, it has to purchase each differentiated product with cash from other households. Also, the cash it obtains from selling its product has to be carried over to the next period.

Consider household $j \in [0, 1]$. We assume that households are identical except that what they produce are differentiated products. The household enters the initial period 0 with the nominal wealth $w_0$, that consists of cash, $\overline{m}$, and public debt, $b_0$,

$$w_0 = \overline{m} + b_0.$$

At the beginning of the period, the government distributes an equal amount of nominal transfers (taxes if negative), $\tau_0$, across households. Then the asset market opens, and there is a complete set of contingent claims. Let $q(s)$ be the price of the contingent claim that pays off one unit of currency if and only if state $s$ occurs at the following date. The budget constraint for the household in the asset market is

$$\hat{m}_0 + \sum_{s=1} q(s) b_1(s) \leq w_0 + \tau_0, \tag{4}$$

where $\hat{m}_0$ is the amount of cash obtained by the household, and $b_1(s)$ is the amount of each elementary security. If $r_0$ is the the nominal interest rate, the no-arbitrage condition implies that

$$\sum_{s=1}^{S} q(s) = \frac{1}{1 + r_0}. \tag{5}$$

The market for goods open next. The purchase of the consumption goods is subject to the cash-in-advance constraint

$$\int_0^1 p_0(i) c_0^j(i)\, di = P_0 c_0^j \leq \hat{m}_0. \tag{6}$$

The household also receives cash by selling its output, $y_0(j)$. Hence, the amount of cash that it carries over to the next period, $m_0$, is

$$m_0 = p_0(j) y_0(j) + \hat{m}_0 - P_0 c_0^j. \tag{7}$$

Given (7), the cash-in-advance constraint (6) is equivalent to the constraint on $m_0$ that

$$m_0 \geq p_0(j) y_0(j). \tag{8}$$

The household enters state $s$ at the second date with nominal wealth

$$w_1(s) = m_0 + b_1(s), \qquad s \in \mathcal{S}. \tag{9}$$

Substituting for $\hat{m}_0$ and $b_1(s)$ from (7) and (9) into (4) yields the flow budget constraint in period 0,

$$P_0 c_0^j + \frac{r_0}{1 + r_0} m_0 + \sum_{s=1}^{S} q(s) w_1(s) \tag{10}$$

$$\leq w_0 + \tau_0 + p_0(j) y_0(j)$$

The household's choice in the first period is subject to the flow budget constraint (10) and the cash constraint (8).

The transactions the household makes at the second date are similar, except for the fact that no uncertainty remains. Let $r_1(s)$ be the nominal interest rate in state $s \in \mathcal{S}$. Then the flow budget constraint and the cash constraint the household faces at state $s$ are given by

$$P_1(s)c_1^j(s) + \frac{r_1(s)}{1+r_1(s)}m_1(s) + \frac{1}{1+r_1(s)}w_2(s) \leq w_1(s) + \tau_1(s) + p_1(s,j)y_1(s,j), \tag{11}$$

and

$$m_1(s) \geq p_1(s,j)y_1(s,j), \tag{12}$$

where $w_2(s)$ is the nominal wealth the household leaves at the end of state $s$ in the second period.

Since the household cannot leave debt,

$$w_2(s) \geq 0 \tag{13}$$

the flow budget constraints (10) and (11) reduce to the single, lifetime budget constraint

$$P_0c_0^j + \frac{r_0}{1+r_0}m_0 + \sum_{s=1}^{S} q(s)\left\{P_1(s)c_1^j(s) + \frac{r_1(s)}{1+r_1(s)}m_1(s)\right\} \leq w_0 + \tau_0 + p_0(j)y_0(j) + \sum_{s=1}^{S} q(s)\{\tau_1(s) + p_1(s,j)y_1(s,j)\}. \tag{14}$$

Note that the cash constraints (8) and (12) are written as

$$\frac{r_0}{1+r_0}m_0 = \frac{r_0}{1+r_0}p_0(j)y_0(j),$$
$$\frac{r_1(s)}{1+r_1(s)}m_1(s) = \frac{r_1(s)}{1+r_1(s)}p_1(s,j)y_1(s,j), \qquad s \in \mathcal{S},$$

because, with $r > 0$, the cash constraint binds; if $r = 0$ both sides of the above equation are zero. Substituting these into the lifetime budget constraint, we obtain

$$P_0c_0^j + \sum_{s=1}^{S} q(s)P_1(s)c_1^j(s) \leq w_0 + \tau_0 + \frac{p_0(j)}{1+r_0}y_0(j) + \sum_{s=1}^{S} q(s)\left\{\tau_1(s) + \frac{p_1(s,j)}{1+r_1(s)}y_1(s,j)\right\} \tag{15}$$

