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The Distribution of Human Capital and Economic Growth*

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This paper analyzes the interaction between the distribution of human capital, technological progress, and economic growth. It argues that the composition of human capital is an important factor in the determination of the pattern of economic development. The study demonstrates that the evolutionary pattern of the human capital distribution, the income distribution and economic growth are determined simultaneously by the interplay between a *local* home environment externality and a *global* technological externality. In early stages of development the local home environment externality is the dominating factor and hence the distribution of income becomes polarized: whereas in mature stages of development the global technological externality dominates and the distribution of income ultimately contracts. Polarization, in early stages of development may be a necessary ingredient for future economic growth. An economy that prematurely implements a policy designed to enhance equality may be trapped at a low stage of development. An underdeveloped economy, which values equality as well as prosperity, may confront a trade-off between equality in the short-run followed by equality and stagnation in the long run, and inequality in the short-run followed by equality and prosperity in the long run.

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1. Introduction

This paper analyses the interaction between the distribution of human capital, technological progress, and economic growth. It demonstrates the significant role that the distribution of human capital may play in the determination of the pattern of economic growth.

The paper rests upon two observations that are largely supported by empirical evidence: (a) An individual's level of human capital is an increasing function of the parental level of human capital - - the home environment externality.[1] (b) Technological progress (or the rate of adoption of new technologies), is positively related to the average level of human capital in society - - the global technological externality.[2] The analysis demonstrates that the interplay between the local home environment externality and the global technological externality governs the evolutionary patterns of the distribution of human capital, the distribution of income, and economic growth. In periods in which the home environment externality is the dominating factor, the distribution of income becomes polarized, whereas in periods in which the global technological externality dominates, income convergence ultimately takes place. In early stages of development the local externality may generate polarization, which provides the necessary seeds for future economic growth, whereas in mature stages of development, the growth process may generate a global technological externality which induces income convergence.[3]

The paper develops a model of a small overlapping generations economy in which individuals within as well as across generations are identical in their preferences and their production technology of human capital. They may differ, however, in the parental level of human capital, and thus in the efficacy of their own investment in human capital. The individual's level of human capital increases with the resources invested in its formation and with the parental level of human capital. Parents have a dual effect on the incentives of their children to invest in human capital. First, parents affect their children directly

[1]The importance of parental educational input in the formation of the human capital of the child has been explored theoretically as well as empirically. The empirical significance of the parental effect has been documented by Coleman et al. (1966), Becker and Tomes (1986), as well as others.

[2]As was argued by Schultz (1975), the ability of individuals to adapt to changing economic environment (i.e., to an environment characterized by technological change) is positively related to their level of human capital.

[3]Global and local externalities have been utilized by several other studies (e.g., Benabou (1996), Iyigun and Owen (1996), Mountford (1996), and Tamura (1996)).

via a home environment that facilitates better schooling for a given level of investment in human capital. Second, parents affect their children indirectly, via their contribution to the average level of human capital in the society as a whole, which in turn affects the magnitude of the labor augmenting technological progress in the next period. This technological progress increases the rate of return on investment in human capital for the children's generation and consequently stimulates further investment in human capital. The interaction between individuals within a dynasty is via the home environment externality; whereas, the interaction across dynasties emerges via the global production externality.

The dynamical system that governs the evolution of human capital within a dynasty may be characterized (for some ranges of technological level) by multiple locally stable steady-state equilibria. In these configurations, the level of human capital in the long run differs across dynasties, despite the suppositions that ability is identical across individuals, capital markets are perfect, and the economy is deterministic.[4] The initial level of human capital of each dynasty determines the evolutionary pattern of human capital within a dynasty and its long run equilibrium level. The (local) home environment externality induces, therefore, the creation of inequality in the distribution of human capital. In contrast, the interaction between dynasties and its impact on the production technology may bring about a qualitative transformation of the dynamical system from one that is characterized by multiple locally stable steady-state equilibria to another with a unique globally stable steady-state equilibrium. Under this scenario, the distribution of human capital ultimately contracts and the long run properties of the distribution of human capital are independent of its initial structure. Thus, in stages of development in which the home environment externality is the dominating factor, the distribution of human capital becomes polarized, whereas in those stages in which the global technological externality dominates, convergence ultimately takes place.

The study therefore suggests that a relatively poor economy that values equity as well as prosperity may confront a trade-off between equality in the short-run followed by equality and stagnation in the long run and inequality in the short-run followed by equality and prosperity in the long run. A wide distribution of human capital may be

[4]The current paper abstracts from the important role that capital markets imperfections play in the investment in human capital. See Galor and Zeira (1993).

essential in order to increase the aggregate level of human capital and output during the early stages of development. Inequality may enable members of the highly educated segments of society to overcome the gravitational forces of a low, stable, steady-state equilibrium and to increase their investment in human capital, whereas equality may trap the society as a whole at a low level of investment in human capital. As the investment in human capital of the highly educated segments of society increases and income inequality widens, the accumulated knowledge trickles down to the less-educated segments of society via a technological progress in production. The return to skill improves, and investment in human capital becomes more beneficial to members of all segments of society. The economy may therefore find it beneficial to subsidize the education of selected groups of individuals that may ultimately generate enough externalities to pull the society as a whole to a state of equity and prosperity. Furthermore, an economy that prematurely implements a policy designed to enhance equality in the distribution of income may be trapped unnecessarily at a low output equilibrium without ever reaching prosperity.

The paper contributes to two important recent research strands within the field of economic growth: human capital and growth (e.g., Lucas (1988), Azariadis and Drazen (1990), Barro (1991), and Mankiw, Romer, Weil (1992), and income inequality and growth (e.g., Galor and Zeira (1993) Benabou (1996) and Durlauf, (1996) and Perotti (1996)). In contrast to the existing literature, the paper analyzes the reciprocal relation between the distribution of human capital and economic growth, suggesting that (a) the composition of human capital, in addition to the average level of human capital, is an important factor in the process of development; (b) the distribution of human capital evolves non-monotonically in the growth process; polarization may be a necessary condition for a take-off; and (c) the observed relationship between income distribution and economic growth may be governed by the relationship between the distribution of human capital and economic growth.

2. The Model

Consider a small open overlapping-generations economy that operates in a perfectly competitive world in which economic activity extends over an infinite discrete time. In every period the economy produces a single homogenous good, using physical capital and efficiency units of labor in the production process. The good can be used for consumption, investment in the acquisition of human capital, or saving. The supply of physical capital in every period is constituted by the aggregate saving of individuals in the economy, in addition to net international borrowing, whereas the supply of efficiency labor in every period is the outcome of the economy's aggregate investment in human capital in the preceding period.

2.1 The Production of Goods

Production occurs within a period according to a constant-returns-to-scale neoclassical production technology that is subject to endogenous technological progress. The output produced at time t, Y_t, is

$$Y_t = F(K_t, \lambda_t H_t) \equiv \lambda_t H_t f(k_t); \quad k_t \equiv K_t/(\lambda_t H_t), \tag{2.1}$$

where K_t and H_t are the quantities of capital and efficiency-labor employed in production at time t, and λ_t is the technological coefficient at time t. Changes in λ_t reflects therefore a labor augmenting technological change at time t. The production function $f(k_t)$ is strictly monotonic increasing, strictly concave satisfying the neoclassical boundary conditions that assure the existence of an interior solution to the producers' profit-maximization problem.

