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Sovereign debt and consumption smoothing☆

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Abstract

This paper shows that whether or not a sovereign can borrow to smooth consumption depends both on how consumption smoothing is achieved, whether by contingent debt issuance or by contingent debt servicing, and on the penalty for debt repudiation. If a sovereign that repudiated its debt could not borrow again, but could continue to save and to dissave, then contingent debt issuance, without contingent debt servicing, cannot support a positive amount of uncollateralized sovereign debt. But, with this same penalty for repudiation, contingent debt servicing supports a positive amount of uncollateralized sovereign debt.

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1. Introduction

The existing literature widely recognizes that sovereign debt is often used to smooth consumption intertemporally in the face of a stochastic income stream.

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The literature, however, reveals no concensus on whether or not this motive for debt issue can support a positive amount of uncollateralized sovereign debt, without a binding commitment to service debt. In the analysis that follows, we show that whether or not a sovereign can borrow to smooth consumption depends both on how consumption smoothing is achieved and on the penalty for debt repudiation.

In some models – see, for example, Eaton and Gersovitz (1981) and Chari and Kehoe (1993) – consumption smoothing is achieved by making debt issuance contingent on the realization of income. Specifically, the sovereign issues additional debt whenever it has a low realization of income and the sovereign retires debt whenever it has a high realization of income. In these models, debt servicing depends on the amount of accumulated debt and the interest rate, but debt servicing is not contingent on the realization of income. In equilibrium, the sovereign always services its debt in full.

In other models – see, for example, Grossman and Van Huyck (1988) – consumption smoothing is achieved by making debt servicing contingent on the realization of income. Specifically, the sovereign issues an amount of debt that depends, *inter alia*, on the probability distribution of income and on the interest rate, but does not depend on the realization of income. To smooth consumption, the sovereign services its debt in full only when it has a high realization of income. In the event of a low realization of income, the sovereign defaults either partially or fully.

In any case, if the sovereign is unable to make a binding commitment to service its debt and also does not collateralize its debt, then in the event of a high realization of income the sovereign will be tempted to repudiate its debt.[1] Models of sovereign debt typically assume that lenders use a two-part strategy to deter the sovereign from repudiating. First, lenders impose a ceiling on the amount of debt that the sovereign can issue. Second, lenders would punish repudiation by denying the sovereign access to further loans.

The ambiguity in the literature involves the exact specification of the options that would remain open to a sovereign that suffered this punishment. Various authors have considered at least three different possibilities, each of which has radically different implications.

At one extreme, Bulow and Rogoff (1989) assume that, even if a sovereign that repudiated its debt could not borrow again, it could continue to smooth

[1] The appropriate definition of repudiation depends on whether debt issuance or debt servicing is contingent on income. Without contingent debt servicing, repudiation is equivalent to failure to service the accumulated debt in full. In contrast, with contingent debt servicing, repudiation occurs only if the sovereign fails to service its debt according to the understood debt-servicing schedule. In their model of contingent debt servicing, Grossman and Van Huyck distinguish repudiation from excusable default, which occurs when the sovereign understandably fails to service its debt in full because of a low realization of income.

consumption by buying a standard insurance policy against low realizations of income. Under this assumption, regardless of whether debt issuance or debt servicing is contingent on income, the ability to borrow to smooth consumption would have no value for the sovereign, and the penalty of no further borrowing would not deter repudiation. Consequently, a sovereign could not issue any uncollateralized debt. This model, however, has limited interest both because standard insurance policies against low realizations of income do not seem to exist and because sovereigns actually seem to issue large amounts of uncollateralized debt.

At another extreme, many discussions abstract from saving by the sovereign. Examples include Eaton and Gersovitz (1981), Section 2.1, Grossman and Van Huyck (1988), Worrall (1990) and Eaton (1993). With no saving, the penalty of no further borrowing would cause the sovereign's future consumption stream to match exactly its realized future income stream. Under this assumption, regardless of whether debt issuance or debt servicing is contingent on income, the penalty of no further borrowing generally would be a sufficient deterrent to repudiation that lenders would allow a positive debt ceiling. This model, however, also has limited interest because it would always seem possible, even in autarky, for a sovereign to save.