Given prices, $P_0$, $P_1(s)$, $r_0$, $r_1(s)$, and $q(s)$, household $j$ chooses $c_0^j$, $c_1^j(s)$, $p_0(j)$, and $p_1(j)$ so as to utility (1) subject to the life-time budget constraint (15), and the demand functions for its product (2) and (3). The lifetime budget constraint should bind at optimum; that is,

$$w_2(s) = 0, \qquad s \in \mathcal{S} \tag{16}$$

The first-order conditions are

$$\frac{u_1\left[c_0^j, \overline{y}_0 - y_0(j)\right]}{u_2\left[c_0^j, \overline{y}_0 - y_0(j)\right]} = \left[1 + r_0\right] \frac{\theta}{\theta - 1} \frac{P_0}{p_0(j)} \tag{17}$$

$$\frac{u_1\left[c_1^j(s), \overline{y}(s) - y_1(s,j)\right]}{u_2\left[c_1^j(s), \overline{y}(s) - y_1(s,j)\right]} = \left[1 + r_1(s)\right] \frac{\theta}{\theta - 1} \frac{P_1(s)}{p_1(s,j)}, \tag{18}$$

$$\frac{\beta u_1\left[c_1^j(s), \overline{y}(s) - y_1(s,j)\right] f(s)}{u_1\left[c_0^j, \overline{y}_0 - y_0(j)\right]} = \frac{q(s) P_1(s)}{P_0}, \tag{19}$$

for all $s \in \mathcal{S}$ and $j \in [0, 1]$.

## 2.2 The monetary-fiscal authority

The flow budget constraints that the monetary-fiscal authority faces are

$$\frac{r_0}{1 + r_0} M_0 + \sum_{s=1}^{S} q(s) W_1(s) = W_0 + T_0, \tag{20}$$

$$\frac{r_1(s)}{1 + r_1(s)} M_1(s) + \frac{1}{1 + r_1(s)} W_2(s) = W_1(s) + T_1(s), \qquad s \in \mathcal{S} \tag{21}$$

where $M_0$ and $M_1(s)$ are money supplies, $W_0$, $W_1(s)$, $W_2(s)$ are the total liabilities of the monetary-fiscal authority, and $T_0$ and $T_1(s)$ are aggregate transfers to the households.

**Monetary Policy.** Monetary policy sets either nominal interest rates, $r_0$ and $r_1(s)$, $s \in \mathcal{S}$, or money supplies, $M_0$ and $M_1(s)$, $s \in \mathcal{S}$.

We assume that fiscal policy is "Ricardian," in the sense used by Woodford (1996) and Benhabib, Schmitt-Grohé and Uribe (2001, 2002), among others. In particular, we assume the following form of fiscal policy, which is a stochastic analogue of the one considered by Benhabib, Schmitt-Grohé and Uribe (2001, 2002). Let $\overline{W}_1(s)$, $s \in \mathcal{S}$, denote the "composition" of the debt portfolio of the monetary-fiscal authority, and $d$ be the "scale" of the debt:

$$W_1(s) = d\overline{W}_1(s), \quad \text{and} \quad \sum_{s \in \mathcal{S}} \overline{W}_1(s) = 1.$$

**Fiscal Policy.** The fiscal authority sets (1) the refinancing rate of the initial liability, $\alpha \in (0,1]$, and (2) the composition of the debt portfolio, $\overline{W}_1(s)$, $s \in \mathcal{S}$. At date 0, given $W_0$, $r_0$, and $M_0$, the transfer, $T(0)$, is determined by

$$T(0) = \frac{r_0}{1+r_0} M_0 - \alpha W_0,$$

and the scale of the debt portfolio, $d$, is determined by the flow budget constraint

$$d = \frac{1}{\sum q(s)\overline{W}_1(s)}(1-\alpha)W_0.$$

At each state $s$ at date 1, $T_1(s)$ is set as

$$T_1(s) = \frac{r(s)}{1+r(s)} M_1(s) - W_1(s),$$

where $W_1(s) = d\overline{W}_1(s)$.

Note that this fiscal policy rule implies that

$$W_2(s) = 0, \qquad s \in \mathcal{S}, \tag{22}$$

for all possible, equilibrium or non-equilibrium, values of $P$, $r$, and $M$.

## 2.3 Equilibrium conditions

Since households are symmetric, the market clearing conditions are given by

$$\begin{gathered} c_0 = c_0^j = y_0(i), \quad c_1(s) = c_1^j(s) = y_1(s,i), \\ P_0 = p_0(j), \quad P_1(s) = p_1(s,j), \\ m_0 = M_0, \quad m_1(s) = M_1(s), \\ w_1(s) = W_1(s), \quad w_2(s) = W_2(s), \end{gathered}$$

for all $i, j \in [0,1]$ and $s \in \mathcal{S}$. Also, consistency requires that

$$\tau_0 = T_0, \quad \tau_1(s) = T_1(s), \quad w_0 = W_0.$$

The no-arbitrage condition (5) implies that the prices of elementary securities, $q(s)$, $s \in \mathcal{S}$, can be written as

$$q(s) = \frac{\mu(s)}{1+r_0}, \qquad s \in \mathcal{S}, \tag{23}$$

for some positive $\mu(s)$, $s \in \mathcal{S}$, that satisfy

$$\sum_{s=1}^{S} \mu(s) = 1.$$

It follows that $\mu$ can be viewed as a probability measure over $\mathcal{S}$, the *nominal equivalent martingale measure*. We shall see that there are no equilibrium conditions that determine $\mu$, regardless of whether monetary policy sets interest rates or money supplies; in other words, $\mu$ is indeterminate. A symmetric equilibrium under interest-rate policy is defined as follows:

**Definition.** Given the initial public liability, $w_0 = W_0$, interest-rate policy, $\{r_0, r_1(s)\}$, and fiscal policy, $\{\alpha, \overline{W}_1(s)\}$, a symmetric equilibrium consists of an allocation, $\{c_0, c_1(s), y_0, y_1(s)\}$, a portfolio of households, $\{m_0, m_1(s), w_1(s), w_2(s)\}$, a portfolio of the monetary-fiscal authority, $\{M_0, M_1(s), W_1(s), W_2(s)\}$, transfers, $\{T_0, T_1(s)\}$, spot-market prices, $\{P_0, P_1(s)\}$, and nominal equivalent martingale measure, $\mu$, such that