Producers operate in a perfectly competitive environment. Given the wage rate and the rate of return to capital at time t, w_t and r_t respectively, producers choose the level of employment of capital, K_t, and labor, H_t, so as to maximize profits. That is, $\{K_t, H_t\} = \text{argmax}\ [\lambda_t H_t f(k_t) - w_t H_t - r_t K_t]$. The producers' inverse demand for factors of production is therefore

$$\begin{aligned} r_t &= f'(k_t); \\ w_t &= \lambda_t[f(k_t) - f'(k_t)k_t] \equiv \lambda_t w(k_t). \end{aligned} \tag{2.2}$$

2.2 Factor Prices

Suppose that the world rental rate is stationary at a level $\overline{r}$. Since the small economy permits unrestricted international lending and borrowing, its rental rate is stationary as well at the rate $\overline{r}$.[5] Namely,

$$r_t = \overline{r}. \tag{2.3}$$

[5] The choice of the framework of a small open economy with perfect capital mobility is based primarily on the observation that interest rates do not decrease significantly in the course of economic growth. In a closed economy, however, there exists another channel for the more educated to affect the incentive

Consequently, the ratio of capital to efficiency units of labor in every time t, k_t, is stationary at a level $f'^{-1}(\bar{r}) \equiv \bar{k}$, and the wage rate per efficiency labor, w_t, is

$$w_t = \lambda_t w(\bar{k}) \equiv \lambda_t \bar{w}. \tag{2.4}$$

2.3 Technological Progress

The level of technology employed at time $t+1$ in the production of goods, λ_{t+1}, advances with the average level of human capital of the previous generation, h_t.

$$\lambda_{t+1} = \max[\lambda(h_t), \lambda_t], \tag{2.5}$$

where $h_t \equiv H_t/N$, and N is the measure of individuals within a generation. The level of technology is a monotonically nondecreasing concave function of the average level of human capital in the preceding period.

Remark 2.1. A labor augmenting technological progress does not alter the wage ratio between high-skill and low-skill labor. Nevertheless, since (as will become apparent in section 2.4) the material cost of education is not affected by this technological progress, the income ratio between high-skill and low-skill labor increases, and the technological progress may be viewed as skilled biased in accordance with the interpretations for most technological progress in recent history (e.g., Katz and Murphy (1992) and Mincer (1996)).

2.4 Consumption, Savings, and Investment in Human Capital

In every period a generation, which consists of a continuum of individuals of measure N, is born.[6] Individuals, within as well as across generations, are identical in their preferences and their production technology of human capital. However, they may differ in their parental level of human capital, and thus in the efficacy of their own investment in education. Individuals live three periods. In the first period, individuals invest real resources as well as their time endowment in the production of human capital. They finance their consumption and investment in human capital via borrowing at the market

scheme of the less-educated. As the more educated increase their investment in human capital, their income increases and consequently savings and capital formation increase as well. The wage per efficiency unit of labor increases and the interest rate decreases. Consequently, the less-educated have a greater incentive to invest in human capital.

[6]For simplicity there exists no population growth. Clearly, the qualitative results of this paper are not sensitive for changes in this assumption.

interest rate. In the second period they supply inelastically (at the competitive market wage) the resulting efficiency units of labor, and allocate the income net of loan repayments between consumption and saving. In the third period, individuals retire, using their savings for consumption.

Individuals' preferences are defined over the vector of consumption in all three periods of their lives, $(c_t^{t,i}, c_{t+1}^{t,i}, c_{t+2}^{t,i}) \equiv c^{t,i}$. [7] The preferences of an individual i who is born at time t (a member i of generation t) are represented by the intertemporal utility function,

$$u^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, c_{t+2}^{t,i}), \tag{2.6}$$

where $c_j^{t,i}$ is the consumption of a member i of generation t in period j, $j = t,\ t{+}1,\ t{+}2$. The utility function is strongly monotonic, strictly quasi-concave, satisfying the conventional boundary conditions that assure the existence of an interior solution for the utility maximization problem.

In the first period of their lives, members of generation t invest in human capital. The acquisition of skills requires real resources. In the absence of income, individuals borrow the necessary capital at the market interest rate $\bar{r}$. A member i of generation t who is born to a parent with h_t^i units of human capital and who, at time t, invests x_t^i units of real resources and one unit of labor in the formation of human capital, acquires h_{t+1}^i units of human capital.[8] These h_{t+1}^i units constitute the individual's labor supply in the second period of life.

$$h_{t+1}^i = \phi(h_t^i, x_t^i). \tag{2.7}$$

Assumption 2.1. $\forall (h_t^i, x_t^i) \in \mathbb{R}_{++}^2$

- $\phi:\ \mathbb{R}_+^2 \to \mathbb{R}_+;\quad \phi \in C^3$.
- $\phi(h_t^i, 0) = \mu \geq 0$.
- $D\phi(h_t^i, x_t^i) >> 0$.
- $\phi_{12}(h_t^i, x_t^i) > 0$ *and* $\phi_{jj}(h_t^i, x_t^i) < 0, \quad j = 1, 2$.
- $\lim_{x_t^i \to 0} \phi_2(h_t^i, x_t^i) = \infty$.

The properties of the production function of human capital reflect some of the important elements that form the foundations for the theory:

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[7]The implications of intergenerational altruism is considered in Section 4.1.

[8]The implications of uncertainty with respect to the outcome of the investment in human capital are analyzed in Section 5.

(a) *The individuals' level of human capital is an increasing function of the parental level of human capital (i.e.,* $\phi_1(h_t^i, x_t^i) > 0$*).*

The importance of the parental educational input in the formation of the human capital of the child has been explored theoretically as well as empirically. The empirical significance of the parental effect has been documented by Coleman et al. (1966), Becker and Tomes (1986), as well as others.

(b) *The individual's level of human capital is an increasing function of the individual's investment of real resources (i.e.,* $\phi_2(h_t^i, x_t^i) > 0$*).*

The importance and the empirical significance of private as well as the public educational inputs is well documented in the literature. For a comprehensive survey of the related literature see Hanushek (1986).

(c) *The production function of human capital is characterized by complementarity between the parental human capital effect and the private resources invested in the production of human capital (i.e.,* $\phi_{12}(h_t^i, x_t^i) > 0$*).*

(d) *There exist diminishing returns to the parental human capital effect and to the level of resources invested in the production of human capital (i.e.,* $\phi_{jj}(h_t^i, x_t^i) < 0 \ \ \forall j, \ j = 1, 2$*).*

Parents have a dual effect on the incentives of their children to invest in human capital. The parent affects the child directly via the home environment that facilitates better schooling for a given level of investment in human capital. This *(local) home environment externality* is captured by the properties of the function $\phi(h_t^i, x_t^i)$ with respect to h_t^i (i.e., the parental level of human capital increases ϕ with diminishing rates). In addition, parents affect their children indirectly via their contribution to the average level of human capital in the society as a whole, which in turn, affects the magnitude of the labor augmenting technological progress in the next period. This technological progress increases the rate of return on investment in human capital for the children's generation and consequently stimulates further investment in human capital. This *(global) technological externality* is captured by the function $\lambda(h_t)$ which is nondecreasing in the average level of human capital of the parents' generation, h_t.[9]

[9] One may consider an alternative formulation in which the average level of human capital affects the production of human capital rather than the production technology of goods. This specification generates similar qualitative results about the evolution of the distribution of human capital along the process of growth, but it eliminates the important interaction with technological progress.

The labor income generated by an individual i from generation t at time $t+1$, I_{t+1}^i, is therefore the wage rate per efficiency-labor at time $t+1$, w_{t+1}, times the number of efficiency units supplied by the individual, h_{t+1}^i.

$$I_{t+1}^i = w_{t+1}h_{t+1}^i = \overline{w}\lambda_{t+1}h_{t+1}^i. \tag{2.8}$$

The labor income, net of loan repayment, is allocated between savings, s_{t+1}^i and consumption $c_{t+1}^{t,i}$. The saving of a member i of generation t at time $t+1$, s_{t+1}^i, is therefore

$$s_{t+1}^i = \overline{w}\lambda_{t+1}h_{t+1}^i - \overline{R}(x_t^i + c_t^{t,i}) - c_{t+1}^{t,i}, \tag{2.9}$$

where $\overline{R} \equiv 1 + \overline{r}$.