(a) given $W_0$ and $\{r_0, r_1(s), M_0, M_1(s)\}$, fiscal policy $\{\alpha, \overline{W}_1(s)\}$ determines transfers $\tau_0 = T_0$ and $\tau_1(s) = T_1(s)$, $s \in \mathcal{S}$, and debt portfolio $\{W_1(s), W_2(s)\}$;

(b) the monetary authority accommodates the money demand, $M_0 = m_0$ and $M_1(s) = m_1(s)$, $s \in \mathcal{S}$;

(c) given interest rates, $r_0$, $r_1(s)$, spot-market prices, $p_0(j) = P(0)$, $p_1(s, j) = P_1(s)$, all $j$, nominal equivalent martingale measure, $\mu$, and transfers, $\tau_0$, $\tau_1(s)$, the household's problem is solved by $c_0^j = c_0$, $c_1^j(s) = c_1(s)$, $y_0(j) = y_0$, $y_1(j, s) = y_1(s)$, $m_0$, $m_1(s)$, $w_1(s)$, and $w_2(s)$;

(d) all market clear.

A symmetric equilibrium under money-supply policy is similarly defined.

### 2.4 Equilibria under interest-rate policy

Consider an interest-rate policy, $\{r_0, r_1(s), s \in \mathcal{S}\}$. To guarantee the existence of an existence of equilibrium, we restrict the boundary behavior of the flow utility function:

**Assumption 2.** *The flow utility function, $u$, satisfies*

$$\lim_{c\to 0} \frac{u_1(c, y-c)}{u_2(c, y-c)} = \infty,$$

*for each $y > 0$.*

The following proposition shows that $P_0$ and $\mu$ are not determined, and hence, there is $S$-dimensional indeterminacy, which is exactly the same result obtained in the case of perfect competition.

**Proposition 1.** *Monetary policy sets interest-rates, $\{r_0, r_1(s), s \in \mathcal{S}\}$. Fiscal policy sets $\alpha \in (0,1]$ and $\overline{W}_1(s)$, $s \in \mathcal{S}$. The initial liability is $w_0 = W_0$. Then*

*(a) a competitive equilibrium exists;*

*(b) the equilibrium allocation $\{c_0, c_1(s), y_0, y_1(s)\}$ is unique;*

*(c) the initial price, $P_0$, and the nominal equivalent martingale measure, $\mu$, are indeterminate: for any $P_0 > 0$ and for any strictly positive probability measure $\mu$, any prices and portfolio $\{P_1(s), M_0, M_1(s), W_1(s)\}$ satisfying*

$$\frac{P_1(s)}{P_0} = \frac{\beta u_1[c_1(s), \overline{y}_1(s) - y_1(s)] f(s)}{u_1[c_0, \overline{y}_0 - y_0]} \frac{1+r_0}{\mu(s)},$$

$$M_0 \geq P_0 c_0, \quad M_1(s) \geq P_1(s) c_1(s), \qquad \text{(equality if } r_0, r_1(s) > 0\text{)},$$

$$W_1(s) = (1-\alpha)(1+r_0) W_0 \frac{\overline{W}_1(s)}{\sum \mu(s) \overline{W}_1(s)}$$

*support the allocation $\{c_0, c_1(s), y_0, y_1(s)\}$.*

*Proof.* Given interest rates $r_0$ and $r_1(s)$, $s \in \mathcal{S}$, the first-order conditions (17)-(18) determines the allocation of resources at each date-event:

$$\frac{u_1\left[c_0, \overline{y}_0 - c_0\right]}{u_2\left[c_0, \overline{y}_0 - c_0\right]} = \left[1 + r_0\right] \frac{\theta}{\theta - 1}$$

$$\frac{u_1\left[c_1(s), \overline{y}_1(s) - c_1(s)\right]}{u_2\left[c_1(s), \overline{y}_1(s) - c_1(s)\right]} = \left[1 + r_1(s)\right] \frac{\theta}{\theta - 1}, \quad s \in \mathcal{S}.$$

Our assumptions on $u$ guarantees the existence and uniqueness of the solutions to these equations. The equilibrium output of each product $j$, $j \in [0,1]$, at each date-event, $y_0(j)$ and $y_1(s,j)$, $s \in \mathcal{S}$, is given by

$$y_0(j) = c_0, \quad \text{and} \quad y_1(s,j) = c_1(s), \quad s \in \mathcal{S}, \;\; j \in [0,1].$$

Fiscal policy sets transfers so that $w_2(s) = W_2(s) = 0$, all $s$. Hence, the allocation is uniquely determined. It is straightforward to see that given any $P_0 > 0$, and $\mu$, the prices and portfolio constructed as in the proposition support the equilibrium allocation. $\square$

Notice that the indeterminacy of $\mu$ implies that the inflation rate, $\pi_1(s) \equiv P_1(s)/P_0$ is indeterminate. Thus, interest-rate policy does not determine the stochastic path of inflation. Note also that the shock in this model could be purely extrinsic. If $r_1(s)$ and $\overline{y}_1(s)$ are identical for all $s$, the economy does not have any uncertainty in "fundamentals." Nevertheless, there are equilibria in which the inflation rate, $P_1(s)/P_0$, varies across states.