Consumption of a member i of generation t at time $t+2$, $c_{t+2}^{t,i}$, is therefore the gross return on the savings from time $t+1$ according to the international interest factor $\overline{R}$.

$$c_{t+2}^{t,i} = \overline{R}[\overline{w}\lambda_{t+1}h_{t+1}^i - \overline{R}(x_t^i + c_t^{t,i}) - c_{t+1}^{t,i}]. \tag{2.10}$$

Given the interest factor, $\overline{R}$, the expected wage rate per efficiency unit of labor, $\overline{w}\lambda_{t+1}$, and the level of the parental human capital, h_t^i, an individual i of generation t chooses the level of investment in human capital, x_t^i, first period consumption, $c_t^{t,i}$, and second period savings s_{t+1}^i, so as to maximize the intertemporal utility function $u^{t,i}$. Namely,

$$\{x_t^i, c_t^{t,i}, s_{t+1}^i\} = \text{argmax } u[c_t^{t,i},\ \overline{w}\lambda_{t+1}\phi(h_t^i, x_t^i) - \overline{R}(x_t^i + c_t^{i,t}) - s_{t+1}^i,\ \ \overline{R}s_{t+1}^i] \tag{2.11}$$

$$\text{subject to} \quad (x_t^i, c_t^{t,i}, s_{t+1}^i) \geq 0.$$

Given the assumptions about the utility function and the production function of human capital, there exists a unique and interior solution to the maximization problem that is characterized by the necessary and sufficient conditions

$$\frac{u_1(c^{t,i})}{u_2(c^{t,i})} = \frac{u_2(c^{t,i})}{u_3(c^{t,i})} = \overline{R}; \tag{2.12}$$

$$\phi_2(h_t^i, x_t^i) = \frac{\overline{R}}{\overline{w}\lambda_{t+1}}. \tag{2.13}$$

Following (2.13), the relation between x_t^i and h_t^i, for a given level of technology λ_{t+1}, is given by Lemma 2.1.

Lemma 2.1. *Under Assumption 2.1, there exists a single-valued function* $\xi : \mathbb{R}_+ \to \mathbb{R}_+$ *such that*

$$x_t^i = \xi(h_t^i;\ \lambda_{t+1}),$$

where $\forall (h_t^i, x_t^i) \in \mathbb{R}_{++}^2$

$$\xi_t'^i; \lambda_{t+1}) \equiv \frac{\partial x_t^i}{\partial h_t^i} = -\frac{\phi_{21}(h_t^i, x_t^i)}{\phi_{22}(h_t^i, x_t^i)} > 0;$$

$$\frac{\partial x_t^i}{\partial \lambda_{t+1}} = -\frac{\overline{R}}{\overline{w}\lambda_{t+1}^2 \phi_{22}(h_t^i, x_t^i)} > 0.$$

Proof. Since $\phi_{22}(h_t^i, x_t^i) < 0$ $\forall (h_t^i, x_t^i) \in \mathbb{R}_{++}^2$ (i.e., $\phi_2(h_t^i, x_t^i)$ is, globally, strictly monotonic in x_t^i), it follows from the *Implicit Function Theorem* that, for a given level of technology, λ_{t+1}, the variable x_t^i can be written as a single-valued function of h_t^i, whose structure is given in Lemma 2.1. □

Thus, for a given level of technology, the amount of real resources invested in human capital by individual i is positively related to the parental level of human capital. Furthermore, regardless of the parental level of human capital, the amount of real resources invested in human capital is positively related to the level of technology and thus (given the nature of technological progress) to the average level of human capital in the preceding generation. This result is consistent with empirical evidence (e.g., Freeman (1971) and Bartel and Lichtenberg (1987)) who has shown that the responsiveness of individuals to changes in the incentives to invest in human capital is strongly positive.

2.5 The evolution of human capital within a dynasty

2.5.1 The General Case

Following (2.7) and Lemma 2.1, the evolution of the investment in human capital of each dynasty i is governed by the nonlinear difference equation

$$h_{t+1}^i = \phi(h_t^i, \xi(h_t^i; \lambda_{t+1})) \equiv \psi(h_t^i; \lambda_{t+1})). \tag{2.14}$$

where h_0^i is historically given.

The structure of the dynastic dynamical system is given by Proposition 2.1.

Proposition 2.1. *Under Assumption 2.1, for a given level of technology* λ, *the evolution of human capital within a dynasty* i, $\{h_t^i\}_{t=0}^{\infty}$, *is governed by an autonomous first-order nonlinear difference equation*

$$h_{t+1}^i = \psi(h_t^i; \lambda)$$

where h_0^i *is given, and*

- $\psi(0; \lambda) = \mu \geq 0$,
- $\psi_t'^i; \lambda) = \phi_1(h_t^i, \xi(h_t^i; \lambda)) + \phi_2(h_t^i, \xi(h_t^i; \lambda))\xi_t'^i; \lambda) > 0 \quad \forall h_t^i > 0$,
- $\psi_t''^i; \lambda) = \phi_{11} - (\phi_{12})^2/\phi_{22} - \frac{\phi_2}{(\phi_{22})^2}\left\{[\phi_{211} - \phi_{212}\frac{\phi_{21}}{\phi_{22}}]\phi_{22} - [\phi_{221} - \phi_{222}\frac{\phi_{21}}{\phi_{22}}]\phi_{21}\right\}$

Proof. Follows from (2.14), Lemma 2.1, and Assumption 2.1. □

Thus, consistent with empirical evidence that suggests a positive correlation between the parental level of human capital and that of the child (Becker and Tomes (1986)), $\psi(h_t^i; \lambda)$ is monotonically increasing in h_t^i. However, its curvature depends on the degree of complementarity between x_t^i and h_t^i and on the third derivatives of the function $\phi(x)$. Despite the assumption of positive and diminishing returns to factor of production, $\psi(h_t^i; \lambda)$ may be concave, convex, or alternating between convexity and concavity.

Lemma 2.2. *Under Assumption 2.1,* $\exists \mu \geq 0$ *such that the difference equation* $h_{t+1}^i = \psi(h_t^i; \lambda)$ *is characterized by multiple locally stable steady-state equilibria if the following conditions are satisfied:*

- $\lim_{h_t^i \to 0} \psi_t'^i; \lambda) = 0$,
- $\lim_{h_t^i \to \infty} \psi_t'^i; \lambda) = 0$,
- $\psi(h_t^i; \lambda) > h_t^i$ *for some* $h_t^i > 0$.

Proof. Since $\psi(h_t^i; \lambda)$ is continuous in h_t^i the lemma follows from the *Intermediate Value Theorem,* noting Figures 1 and 2. □

Section 2.5.2, provides a simple and plausible example of a production function of human capital that generates multiple steady-state equilibria in the evolution of human capital within a dynasty. For a generic range of the parameter values, the example satisfies Assumption 2.1, as well as the sufficient conditions in Lemma 2.2.

Lemma 2.2 provides a set of sufficient conditions for the existence of multiple steady-state equilibria in the dynamical system that characterizes the evolution of human capital within a dynasty, given a stationary technology. Thus, if additional structure is imposed on the dynamical system, $h^i_{t+1} = \psi(h^i_t; \lambda)$, so as to incorporate these sufficient conditions, the dynamical system that describes the evolution of human capital within a dynasty is characterized by multiple steady-state equilibria for some ranges of the parameter μ and λ. As depicted in Figure 1, for a sufficiently small level of μ there exists a technological level, λ^1, such that the system is characterized by multiple locally stable steady state equilibria: $h^a(\lambda^1)$ and $h^c(\lambda^1)$ are locally stable whereas $h^b(\lambda^1)$ is unstable. Furthermore, for the same level of μ, there exist a sufficiently low and a sufficiently high technological levels, λ^0 and λ^2, respectively, such that the system is characterized by a unique and globally stable steady-state equilibrium, $h(\lambda^0)$ and $h(\lambda^2)$, respectively; the latter is depicted in Figure 2.[10]

In light of the feasibility of multiple steady-state equilibria of the dynamical system that describes the evolution of human capital within a dynasty, the fate of a dynasty may depend on its initial conditions.

Proposition 2.2. *Consider the dynamical system $h^i_{t+1} = \psi(h^i_t; \lambda)$. If the system is characterized by:*

- *three steady-state equilibria ($h^a(\lambda)$ and $h^c(\lambda)$ are locally stable and $h^b(\lambda)$ is unstable), then*

$$\lim_{t\to\infty} h^i_t = \begin{cases} h^a(\lambda) & if \quad h^i_0 \in [0, h^b(\lambda)) \\ h^c(\lambda) & if \quad h^i_0 \in (h^b(\lambda), \infty) \end{cases}$$

- *unique steady-state equilibrium, $h(\lambda)$, then*

$$\lim_{t\to\infty} h^i_t = h(\lambda) \quad \forall h^i_t \geq 0.$$

Proof. Follows immediately from Figure 1. □

Thus the level of human capital in the long run may differ across dynasties despite the

[10]If $\mu > 0$, then the low human capital steady-state equilibrium is strictly positive, whereas if $\mu = 0$ it is zero.

suppositions that: ability is identical across individuals, capital markets are perfect, and the economy is deterministic.

For a given technological level, the dynamical system that governs the evolution of human capital within a dynasty may be characterized by multiple locally stable steady-state equilibria. Hence, as depicted in Figure 1, the historical human capital level of each dynasty determines the evolution pattern of human capital within a dynasty and its long run equilibrium level. The (local) home environment externality induces therefore the creation of inequality in the distribution of human capital. Convergence of human capital levels across dynasties, as depicted in Figure 2 may occur, however, at a higher range of technological level under which the dynamical system is characterized by a unique and globally stable steady-state equilibrium.[11]

The interaction between dynasties affects the economy's technological capabilities and may, therefore, bring about a convergence of the levels of human capital across dynasties via a qualitative transformation of the dynamical system from one that is characterized by multiple locally stable steady-state equilibria to another with a unique globally stable steady-state equilibrium. This interplay between the (local) home environment externality and the (global) technological externality is analyzed in section 3.

2.5.2 Example

This subsection provides a simple example that demonstrates that indeed, as argued generally in the previous subsection, for some range of technological levels, the dynamical system that describes the evolution of human capital within a dynasty is characterized by multiple locally stable steady-state equilibria, whereas for other ranges the dynamical system is characterized by a unique and globally stable steady-state equilibrium.

Suppose that the production function of human capital of member i of generation t is given by a modified Cobb-Douglas form, where the exponent associated with the parental human capital diminishes as the level of parental human capital increases. This production function is characterized by diminishing complementarity between h_t^i and

[11] For additional configurations see Remarks 3.2 and 3.3 and section 3.2.4.

x_t^i.[12]

$$h_{t+1}^i = \mu + (h_t^i)^{\alpha(h_t^i)}(x_t^i)^{\beta}, \tag{2.15}$$

where $\mu \geq 0,\ \beta \in (0,1),$ and $\alpha(h_t^i) > 0$ and $\alpha_t'^i) \leq 0\ \forall h_t^i > 0.$

Remark 2.2. As is apparent in light of Proposition 2.1, diminishing complementarity is not an essential feature of a production function of human capital that is associated with multiple steady-state equilibria. Furthermore, deviations from constant-returns-to-scale are not essential in order to generate multiple steady-state equilibria (e.g., Galor and Ryder (1989), Galor (1996), Azariadis (1996)). Section 5 demonstrates the feasibility of multiple steady-state equilibria in the presence of an uncertain production process.

Following the general analysis in Section 2.4, the optimal investment in human capital of a member i of generation t is:

$$x_t^i = \left[\frac{\beta \overline{w} \lambda(h_t)}{\overline{R}}\right]^{\frac{1}{1-\beta}} (h_t^i)^{\frac{\alpha(h_t^i)}{1-\beta}}. \tag{2.16}$$

The dynamical evolution of dynasty i is therefore characterized by the nonlinear difference equation:

$$h_{t+1}^i = \mu + \left[\frac{\beta \overline{w} \lambda(h_t)}{\overline{R}}\right]^{\frac{\beta}{1-\beta}} (h_t^i)^{\frac{\alpha(h_t^i)}{1-\beta}} \equiv \mu + Z(h_t)(h_t^i)^{\frac{\alpha(h_t^i)}{1-\beta}} \equiv \psi(h_t^i; \lambda(h_t)), \tag{2.17}$$

where h_t^i is historically given, and

$$\psi_t'^i; \lambda(h_t)) \equiv \frac{\partial h_{t+1}^i}{\partial h_t^i} = Z(h_t)(\frac{1}{1-\beta})(h_t^i)^{\frac{\alpha(h_t^i)+\beta-1}{1-\beta}}[\alpha_t'^i) h_t^i ln(h_t^i) + \alpha(h_t^i)]. \tag{2.18}$$

Consequently, in order to guarantee that the parental level of human capital is positively correlated to that of the child (Becker and Tomes (1986)), the elasticity of α with respect to h_t^i must be small enough in a well defined sense. Namely,

$$\eta_{\alpha,h_t^i} \equiv \frac{-\alpha_t'^i)\ h_t^i}{\alpha(h_t^i)} \leq \frac{1}{ln\ (h_t^i)} \quad \forall h_t^i > 1. \tag{2.19}$$

[12]This example can be easily generalized to a modified CES production function of the form

$$h_{t+1}^i = \mu + [b(x_t^i)^{\rho} + (1-b)(h_t^i)^{\rho\alpha(h_t^i)}]^{\frac{1}{\rho}}.$$

Assumption 2.2.

- $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}; \quad \alpha \in C^1$
- $\alpha_t'^{i}) \leq 0 \quad \forall h_t^i > 0$
- $\lim_{h_t^i \rightarrow 0} \alpha(h_t^i) + \beta > 1$ *and* $\lim_{h_t^i \rightarrow \infty} \alpha(h_t^i) + \beta < 1$
- $\eta_{\alpha,h_t^i} \equiv -\frac{\alpha_t'^{i})h_t^i}{\alpha(h_t^i)} \leq \frac{1}{ln(h_t^i)} \quad \forall h_t^i > 1$
- $\alpha_t'^{i})ln(h_t^i) < \infty \quad \forall h_t^i < 1.$

Lemma 2.3. *Under Assumption 2.2,* $\exists \mu \geq 0$ *such that the difference equation* $h_{t+1}^i = \psi(h_t^i; \lambda)$ *is characterized by multiple steady-states equilibria.*

Proof. Given Assumption 2.5, the first two conditions in Lemma 2.2 are satisfied. For a large set of parameters, the third condition in the proposition is satisfied as well. Thus, the lemma is a corollary of Lemma 2.2. □

3. The Evolution of the Distribution of Human Capital

This section analyzes the evolution of the *distribution* of human capital in the economy. The analysis is conducted in two stages. In the first stage, the analysis focuses on the evolution of this distribution in the absence of interaction across dynasties (i.e., abstracting from the effect of the average level of human capital on the technological level in the economy and thus viewing technology as stationary across time). This stage captures the implications of the effect of the (local) home environment externality on the distribution of human capital. In the second stage, building upon the results derived under the stationary technology case, the analysis proceeds to characterize the evolution of the distribution of human capital in the presence of endogenous technological progress induced by the average level of human capital in society. This stage captures the interaction between the (local) home environment externality and the (global) technological externality and its implications for the evolution pattern of this distribution.

3.1 Stationary Technology

Suppose that the distribution of human capital of the parent generation at time 0 is given by the density function $g_0(h_0^i)$ defined over the nonnegative real line.[13] Given the

[13] A mechanism that may generate the initial distribution of human capital is explored in Section 5.

population size of each generation, N, it follows that

$$\int_0^\infty g_0(h_0^i)dh_0^i = N. \tag{3.1}$$

Furthermore, for a stationary technology λ, if the dynamical system is characterized by multiple steady-state equilibria, then the number of low skilled dynasties in the long run, L^u, is

$$L^u = \int_0^{h^b(\lambda)} g_0(h_0^i)dh_0^i, \tag{3.2}$$

whereas the number of high skilled dynasties in the long run, L^s, is

$$L^s = \int_{h^b(\lambda)}^\infty g_0(h_t^i)dh_0^i; \tag{3.3}$$

Clearly, $L^u + L^s = N$.

Thus, for a given technology and for a given initial distribution of human capital, the level of human capital at the unstable steady-state equilibrium, $h^b(\lambda^1)$, determines the decomposition of dynasties that gravitate towards the high skilled steady-state equilibrium and those that gravitate towards the low skilled steady-state equilibrium.