The reason why $P(0)$ and $\mu$ are indeterminate is simple, and closely related to the fact that only relative prices are determined in equilibrium. Look at equation (14). The relative prices between consumption and real balances are $r_0/[1+r_0]$ and $r_s/[1+r_s]$, which are set by monetary policy. Given these prices, the equilibrium real balances, $M_0/P_0$ and $M_1(s)/P_1(s)$, are determined. Also, the intertemporal relative prices of consumption, $q(s)P_1(s)/P_0$, are determined in equilibrium, which gives $S$ restrictions on $q(s)$, $P_1(s)$, and $P_0$ ($2S + 1$ prices). In addition, the no-arbitrage condition (23) imposes one restriction on $q(s)$. There are no further restrictions. Hence, there are $(S+1)$ equations in $(2S+1)$ variables, which leads to indeterminacy of degree $S$, and $P_0$ and $\mu$ are undetermined.

It is straightforward to extend the model to $T$ periods ($T$ may be infinity). Let $s_t \in \mathcal{S}$ be the shock realized at date $t$, and $s^t = (s_0, \dots, s_t) \in \mathcal{S}^t$ be the date-event (or history). Suppose again that there exists a complete set of elementary securities. Let $q_0(s^t)$ be the date-0 price of the elementary security that pays off one unit of currency if and only if $s^t$ occurs. Then, there exists a probability measure, $\mu$, over the set of date-events such that

$$q_0(s^t) = \prod_{v=0}^{t-1} \frac{1}{1 + r_v(s^v)} \mu(s^t),$$

for all $s^t \in \mathcal{S}^t$ and $t = 0, 1, \dots, T$. In the $T$-period economy, the initial price $P_0$, and the nominal equivalent martingale measure, $\mu$, are not determined. The degree of indeterminacy is, therefore, $S^T$, which is equal to the number of terminal nodes.

Indeterminacy may go away if the fiscal policy rule is such that the monetary-fiscal authority may leave non-zero debt for some (non-equilibrium) values of $P$

and $M$ (that is, if it is "non-Ricardian"), as discussed, for example, in Woodford (1994, 1996), Benhabib, Schmitt-Grohé and Uribe (2001), and Nakajima and Polemarchakis (2001).

### 2.5 Equilibria under money-supply policy

Consider a money-supply policy, $\{M_0, M_1(s)\}$. Define $c_0^*$ and $c_1^*(s)$, $s \in \mathcal{S}$, implicitly by

$$\frac{u_1\big[c^*, \overline{y}_0 - c_0^*\big]}{u_2\big[c^*, \overline{y}_0 - c_0^*\big]} = \frac{u_1\big[c^*, \overline{y}_1(s) - c_1^*(s)\big]}{u_2\big[c^*, \overline{y}_1(s) - c_1^*(s)\big]} = \frac{\theta}{\theta - 1}.$$

Such a $c^*$ exists and is unique under Assumption 1. Note that $c^*$ is the level of consumption when the nominal interest rate is zero. To guarantee the existence of an equilibrium, we impose an alternative boundary restriction on the flow utility function:

**Assumption 3.** *The flow utility function, $u$, has the property that for all $y > 0$,*

$$\lim_{c \to 0} c u_1(c, y - c) = 0,$$

*and the function $c u_1(c, y - c)$ is monotonically increasing in the interval $(0, c^*(y))$.*

The next proposition shows that under money-supply policy there is the same degree of indeterminacy as under interest-rate policy but indeterminacy is real.

**Proposition 2.** *Monetary policy sets the money-supply, $\{M_0, M_1(s), s \in \mathcal{S}\}$. Fiscal policy sets $\alpha \in (0, 1]$ and $\overline{W}_1(s)$, $s \in \mathcal{S}$. The initial liability is $w_0 = W_0$. Then*

*(a) a competitive equilibrium exists;*

*(b) the initial price, $P_0$, and the nominal equivalent martingale measure, $\mu$, are indeterminate. For any strictly positive $P_0$ and $\mu$, there exists a unique competitive equilibrium corresponding to them.*

*(c) the indeterminacy regarding $P_0$ and $\mu$ is real: different $P_0$ or different $\mu$ are associated with different allocations as well as different inflation rates.*

*Proof.* Let the initial price $P_0$ and the strictly positive probability measure $\mu$ be arbitrarily given. Let $M_0$ and $M_1(s)$, $s \in \mathcal{S}$, be the money supplies chosen by the policy. Given $M_0$ and $P_0$, $c_0$, $y_0$, and $r_0$ are determined by

$$c_0 = \min\left\{\frac{M_0}{P_0}, c_0^*\right\}, \quad 1 + r_0 = \frac{u_1[c_0, \overline{y}_0 - c_0]}{u_2[c_0, \overline{y}_0 - c_0]} \frac{\theta - 1}{\theta},$$

and $y_0 = \overline{y}_0 - c_0$. At state $s$ in the second period, if

$$M_1(s) > \frac{\beta c_1^*(s) u_1[c_1^*(s), \overline{y}_1(s) - c_1^*(s)]}{u_2[c_0, \overline{y}_0 - c_0]} \frac{f(s)}{\mu(s)} P_0,$$

then let $c_1(s) = c_1^*(s)$. Otherwise, $c_1(s)$ is a solution to

$$M_1(s) = \frac{\beta c_1(s) u_1[c_1(s), \overline{y}_1(s) - c_1(s)]}{u_2[c_0, \overline{y}_0 - c_0]} \frac{f(s)}{\mu(s)} P_0.$$