Proposition 3.1. *Under Assumptions 2.1 - 2.2, if the technology is stationary at a level* λ, *and if the dynamical system* $h_{t+1}^i = \psi(h_t^i;\lambda)$ *is characterized by*

- *three steady-state equilibria* ($h^a(\lambda)$ *and* $h^c(\lambda)$ *are locally stable and* $h^b(\lambda)$ *is unstable), then*

$$\lim_{t\to\infty} g_t(h_t^i) = \begin{cases} h^a(\lambda) & \textit{with a mass of} \quad L^u \\ h^c(\lambda) & \textit{with a mass of} \quad L^s \end{cases}$$

- *unique steady-state equilibrium,* $h(\lambda)$, *then*

$$\lim_{t\to\infty} g_t(h_t^i) = h(\lambda) \quad \textit{with a mass of} \quad N$$

Proof. Follows immediately from Figures 1 and 2. □

Thus, for a given stationary technological level, the initial distribution of human capital may determine the long run distribution of human capital despite the suppositions that: ability is identical across individuals, capital markets are perfect, and the economic environment is deterministic. If the dynamical system that governs the evolution of human capital within a dynasty is characterized by multiple locally stable steady-state equilibria, and if the initial density function is strictly positive in the basin of attraction of these steady-state equilibria, then, as depicted in Figure 1, the density function converges to a 2 point distribution with masses that are pre-determined at the starting point of the economy. However, if the dynamical system is characterized by a unique steady-state equilibrium, then, as depicted in Figure 2, the density function converges to a single mass point distribution, regardless of the initial distribution of human capital. Hence, the (local) home environment externality may induce inequality in the distribution of human capital.

Remark 3.1. The intergenerational persistence in human capital levels is generated in this paper by the home environment effect. An alternative model in which borrowing constraints prevent children from low income families from investing in human capital (Galor and Zeira (1993)) generates an observationally equivalent prediction. Nevertheless, the policy implications of the two models differ. If constrained borrowing are the dominant effect, an opening of the missing capital market, if feasible, may eliminate the intergenerational linkage in human capital levels, whereas if the home environment effect is the dominant component, neighborhood effects (e.g., Case and Katz (1991) may be required in order to mitigate the extent of this intergenerational linkage.

3.2 Endogenous Technological Progress

This section analyses the evolution of the distribution of human capital in the presence of endogenous technological progress induced by the average level of human capital in society. The analysis demonstrates that the interplay between the (local) home environment externality (discussed in previous sections) and the (global) technological externality is a fundamental factor in the characterization of the evolution of this distribution. The interaction between dynasties and its impact on the production technology may bring about a qualitative change in the dynamical system that may result in the convergence of the distribution of human capital to a single mass point. That is, as a result of the interaction between dynasties, the long run distribution of human capital may be independent of the initial distribution of human capital.

In order to demonstrate the role of the interplay between the local and the global externalities, in a rather simple manner, the analysis focuses initially on technological progress that is associated with a threshold externality. The study then proceeds with the characterization of the evolution pattern of the distribution of human capital under smooth technological progress.

3.2.1 Threshold Externalities

Suppose that technological progress is of the following threshold externality form:[14] the production technology is stationary at a level λ^1 as long as the average level of human capital is below some threshold level, $\hat{h}$, whereas once $\hat{h}$ is reached the level of technology jumps to a higher stationary level, λ^2, and remains at this level as long as h_t remains above $\hat{h}$. Namely,

Assumption 3.1

- $\lambda_{t+1} = \lambda(h_t) = \begin{cases} \lambda^1 & \text{if} \quad h_t < \hat{h} \\ \lambda^2 & \text{if} \quad h_t \geq \hat{h}. \end{cases}$

Suppose that the distribution of human capital (of the parent generation) at time 0 is given by the density function $g_0(h_0^i)$ defined over the non-negative real line. Suppose further that the average level of human capital at time zero is below the technological threshold $\hat{h}$. Hence,

Assumption 3.2

- $\int_0^\infty h_0^i g_0(h_0^i) dh_0^i < \hat{h}$.

In light of (2.14) and Assumptions 3.1 and 3.2, it follows that the technological level at time 0 is λ^1, and the dynamical system that describes the evolution of each dynasty is $h_{t+1}^i = \psi(h_t^i; \lambda^1)$.

Consider Figures 1 - 3. As long as the average level of human capital within a period is below $\hat{h}$, a constant number of individuals, $L^s = \int_{h^b(\lambda)}^\infty g_0(h_t^i) dh_0^i$, within each generation (members of group L^s) gravitate towards the high human capital steady-state equilibrium, $h^c(\lambda^1)$, where the wage income is $\overline{w}\lambda^1 h^c(\lambda^1)$. Similarly, a constant number

[14] Threshold externalities are used in Azariadis and Drazen (1990) who consider a Hicks neutral technological change that is induced by the capital-labor ratio at the same period. Namely, $\lambda_{t+1} = \lambda(k_{t+1})$.

of individuals, $L^u = \int_0^{h^b(\lambda)} g_0(h_0^i)dh_0^i$, within each generation (members of group L^u) gravitate towards the low human capital steady-state equilibrium, $h^a(\lambda^1)$, where the wage income is $\overline{w}\lambda^1 h^a(\lambda^1)$.[15] As long as the level of technology remains unchanged, the composition of dynasties within each group is stationary. The economy experiences, therefore, a polarization in the distribution of human capital as well as in the distribution of income.

Suppose that there exists a time period $t^* > 0$ in which the average level of human capital exceeds the technological threshold $\hat{h}$ and consequently the level of technology increases from λ^1 to λ^2.

Assumption 3.3

- $\int_0^\infty h_t^i g_t(h_t^i)dh_t^i > \hat{h}, \quad for\ some\ t.$

In light of Assumption 3.3 , it follows that the dynamical system that describes the evolution of each dynasty from time t^* is $h_{t+1}^i = \psi(h_t^i; \lambda^2)$.

This technological progress increases the wage rate per efficiency unit of labor in the economy as a whole. It causes a qualitative change in the nature of the dynamical system that eliminates the low and stable steady-state equilibrium and generates a dynamical system that is characterized by a unique and globally stable steady-state equilibrium. Consequently, it induces an increase in the optimal investment in human capital by all individuals relative to the desirable level in the absence of technological progress. As depicted in Figure 3, the two groups evolve along the same dynamical system and converge to a higher steady-state equilibrium, $h(\lambda^2)$ where aggregate output is higher and the wage income is identical across the two dynasties. Thus, after a certain point in time, growth of output is accompanied by decreasing income inequality.[16]

Proposition 3.2. *Under Assumptions 2.1, 3.1-3.3, if the dynamical system* $h_{t+1}^i = \psi(h_t^i; \lambda^1)$ *is characterized by three steady-state equilibria* ($h^a(\lambda)$ *and* $h^c(\lambda)$ *are locally stable and* $h^b(\lambda)$ *is unstable), and if the dynamical system* $h_{t+1}^i = \psi(h_t^i; \lambda^2)$ *is characterized by a unique globally stable steady-state equilibria* $h(\lambda^2)$, *then*

- *in earlier stages of development the distribution of human capital becomes more*

[15] The segment of the dynasties that is located at time 0 at the unstable steady-state equilibrium is of measure zero and is therefore ignored.

[16] See Remark 3.2 for the case in which the basic level of human capital, μ, is equal to zero.

polarized, whereas in later stages of development the degree of polarization diminishes;

- $\lim_{t\to\infty} g_t(h_t^i) = h(\lambda^2)$ *with a mass of* N

Proof. Follows from Figure 3 under the above assumptions. □

Thus, the relative magnitude of the local home environment externality and the global technological externality determines the qualitative nature of the evolution pattern of the distribution of human capital (and thus the distribution of income) along the process of economic development. In particular, in the above case, in earlier stages of development, when technology is stationary, the individual's specific, local home environment externality is the dominating factor, and consequently, as discussed in section 3.1, the distribution of human capital becomes polarized. In later stages of development however, the global technological externality dominates and the distribution of human capital contracts.

3.2.2. Smooth Technological Progress

Suppose that the level of technology at time $t+1$, λ_{t+1}, advances smoothly with the average level of human capital of the previous generation, h_t. Given the average level of human capital at time 0, h_0, the level of technology at time 1, λ_1, is uniquely determined; $\lambda_1 = \lambda(h_0)$. Modifying (2.14) accordingly, it follows that for a given initial level h_0^i, the level of human capital of dynasty i at time 1 is uniquely determined by the function $h_1^i = \psi(h_0^i; \lambda_1)$. Thus, once the initial condition is specified, h_1^i, is uniquely determined.

Suppose that $\psi(h_t^i; \lambda_1)$ is characterized by multiple steady-state equilibria. As follows from (2.5), $\lambda_2(h_1) \geq \lambda_1(h_0)$. Modifying (2.14) accordingly, the level of human capital of dynasty i at time 2 is governed by $h_2^i = \psi(h_1^i; \lambda_2)$. Members of group L^u as well as members of group L^s find it beneficial to increase their investment in human capital. Given the curvature of $\psi(h_t^i; \lambda_2)$, and the fact that an increase in λ increases both the first and the second derivative of $\psi(h_t^i; \lambda)$ with respect to h_t^i, it follows that the increase of the level of investment in human capital of members of group L^s is larger than the increase of members of group L^u. Thus, output increases in conjunction with an increase in the polarization in the distribution of human capital and hence in the polarization of the distribution of income as well.

In light of Assumption 3.3. there exists a time period t^* from which the dynamical system, $h_{t+1}^i = \psi(h_t^i; \lambda_3)$, is characterized by a unique steady-state equilibrium. Given the strict concavity of $\lambda(h_t)$, it follows that $\lim_{t\to\infty} h_t^i = h^c(\lambda_t)$. Thus after a certain period the growth in output is accompanied by a decrease in the polarization of the distribution of human capital and the distribution of income.

Remark 3.2. If the basic level of human capital, μ, is equal to zero then technological progress gradually decreases the size of the less-educated population. However, the low skilled steady-state equilibrium remains a viable equilibrium, and consequently, a segment of the society will remain in a poverty trap forever. The size of this segment declines over time, however.

Remark 3.3. Suppose that the initial technological level generates a dynamical system that is characterized by a unique and globally stable (low human capital) steady-state equilibrium. Suppose further that a moderately higher technological level is associated with multiple steady-state equilibria, whereas a significantly higher technological level generates a dynamical system with a unique and globally stable (high skilled) steady-state equilibrium. In light of the analysis above, it follows that the distribution of human capital and the associated wage differential between skilled and unskilled individuals may follow the following pattern: In earlier stages of development, the human capital distribution contracts, in intermediate stages of development, the distribution expands and converges towards a bi-modal distribution, and in late stages of development this distribution contracts once again and converges to a single mass point.

Proposition 3.3. *Under Assumptions 2.2 and 3.1-3.3, in time periods in which the dynamical system $h_{t+1}^i = \psi(h_t^i; \lambda_t)$ is characterized by multiple steady-state equilibria, the distribution of human capital gravitates towards increased polarization, whereas in periods in which the dynamical system is characterized by a unique globally stable steady-state equilibrium the distribution of human capital gravitates towards diminished polarization.*

Proof. Follows from Figure 3 under the above assumptions. □

Thus, the relative magnitude of the local home environment externality and the global technological externality determines the qualitative nature of the evolution pattern of the distribution of human capital (and thus the distribution of income) along the process of economic development. In stages in which the individual's specific, local home environment externality is the dominating factor, the distribution of human capital and consequently the wage differential between skilled and unskilled labor becomes polarized, whereas in stages in which the global technological externality dominates the distribution of human capital, the distribution of income, and the wage differential between skilled and unskilled labor converge.

Remark 3.4. If the technological progress improves the productivity of high-skill individuals more than that of low-skill individuals the qualitative nature of the results does not alter. This skill-biased technological progress may result in an adverse bifurcation that fosters inequality for a longer time period. However, in the long run it fosters equality as well.

3.2.3. Inequality as a vehicle in the development of less developed economies

This section demonstrates that for a generic range of initial distributions of human capital, a sufficient level of inequality in the distribution of human capital may be an essential element of economic development in a less-educated, less developed economy.

Suppose that technological progress is associated with a threshold externality along the lines of Assumption 3.1.[17] Suppose that the distribution of human capital (of the parent generation) at time 0, as given by the density function $g_0(h_0^i)$, is such that the average level of human capital at time zero is below the technological threshold $\hat{h}$ (i.e., Assumption 3.2 is satisfied). Suppose further that there exists *no* time period $t > 0$ in which the average level of human capital exceeds the technological threshold $\hat{h}$. Hence, in the absence of intervention, the level of technology remains λ^1, and the potential for a substantial technological progress to the level λ^2 is not materialized. Namely, the following assumption is satisfied:

Assumption 3.4.

- $\int_0^\infty h_t^i g_t(h_t^i) dh_t^i < \hat{h}, \quad \forall t.$

Proposition 3.4. *Under Assumptions 2.2, 3.1- 3.4, if the dynamical system* $h_{t+1}^i = \psi(h_t^i; \lambda^1)$ *is characterized by three steady-state equilibria* ($h^a(\lambda)$ *and* $h^c(\lambda)$ *are locally stable and* $h^b(\lambda)$ *is unstable), and if the dynamical system* $h_{t+1}^i = \psi(h_t^i; \lambda^2)$ *is characterized by a unique globally stable steady-state equilibria* $h(\lambda^2)$, *then*

[17] Clearly, in light of the discussion in the previous section, the qualitative results are applicable to a smooth technological progress as well.

- *the distribution of human capital becomes more polarized, and the economy's level of output is bounded by the lower technological level* λ^1)
- *a redistribution of income towards the middle class in period 0 may generate a lower level of polarization and a higher level of output, in the long run.*

Proof. A corollary of Proposition 3.3 and Assumption 3.4. □

Proposition 3.4 and Figure 4 suggest therefore that a relatively poor economy which values equity as well as prosperity may confront a difficult trade-off between equity in the short-run and equity and prosperity in the long run. Inequality in the distribution of human capital may enable members of the highly educated segments of society to overcome the gravitational forces of a low, stable equilibrium and to increase their investment in human capital, whereas, equality, may trap the society as a whole at a low level of investment in human capital. As the investment in human capital of the highly educated segments of society increases and income inequality widens, the accumulated knowledge trickles down to the less educated segments of society via a technological progress in production. The return to skill improves, and investment in human capital becomes more beneficial to members of all segments of society. The economy may find it beneficial to subsidize the education of a selected group of individuals that will ultimately generate enough externalities to pull the society as a whole to a state of equity and prosperity. Furthermore, an economy that may implement prematurely a policy that is designed to enhance equality in the distribution of income may be trapped unnecessarily at a low output equilibrium without ever reaching prosperity.

Remark 3.5. For some generic range of initial distributions of human capital, a relatively educated economy may find that equality enhances economic growth.

Clearly as can be derived from Proposition 3.2, for some initial distribution of human capital the model can demonstrate that at the early stage of development, an increase in the aggregate level of investment in human capital may not be feasible unless the distribution of human capital (and consequently the distribution of income) is unequal. Inequality enables members of families from the highly educated segments of society to overcome the gravity of a low stable equilibrium and to increase their investment in human capital, whereas, equality, may trap the society as a whole at a low level of investment in human capital. Thus, inequality may be essential in order to increase the aggregate level of human capital and output during the early stages of growth. As the investment in human capital of the upper segments of society increases and income inequality widens, the accumulated knowledge trickles down to the lower segments of society via a technological progress in production. The wage for efficiency unit of labor increases and investment in human capital becomes more beneficial to members of all segments of society. In particular, members of the less-educated segments, who initially invested relatively little in the formation of human capital, find it beneficial to increase their investment. Due to diminishing returns to the family-specific external effects, the rate of investment (at a certain stage) becomes higher among members of the lower segments of society. Thus, in accordance with the Kuznets hypothesis, during early stages of development, output growth is associated with increased income inequality whereas in the later stages output growth is accompanied by a more equal distribution of human capital and income. Furthermore, output growth is accompanied in early stages of development by a widening wage differential between skilled and unskilled labor and in a later stages this wage differential declines.

3.2.4 Periodicity in the Non-Monotonic Evolution of the Returns to Skills

This section demonstrates that the non-monotonic evolutionary pattern of the distribution of human capital, (i.e., expansion followed by contraction) may reoccur periodically. The labor-augmenting technological progress, $\lambda(h_t)$, discussed in section 2.2, is replaced with an exogenous sequence of technologies (arriving less often than once a period) $\{\lambda_j(h_t)\}_{t=0}^{\infty}$, each of which is subject to gradual improvements that are monotonic increasing in the average level of human capital in society. The level of technology is therefore a piece-wise strictly concave function of the average level of human capital in society, with upward discrete jumps every several periods.[18]

In this setting, a major technological advance may generate a phase of widening inequality. This phase, commencing on impact, may continue for a while. As technologies age, however, the distribution of earnings narrows. To the extent that new technologies arrive when the distribution of human capital over low human capital levels is nondegenerate, the nonmonotonic effect of a major technological progress on income distribution is bound to reoccur whenever the new technologies complement human capital. The

[18]The qualitative analysis does not alter if the arrival rate is stochastic rather than deterministic and if the arrival rate is a function of the average level of human capital.

extent and duration of rising inequality depend on the initial distribution as well.