The unique existence of a solution is guaranteed by Assumption 3. Given $c_1(s)$, $y_1(s) = \overline{y}_1(s) - c_1(s)$,

$$P_1(s) = \frac{\beta u_1[c_1(s), \overline{y}_1(s) - c_1(s)]}{u_2[c_0, \overline{y}_0 - c_0]} \frac{f(s)}{\mu(s)} P_0,$$

and

$$1 + r_1(s) = \frac{u_1[c_1(s), \overline{y}_1(s) - c_1(s)]}{u_2[c_1(s), \overline{y}_1(s) - c_1(s)]} \frac{\theta - 1}{\theta}.$$

Given the path of nominal interest rates, $\{r_0, r_1(s)\}$, the debt portfolio, $\{W_1(s)\}$, is determined as in the proof of the previous proposition. □

As in the case with interest-rate policy, the shock could be purely extrinsic. That is, even when the second-period money supply does not depend on $s$, there are equilibria in which the allocation and inflation rate both vary across states. Also, the same remark applies for the $T$-period extension.

Given recent discussions on the "liquidity trap," the following corollary of the proposition would be of some interest.[8] It says that, as long as money supply does not decrease too much in the second period, there always exists an equilibrium in which the nominal interest rate equals zero at all date-events.

**Assumption 4.** *For all $s$,*

$$\frac{M_1(s)}{M_0} \geq \max_s \frac{\beta u_1[c_1^*(s), \overline{y}_1(s) - c_1^*(s)]}{u_2[c_0^*, \overline{y}_0 - c_0^*]}.$$

**Corollary 3.** *Consider money-supply policy, $\{M_0, M_1(s), s \in \mathcal{S}\}$. Fiscal policy sets $\alpha \in (0, 1]$ and $\overline{W}_1(s)$, $s \in \mathcal{S}$. The initial condition is $w_0 = W_0$. Suppose that Assumptions 1 and 4 are satisfied. Then, there exists a competitive equilibrium in which the nominal interest rate is identically zero, $r_0 = r_1(s) = 0$, all $s$.*

[8]For example, Benhabib, Schmitt-Grohé and Uribe (2002) and Woodford (1999). Note, however, that in this model, a zero interest rate equilibrium is (constrained) optimal.

*Proof.* Choose any $P_0 \leq M_0/c_0^*$ and let $\mu = f$. Then, it is straightforward to see that the following allocation and price system constitute an equilibrium.

$$c_0 = c_0^*, \qquad c_1(s) = c_1^*(s),$$

$$y_0 = \overline{y}_0 - c_0^*, \qquad y_1(s) = \overline{y}_1(s) - c_1^*(s),$$

$$P_1 = P_0 \frac{\beta u_1[c_1^*(s), \overline{y}_1(s) - c_1^*(s)]}{u_2[c_0^*, \overline{y}_0 - c_0^*]},$$

and

$$r_0 = r_1(s) = 0.$$

Assumption 4 guarantees that

$$\frac{M_1(s)}{P_1(s)} \geq c_1^*(s),$$

for all $s$. □

# 3 Prices Set in Advance

We have seen that if prices are changed freely at each date-event then equilibrium prices are not determined in spite of the fact that prices are set by monopolists. Now suppose that we are interested in an equilibrium with the property that the second-period prices, $P_1(s)$, are the same across states:

$$P_1(s) = P_1, \qquad s \in \mathcal{S}.$$

Such an equilibrium may appear similar to a "sticky-price equilibrium" in that the second-period price, $P_1(s)$, does not depend on the realization of the shock, $s$, and hence it is interpreted as "predetermined." The question asked in this section is whether or not such a requirement leads to the uniqueness of equilibrium. In fact, among equilibria considered in the economy with flexible prices, there is a unique $\mu$ with such a property. To see this, consider interest-rate policy, and let $P_1$ be the price level in the second period, which does not depend on states $s$. Remember that interest-rate policy determines an allocation uniquely. Then the inflation rate, $P_1/P_0$, is uniquely determined by the condition that

$$\frac{P_1}{P_0} = \sum_{s=1}^{S} \frac{\beta u_1\big[c_1(s), \overline{y}_1(s) - c_1(s)\big] f(s)}{u_1\big[c_1(s), \overline{y}_1(s) - c_1(s)\big]}.$$

Given $P_1/P_0$, $\mu(s)$ is, in turn, determined by

$$\mu(s) = \frac{\beta u_1\big[c_1(s), \overline{y}_1(s) - c_1(s)\big] f(s)}{u_2\big[c_1(s), \overline{y}_1(s) - c_1(s)\big]} \frac{P_0}{P_1}.$$

However, this only means the uniqueness of "flexible-price equilibrium" with the property that the second-period price is identical across states. It does not imply the uniqueness of "sticky-price equilibrium" in which all monopolists explicitly take into account the constraint that the second-period price must be set in advance. Indeed, we shall see that sticky-price equilibrium is indeterminate, and, furthermore, the degree of indeterminacy equals the number of terminal nodes, $S$, which is exactly the same as the one associated with flexible-price equilibrium.