The study may shed new light on the evolution of the wage structure in the United States during the twentieth century as documented by Goldin and Margo (1992) and Katz and Murphy (1992). These studies reveal that the wage differential between skilled and unskilled labor had widened until the thirties, had narrowed down from the forties until the sixties and have widened again in the last two decades. This nonmonotonic pattern of the evolution of wages is in accordance with the prediction of our model.[19] If in light of empirical observations that suggest that major technological breakthroughs are usually associated with energy-saving technologies (Landes (1989)), or with a 'general-purpose technology' with network externalities (David (1990)) then, in accordance with our theory, one may identify the source of these two waves of widening inequality with major technological advancements. The first widening in the distribution is due to the increase in industrial use of network electricity, while the second wave is due to the soaring use of electronics (Krueger (1993)).

The type of technological progress we analyzed above benefits individuals with high level of human capital more than those with low level of human capital. However, if the technological progress is skilled biased of the type suggested by, Bremen, Bound, and Griliches (1995) i.e., technological progress that reduces the return to less-skilled individuals while increasing that of skilled individuals, then a poverty trap of the kind analyzed in section 2 may be reintroduced. In this case the distribution of earnings will converge much later. Thus, the polarization in earnings in recent decades may persist for a longer period.

4. Intergenerational Altruism

This section analyzes the robustness of the evolution pattern of the distribution of human capital and economic growth as derived in the previous section for the introduction of intergenerational altruism. A priori, one may wonder whether the path dependence of the evolution of human capital within a dynasty is an outcome of the abstraction from intergenerational altruism, and whether the inability of some dynasties to overcome the gravitation forces of the low human capital, steady-state equilibrium is caused by the shortsight of individuals. This subsection demonstrates that the qualitative results established in the previous section are independent of the assumption of no intergenerational

[19] Alternative theoreis are provided by Mincer (1996), and Galor and Tsiddon (1996).

altruism. They will not be altered under a range of specifications of intergenerational altruistic motives.

4.1 The "Joy of Giving" Altruism

Suppose that parents generate utility from the act of intergenerational transfers to their children rather than form the impact of this transfer on the level of utility of their children. This bequest motive was first suggested by Yaari (1965) and have been popularized recently by the emerging literature on income distribution and economic growth (e.g., Galor and Zeira (1993)). As will become apparent, the dynamical system that characterizes the evolution of human capital under this modification is identical qualitatively to the nonaltruistic dynamical system analyzed in Sections 2 and 3.

Suppose that the preferences of a member i generation t are represented by the intertemporal utility function

$$u^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, c_{t+2}^{t,i}, b_{t+1}^i), \tag{4.1}$$

defined over the individual's consumption in periods t, $t+1$, and $t+2$, $c^{t,i} \equiv (c_t^{t,i}, c_{t+1}^{t,i}, c_{t+2}^{t,i})$, and over an intergenerational transfer to the child, b_{t+1}^i, in period $t+1$,[20] (i.e., in the first period of the child's life, when in the absence of intergenerational transfer borrowing is necessary in order to finance the investment in human capital and consumption).

Given Assumption 2.1 and accounting for the changes in the structure of the utility function, there exists a unique and interior solution to the maximization problem that is characterized by the necessary and sufficient conditions

$$\frac{u_1(c^{t,i}, b_{t+1}^i)}{u_2(c^{t,i}, b_{t+1}^i)} = \frac{u_2(c^{t,i}, b_{t+1}^i)}{u_3(c^{t,i}, b_{t+1}^i)} = \frac{\overline{R}u_2(c^{t,i}, b_{t+1}^i)}{u_4(c^{t,i}, b_{t+1}^i)} = \overline{R}; \tag{4.3}$$

$$\phi_2(h_t^i, x_t^i) = \frac{\overline{R}}{\overline{w}\lambda_{t+1}}. \tag{4.4}$$

Following (4.4), the relation between x_t^i and h_t^i, (for a given level of technology λ_{t+1}) is given in Lemma 2.1. Furthermore, the dynamical system is characterized by Proposition 2.1. Thus, the entire qualitative analysis of Sections 2 and 3 follows.

[20] Clearly, intergenerational transfer in period $t+2$ will not change the qualitative results.

4.2 Parental Altruism With Respect to the Level of Utility of the Child

Suppose that parents generate utility from the utility of their children (Barro (1974)). Namely,

$$u^{i,t} = u(c^{i,t}, u^{i,t+1}). \tag{4.5}$$

This economy (with some additional structure) is equivalent to an economy with a distribution of infinitely-lived dynasties each of which determines at time 0 the optimal sequences of the evolution of human capital $\{h_t^i\}_{t=0}^{\infty}$ and consumption $\{c^{i,t}\}_{t=0}^{\infty}$ for the entire dynasty. Unlike the nonaltruistic model in which from the viewpoint of the individual there exists diminishing marginal returns to investment in human capital throughout the feasible range, in the dynastic model the representative individual internalizes the home environment externalities associated with investment in human capital and therefore confronts with a production technology that is characterized by variable returns to scale. At low levels of human capital there exists increasing returns to scale, while as the level of human capital increases sufficiently the production function exhibits decreasing returns to scale. Thus, despite the internalization of the parent-child externality, multiplicity of equilibria remains a viable possibility. Furthermore, the global externality is clearly not internalized by individual dynasties and thus the possibility of bifurcation of equilibria remains viable as well.

The unstable steady-state equilibrium in the nonaltruistic model (that represents the minimal level of parental human capital that permits higher human capital investment by the child) is associated with a higher level of human capital than that in the altruistic model. If a dynasty is sufficiently close to the unstable steady-state equilibrium a minimal sacrifice in current consumption may bring about a large increment in the utility of all future members of the dynasty. However, for a sufficiently large discount of the utility of future members of the dynasty, gravitation towards the lower steady-state equilibrium of some dynasties is inevitable.

5. Uncertainty

This section introduces uncertainty about the outcome of individuals' investment in human capital. The incorporation of a stochastic environment has several virtues. First, the evolution of dynasties is no longer fully determined by the initial conditions of the dynasty. Thus, as is observed empirically, individuals may be able to overcome adverse initial conditions. Second, a stochastic environment embodies a mechanism that separates dynasties and generates endogenously a non-degenerate human capital distribution. Third, the dynamical system that characterizes the evolution of human capital within a dynasty may be characterized by multiple steady-state equilibria under conditions that would generate a unique globally stable steady-state equilibrium in the deterministic case. In particular, multiplicity of equilibria is feasible if the production function of human capital exhibits constant or decreasing-returns-to-scale in *all* inputs.

Suppose that the realization of individuals' investment in human capital is stochastic due to uncertainty about the effect of the parental level of human capital.[21] The parental input has a strong effect (associated with the parameter δ_1) with probability $p(h_t^i)$, and a weaker effect (associated with the parameter δ_2) with probability $[1-p(h_t^i)]$. Suppose further that the probability of a large parental effect, $p(h_t^i)$, increases with the parental level of human capital. The parental level of human capital affects, therefore, the child's expected level of human capital via an additional channel to the ones discussed in section 2.4. It affects the probability of a high realization of the investment in human capital.

Suppose that the production function of human capital is of a Cobb-Douglas type.[22] It follows that the expected level of human capital for a member i of generation t is:

$$E(h_{t+1}^i) = \{p(h_t^i)\delta_1 h_t^i + [1-p(h_t^i)]\delta_2 h_t^i\}^\alpha (x_t^i)^\beta \tag{5.1}$$

where $\delta_1 > \delta_2 > 0$, $\alpha, \beta > 0$, *and* $\alpha + \beta \leq 1$.[23]

Assumption 5.1.

- $p(h_t^i) : \mathbb{R}_+ \to [0,1]; \quad p(h_t^i) \in C^2$.
- $p_t'^i) \geq 0$.

Assumption 5.2.

- *individuals are risk neutral.*[24]

Since individuals are risk-neutral it follows from (5.1) and the analysis in section 2.4

[21]Uncertainty with respect to the entire investment process could have been considered without altering the qualitative results.