Suppose that the initial prices $p_0(j)$, $j \in [0,1]$, are given and identical for all $j$:

$$p_0(j) = \overline{p}, \qquad j \in [0,1].$$

It follows that $P_0 = \overline{p}$. In the first period, each household $j \in [0,1]$ chooses the second-price, $p_1(s,j)$, before observing the shock $s$. It follows that the second-period price is identical across state, so that it is written as

$$p_1(s,j) = p_1(j), \qquad j \in [0,1],$$

for some $p_1(j)$. Given $\overline{p}$ and $p_1(j)$, the household must supply product $j$ by the amount equal to the demand:

$$y_0(j) = \left(\frac{\overline{p}}{P_0}\right)^{-\theta} c_0, \tag{24}$$

$$y_1(s,j) = \left(\frac{p_1(j)}{P_1(s)}\right)^{-\theta} c_1(s). \tag{25}$$

Given prices, $P_0$, $P_1(s)$, $r_0$, $r_1(s)$, $\mu(s)$, and $\overline{p}$, household $j$ chooses $c_0^j$, $c_1^j(s)$, and $p_1(j)$ so as to maximize the lifetime expected utility (1) subject to the demand functions (24)-(25) and the life-time budget constraint:

$$\begin{aligned} P_0 c_0^j + \sum_{s=1}^{S} \frac{\mu(s)}{1+r_0} P_1(s) c_1^j(s) \\ \le w_0 + \tau_0 + \frac{1}{1+r_0} \overline{p} y_0(j) \\ + \sum_{s=1}^{S} \frac{\mu(s)}{1+r_0} \left\{ \tau_1(s) + \frac{1}{1+r_1(s)} p_1(j) y_1(s,j) \right\}. \end{aligned} \tag{26}$$

The first-order conditions with respect to $c_0$ and $c_1(s)$ lead to

$$\frac{\beta u_1\left[c_1^j(s), 1-y_1(s,j)\right]f(s)}{u_1\left[c_0^j, 1-y_0(j)\right]} = \frac{P_1(s)}{P_0}\frac{\mu(s)}{1+r_0}, \qquad s\in\mathcal{S}. \tag{27}$$

The first-order condition with respect to $p_1(j)$ is given by

$$\frac{\sum_{s=1}^S u_1\left[c_1^j(s), \overline{y}(s)-y_1(s,j)\right]f(s)/\left[1+r_1(s)\right]}{\sum_{s=1}^S u_2\left[c_1^j(s), \overline{y}(s)-y_1(s,j)\right]f(s)} = \frac{\theta}{\theta-1}, \tag{28}$$

where we have used the equilibrium condition that $p_1(j) = P_1$, all $j\in[0,1]$.

As in the previous section, households are all symmetric. The market clearing conditions are the same as in the previous section.

Consider interest-rate policy, $\{r_0, r_1(s), s\in\mathcal{S}\}$. The next proposition shows that there is $S$-dimensional indeterminacy in this economy, and that indeterminacy is indexed by $P_1$ and $\mu$, just as in the economy with flexible prices. Here, however, indeterminacy is real.

**Proposition 4.** *Second-period prices are set in advance. Interest-rate policy is, $\{r_0, r_1(s), s\in\mathcal{S}\}$. Fiscal policy sets $\alpha\in(0,1]$ and $\overline{W}_1(s)$, $s\in\mathcal{S}$. The initial liability is $w_0 = W_0$ and the initial price level is $P_0 = \overline{p}$. Then*

*(a) a competitive equilibrium exists;*

*(b) the price level in the second period, $P_1$, and the nominal equivalent martingale measure, $\mu$, are indeterminate;*

*(c) the indeterminacy is real: different $P_1$ or different $\mu$ are associated with different allocations.*

*Proof.* Let $P_1$ and $\mu$ be given. Then the first-order conditions (27) imply that equilibrium consumption, $c_0$ and $c_1(s)$, $s\in\mathcal{S}$, should satisfy

$$\frac{\beta u_1\left[c_1(s), \overline{y}_1(s)-c_1(s)\right]f(s)}{u_1\left[c_0, \overline{y}_0-c_0\right]} = \frac{P_1}{P_0}\frac{\mu(s)}{1+r_0}, \qquad s\in\mathcal{S}.$$

Under our assumptions, these equations can be solved for $c_1(s)$ as strictly increasing functions of $c_0$. Write them as

$$c_1(s) = \phi_s(c_0), \qquad s\in\mathcal{S},$$

where $\lim_{c\to 0}\phi_s(c) = 0$ and $\lim_{c\to \overline{y}_0}\phi_s(c) = \overline{y}_1(s)$. The first-order condition (28) then implies that

$$\frac{\sum_{s=1}^{S} u_1\big[\phi_s(c_0), \overline{y}_1(s) - \phi_s(c_0)\big] f(s)/\big[1 + r_1(s)\big]}{\sum_{s=1}^{S} u_2\big[\phi_s(c_0), \overline{y}_1(s) - \phi_s(c_0)\big]} = \frac{\theta}{\theta - 1}.$$

Under our assumptions, there is a unique $c_0$ that satisfies this equation. This completes the proof. □

As in the previous section, considering money-supply policy instead of interest-rate policy does not change the degree of indeterminacy. In the $T$-period economy, $P_1$ and $\mu$ are not determined. The degree of indeterminacy is therefore $S^T$, which is equal to the number of terminal nodes, just as in the two-period economy.

# 4 Staggered Price Setting

In the previous section we have seen that the degree of indeterminacy remains the same even when the second-period price is set in advance in the first period. In this section, we see that the result is unchanged when prices are set in a staggered fashion.

Suppose that at the beginning of the initial period each household is allocated into one of two groups. Households in the first group must set the first-period price of its product, $p_0(j)$, at $\overline{p}$, but they can charge the second-period price, $p_1(s, j)$, freely. Households in the second group, on the other hand, can charge the first-period price freely, but they must charge the same price in the second period, thus $p_0(j) = p_1(s, j)$, all $s \in \mathcal{S}$. The allocation of households into these two groups is done stochastically, and the probability that each household is allocated to each group is 1/2. We assume that there is perfect risk sharing among households.

For simplicity, we restrict the form of the flow utility function.

**Assumption 5.** *The flow utility function is additively separable: $u(c) + v(\overline{y} - y)$, and*

$$\lim_{c\to 0} cu'(c) > 0.$$

The lifetime expected utility of household $j$ is then written as

$$\frac{1}{2}\Big\{u\big[c_0^1\big]+v\big[\overline{y}_0-y_0^1(j)\big]+\beta\sum_{s=1}^{S}\Big(u\big[c_1^1(s)\big]+v\big[\overline{y}_1(s)-y_1^1(s,j)\big]\Big)\Big\}$$
$$+\frac{1}{2}\Big\{u\big[c_0^2\big]+v\big[\overline{y}_0-y_0^2(j)\big]+\beta\sum_{s=1}^{S}\Big(u\big[c_1^2(s)\big]+v\big[\overline{y}_1(s)-y_1^2(s,j)\big]\Big)\Big\}$$

where $c_0^i$ and $c_1^i(s)$, $i=1,2$, are consumption when the household is allocated to group $i$, and $y_0^i(j)$ and $y_1^i(s,j)$ are production of product $j$. Let $p_1(s,j)$ be the price charged in the second period at state $s$ if the household is allocated to the first group; $p_0(j)$ be the price charged in both periods if the household is allocated to the second group. It follows that $y_0^i(j)$ and $y_1^i(s,j)$ are given by

$$y_0^1(j)=\left(\frac{\overline{p}}{P_0}\right)^{-\theta}c_0,\qquad y_1^1(s,j)=\left(\frac{p_1(s,j)}{P_1(s)}\right)^{-\theta}c_1(s),\quad s\in\mathcal{S},$$
$$y_0^2(j)=\left(\frac{p_0(j)}{P_0}\right)^{-\theta}c_0,\qquad y_1^2(s,j)=\left(\frac{p_0(j)}{P_1(s)}\right)^{-\theta}c_1(s),\quad s\in\mathcal{S},$$

where $c_0$ and $c_1(s)$ denote aggregate consumption.

Since there is perfect risk sharing among households, consumption is identical between the two groups:

$$c_0^1=c_0^2=c_0,\quad\text{and}\quad c_1^1(s)=c_1^2(s)=c_1(s),\quad s\in\mathcal{S}.$$

The first-order conditions with respect to $c_0$ and $c_1(s)$ lead to

$$\frac{\beta u'\big[c_1(s)\big]f(s)}{u'\big[c_0\big]}=\frac{P_1(s)}{P_0}\frac{\mu(s)}{1+r_0}. \tag{29}$$

The first-order condition with respect to the second-period price charged by the household in the first group, $p_1(s,j)$, is given as: for each $s\in\mathcal{S}$,

$$v'\big[\overline{y}_1(s)-y_1^1(s,j)\big]=u'\big[c_1(s)\big]\frac{p_1(s,j)}{P_1(s)}\frac{\theta-1}{\theta}\frac{1}{1+r_1(s)}. \tag{30}$$

The first-order condition with respect to the price charged in both periods by the second group of households, $p_0(j)$, is

$$y_0^2(j)\left(v'\big[\overline{y}_0-y_0^2\big]-u'\big[c_0\big]\frac{p_0(j)}{P_0}\frac{\theta-1}{\theta}\frac{1}{1+r_0}\right) \tag{31}$$
$$+\beta\sum_{s=1}^{S}y_1^2(s,j)\left(v'\big[\overline{y}_1(s)-y_1^2(s)\big]-u'\big[c_1(s)\big]\frac{p_0(j)}{P_1(s)}\frac{\theta-1}{\theta}\frac{1}{1+r_1(s)}\right)f(s)=0.$$

In a symmetric equilibrium, households in the same group choose the same prices, so that we can write

$$p_0(j) = p_0, \qquad p_1(s,j) = p_1(s),$$
$$y_0^1(j) = y_0^1, \qquad y_0^2(j) = y_0^2,$$
$$y_1^1(s,j) = y_1^1(s), \qquad y_1^2(s,j) = y_1^2(s).$$

By definition, the price levels, $P_0$ and $P_1(s)$, are given by

$$P_0 = \left[\frac{1}{2}\overline{p}^{1-\theta} + \frac{1}{2}p_0^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{32}$$

$$P_1(s) = \left[\frac{1}{2}p_0^{1-\theta} + \frac{1}{2}p_1(s)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{33}$$

Note that $P_1(s)/P_0$ is an increasing function of $p_1(s)/P_1(s)$, and $p_0/P_1(s)$ is a decreasing function of $p_1(s)/P_1(s)$. Production of differentiated products is given by

$$y_0^1 = \left(\frac{\overline{p}}{P_0}\right)^{-\theta} c_0, \qquad y_1^1(s) = \left(\frac{p_1(s)}{P_1(s)}\right)^{-\theta} c_1(s), \quad s \in \mathcal{S}, \tag{34}$$

$$y_0^2 = \left(\frac{p_0}{P_0}\right)^{-\theta} c_0, \qquad y_1^2(s,j) = \left(\frac{p_0}{P_1(s)}\right)^{-\theta} c_1(s), \quad s \in \mathcal{S}. \tag{35}$$

Consider interest-rate policy, $\{r_0, r_1(s), s \in \mathcal{S}\}$. The next proposition shows that this economy, once again, has $S$-dimensional indeterminacy, indexed by the initial price $P_0$ and the nominal equivalent martingale measure $\mu$.