[22]the qualitative results remain unchanged in the presence of the general production function specified in Assumption 2.1. To economize on space a particular function is adopted.

[23]Note that $\alpha + \beta \leq 1$ implies that the production function of human capital is not associated with increasing returns to scale.

[24]Risk aversion does not alter the qualitative nature of the analysis. The choice of risk-neutrality was made simply out of space considerations.

that the optimal investment of individual i of generation t is

$$x_t^i = \left[\frac{\alpha\overline{w}\lambda(h_t)}{\overline{R}}\right]^{\frac{1}{1-\beta}} [\delta_2 + \delta p(h_t^i)]^{\frac{\alpha}{1-\beta}} (h_t^i)^{\frac{\alpha}{1-\beta}}, \tag{5.2}$$

where $\delta \equiv \delta_1 - \delta_2 > 0$.

Lemma 5.1. *Under Assumptions 2.1, 5.1, and 5.2, for a given level of technology* λ, *the expected level of human capital of a member* i *of generation* t *is*

$$E(h_{t+1}^i;\lambda) = z(\lambda)[\delta_2 + \delta p(h_t^i)]^{\frac{\alpha}{1-\beta}} (h_t^i)^{\frac{\alpha}{1-\beta}},$$

where $z(\lambda) \equiv [\alpha\overline{w}\lambda/\overline{R}]^{\frac{\beta}{1-\beta}}$, *and*

$$E'^{i}_{t+1};\lambda) = z(\lambda)(\alpha/1-\beta)[\delta_2 + \delta p(h_t^i)]^{\frac{\alpha+\beta-1}{1-\beta}} (h_t^i)^{\frac{\alpha+\beta-1}{1-\beta}} \{\delta_2 + \delta[p(h_t^i) + h_t^i p'^{i}_t)]\} > 0;$$
$$E''^{i}_{t+1};\lambda) \gtrless 0.$$

Proof. Follows from (5.1) and (5.2). □

The function $E(h_{t+1}^i;\lambda)$ is monotonic nondecreasing with either a positive or a negative second derivative. Hence it may follow a concave-convex-concave pattern.[25] For example, if the production function of human capital satisfies constant returns to scale, $\delta_2 = 0$, $2\alpha > 1-\beta$, $p(h_t^i) = h_t^i/a$, $\forall h_t^i \in [0,a]$, $a \in (0, h^H(\lambda) - \epsilon)$, and $p(h_t^i) = 1$ otherwise, then the function $E(h_{t+1}^i;\lambda)$ has *necessarily* three fixed points.

Since the children's investment decisions are based upon the actual outcome of the parental investment in human capital, rather then on the expected value, the dynamical evolution of dynasty i cannot be analyzed and depicted on the basis of Lemma 5.1. Nevertheless, Lemma 5.1, is instrumental in analyzing the nature of the limiting distribution of human capital.

Proposition 5.1. *Under Assumptions 2.1, 5.1, and 5.2, for a given level of technology* λ, *the evolution of human capital within dynasty* i, $\{h_t^i\}_{t=0}^{\infty}$, *is governed by an autonomous first-order stochastic nonlinear difference equation*

$$h_{t+1}^i = \begin{cases} z(\lambda)\delta_2^{\frac{\alpha}{1-\beta}}(h_t^i)^{\frac{\alpha}{1-\beta}} \equiv \psi^L(h_t^i;\lambda) & \text{with probability } [1-p(h_t^i)] \\ z(\lambda)\delta_1^{\frac{\alpha}{1-\beta}}(h_t^i)^{\frac{\alpha}{1-\beta}} \equiv \psi^H(h_t^i;\lambda) & \text{with probability } p(h_t^i) \end{cases}$$

[25] The convex segment is generated if the derivative of $p(h_t^i)$ is sufficiently large relative to the degree of decreasing return to scale, $(\alpha+\beta)$.

where for j=L,H, $\psi(h_t^i;\lambda) > 0$ *and* $\psi^j(h_t^i;\lambda) \leq 0,\ \forall h_t^i \geq 0,$ *and with probability 1,*

$$\lim_{t\to\infty} h_{t+1}^i \begin{cases} = h^L(\lambda) & \textit{if} \quad p(h_t^i) = 0 \quad \forall h_t^i \geq 0 \\ = h^H(\lambda) & \textit{if} \quad p(h_t^i) = 1 \quad \forall h_t^i \geq 0 \\ \in [h^L(\lambda), h^H(\lambda)] & \textit{otherwise} \end{cases}$$

Proof. Follows from Lemma 5.1 and Figure 5, noting that under the above assumptions there exists a unique stable set, $[h^L(\lambda), h^H(\lambda)]$. □

Corollary 5.1.

$$\psi^L(h_t^i;\lambda) \leq E(h_{t+1}^i;\lambda) \leq \psi^H(h_t^i;\lambda).$$

Proof. Follows from Lemma 5.1, Proposition 5.1 and Figure 5. □

Thus, for any given level of h_t^i, the level of h_{t+1}^i is given by $\psi^H(h_t^i;\lambda)$, in the case of a strong parental effect, or by $\psi^L(h_t^i;\lambda)$ in the case of a weak parental effect. Given the concavity of $\psi^L(h_t^i;\lambda)$ and $\psi^H(h_t^i;\lambda)$, the dynamical system has a unique stable set $[h^L(\lambda), h^H(\lambda)]$ that is globally attractive. As long as $E(h_t^i;\lambda)$ has three fixed points, polarization occurs within the stable set (in a stochastic sense) with some mobility between the poles.

Consider Figure 6(a). Suppose that at period zero a continuum of dynasties of measure N are located at h_0^i. As follows from the law of large numbers a proportion $p(h_0^i)$ of the dynasties has a strong parental effect, whereas a proportion $[1 - p(h_0^i)]$ has a weak parental effect. Dynasties may evolve in a stochastic non-monotonic fashion. Those that fall below $h^M(\lambda^1)$ are less likely to recover. Their incentive to invest is lower since (as follows from Assumption 5.1) they are less likely to be successful in the production of human capital. Thus they approach (in a stochastic sense) $h^L(\lambda^1)$. Dynasties that are above $h^M(\lambda^1)$ converge (in a stochastic sense) towards $h^H(\lambda^1)$. As long as the technology remains stationary, the economy approaches stochastically a bi-modal distribution with increasing mass near the two poles, $h^L(\lambda^1)$ and $h^H(\lambda^1)$ and with a positive rate of transition across the two groups.

Consider Figure 6(b). Suppose that in accordance with Assumptions 3.1 - 3.4 the average human capital in the economy eventually increases and generates a labor-augmenting technological progress that improves the level of technology from λ^1 to λ^2. Consequently, as follows from Lemma 5.1 and Proposition 5.1, the curves $\psi^H(h_t^i;\lambda)$, $\psi^L(h_t^i;\lambda)$, and $E(h_t^i;\lambda)$ rotates upward. Suppose that the change in λ tilts $E(h_t^i;\lambda)$ in a way that eliminates the two fixed points. Hence the incentives for all individuals to invest in human capital are sufficiently strong and the distribution becomes unimodal near $h(\lambda^2)$.

6. Concluding Remarks

This paper analyzes the interaction between the distribution of human capital, technological progress, and economic growth. The study illustrates the important role of the distribution of human capital in the evolution of economies. It demonstrates that the interplay between a local home environment externality and a global technological externality governs the evolution of the distribution of human capital, the distribution of income, the wage differential between skilled and unskilled labor, and economic growth. In stages of development in which the home environment externality is the dominating factor, the distribution of human capital and the wage differential between skilled and unskilled labor become polarized, whereas in stages in which the global technological externality dominates convergence ultimately takes place.

The study suggests that a relatively poor economy that values equity as well as prosperity may confront a trade-off between equality in the short-run followed by equality and stagnation in the long run, and inequality in the short-run followed by equality and prosperity in the long run. The economy may find it beneficial to subsidize the education of a selected group of individuals that will ultimately generate enough externalities to pull the society as a whole to a state of equality and prosperity. Furthermore, an economy that prematurely implements a policy designed to enhance equality in the distribution of income may be trapped unnecessarily at a low output equilibrium without ever reaching prosperity.

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