**Proposition 5.** *Price setting is staggered. Interest-rate policy is $\{r_0, r_1(s), s \in \mathcal{S}\}$. Fiscal policy sets $\alpha \in (0,1]$ and $\overline{W}_1(s)$, $s \in \mathcal{S}$. The initial conditions are $w_0 = W_0$ and $\overline{p}$. Suppose that Assumptions 1 and 5 are satisfied. Then*

- *(a) a competitive equilibrium exists;*
- *(b) the initial price level, $P_0$, and the nominal equivalent martingale measure, $\mu$, are indeterminate;*
- *(c) the indeterminacy is real: different $P_0$ or different $\mu$ are associated with different allocations.*

*Proof.* Let $P_0$ and $\mu$ be given. Note that $P_0$ determines $p_0$ by (32). Consider the first-order conditions (30):

$$v'\left(\overline{y}_1(s) - \left(\frac{p_1(s)}{P_1(s)}\right)^{-\theta}\right) c_1(s) = u'\big[c_1(s)\big]\frac{p_1(s)}{P_1(s)}\frac{\theta-1}{\theta}\frac{1}{1+r_1(s)},$$

for each $s \in \mathcal{S}$. These equations imply that $c_1(s)$ is a strictly increasing function of $p_1(s)/P_1(s)$. Given the fact that $P_1(s)/P_0$ is a strictly increasing function of $p_1(s)/P_1(s)$, the first-order conditions (29) determine $p_1(s)/P_1(s)$, as a strictly increasing function of $c_0$. Then, consider (31), and note that the left-hand side of this equation is strictly increasing in $c_0$ and becomes negative as $c_0 \to 0$. Hence, there is a unique $c_0$. Note that $c_1(s)$, $p_1(s)/P_1(s)$, and $P_1(s)/P_0$ are functions of $c_0$ and derived above. This completes the proof. $\square$

As in the previous Sections, money-supply policy does not change the degree of indeterminacy. In the $T$-period economy, indeterminacy is indexed by the initial price level, $P_0$, and the nominal equivalent martingale measure, $\mu$. The dimension of indeterminacy is therefore $S^T$, as in the economies studies we considered earlier.

## 5 Conclusion

We have considered various specifications of a monetary economy, and we have seen that in every one competitive equilibria display indeterminacy of dimension equal to the number of terminal nodes. It is immaterial whether prices are flexible, set in advance, or set in a staggered way; whether monetary policy sets interest rates or money supplies; or whether money derives its value from cash-in-advance constraints — money-in-the-utility generates the same degree of indeterminacy.

## References


[1] Balasko, Y. and D. Cass (1989), "The structure of financial equilibrium, I: exogenous yields and unrestricted participation," *Econometrica*, 57, 135-162.

[2] Benhabib, J., and R. E. A. Farmer, 1999, "Indeterminacy and Sunspots in Macroeconomics," in J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Volume 1A, Elsevier, 387-448.

[3] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2001), "Monetary policy and multiple equilibria," *American Economic Review*, 91, 167-186.

[4] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2002), "Avoiding liquidity traps," *Journal of Political Economy*, 110, 535-563.

[5] Bloise, G., J. H. Drèze and H. M. Polemarchakis (2000a), "Monetary equlibria over a finite horizon," Mimeo., CORE, Université Catholique de Louvain.

[6] Bloise, G., J. H. Drèze and H. M. Polemarchakis (2000b), "Monetary equlibria over an infinite horizon," Mimeo., CORE, Université Catholique de Louvain.

[7] Cass, D. (1985), "On the 'number' of equilibrium allocations with finacial markets," Working Paper No. 8500, CARESS, University of Pennsylvania.

[8] Chari, V. V. and P. J. Kehoe (1999), "Optimal fiscal and monetary policy," in J. B. Taylor and M. Woodford (eds.) *Handbook of Macroecnomics*, Volume 1C, Elsevier, 1671-1743.

[9] Geanakoplos, J. D. and A. Mas-Colell (1989), "Real indeterminacy with financial assets," *Journal of Economic Theory*, 47, 22-38.

[10] Lucas, R. E. and N. L. Stokey (1987), "Money and interest rates in a cash-in-advance economy," *Econometrica*, 55, 491-513.

[11] Nakajima, T. and H. M. Polemarchakis (2001), "Money and prices under uncertainty," Mimeo., Brown University.

[12] Woodford, M. (1994), "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic Theory*, 4, 345-380.

[13] Woodford, M. (1996), "Control of the public debt: A requirement for price stability?" NBER working paper no. 5684.

[14] Woodford, M. (1999), "Price-level determination under interest-rate rules," Mimeo., Princeton University.