In contrast to both of these uninteresting models, other discussions – see, for example, Eaton and Gersovitz (1981), Section 2.2, Eaton et al. (1986) and Chari and Kehoe (1993) – have suggested a more realistic specification of the penalty for repudiation, according to which a sovereign that repudiated its debt could not borrow again but could continue to save and to dissave. We now show that under this specification of the penalty for repudiation contingent debt issuance, without contingent debt servicing, cannot support a positive amount of sovereign debt.

2. Contingent debt issuance

Assume that debt issuance is contingent on the realization of income. Moreover, as long as the sovereign services its debt and as long as it has not reached a ceiling on accumulated debt, it can continue to issue new debt at a constant interest rate. Thus, if the sovereign services its debt in period t and plans to continue to service its debt in all future periods, then consumption in every period $t+i$, denoted by c_{t+i}, is given by

$$c_{t+i} = y_{t+i} - rb_{t+i-1} + b_{t+i} - b_{t+i-1}, \quad i = 0,1,2,\dots, \tag{1}$$

where y_{t+i} is the stochastic realization of income in period $t+i$, r is the interest rate, b_{t+i-1} is the amount of debt accumulated through period $t+i-1$, and $b_{t+i} - b_{t+i-1}$ is either the amount of additional debt issued or the amount of debt retired in period $t+i$.

If the realization of income in any period is large enough that consumption smoothing would call either for debt to be retired or for debt servicing to exceed new debt issued, then consumption in that period would be larger if the sovereign were to repudiate its accumulated debt than if the sovereign services its debt. In this event, the sovereign would be tempted to repudiate its debt and this temptation would be greater the larger the amount of accumulated debt. Without either collateralization or a binding commitment to service debt, the only deterrent to repudiation is that repudiation would preclude future borrowing. But, given that the value of the ability to smooth consumption is finite, the value of the ability to borrow in the future is also finite. Accordingly, there is a maximum amount of accumulated debt such that the temptation to repudiate would not outweigh the deterrent to repudiation. Let $\bar{b}$ denote this amount of accumulated debt. Lenders will not knowingly allow the sovereign's accumulated debt to exceed $\bar{b}$.

Suppose that $b_{t-1} = \bar{b}$, that is, suppose that the sovereign already has reached its debt ceiling. If $b_{t-1} = \bar{b}$, and if the sovereign services its debt in period t, then Eq. (1) implies that present and future consumption are subject to the following constraints:

$$E_t \sum_{\tau=t}^{\infty} (1+r)^{t-\tau}(y_\tau - c_\tau) = \bar{b} \tag{2}$$

and

$$E_t \sum_{\tau=t}^{n} (1+r)^{t-\tau}(c_\tau - y_\tau) \le 0, \quad \text{for all values of } n = t, t+1, \ldots, \tag{3}$$

where E_t denotes an expectation conditional on information available in period t.

Condition (2) is a solvency constraint. It says that, if the sovereign services its accumulated debt, then the expected present value of present and future consumption is less than the expected present value of present and future income by the amount of present accumulated debt.

Condition (3) represents the assumption that the sovereign has reached its debt ceiling and cannot accumulate any more debt. Condition (3) says that over any horizon the expected present value of present and future consumption cannot exceed the expected present value of present and future income. Condition (3) allows the sovereign to borrow and to consume in excess of its income in some future period if and only if it has first repaid some debt.

Assume further that, if the sovereign were ever to fail to service its debt, then it could never issue any new debt, although it could continue to save and to dissave. Thus, if the sovereign were not to service its debt in period t, then present and future consumption would be subject to the following constraints:

$$E_t \sum_{\tau=t}^{\infty} (1+r)^{t-\tau}(y_\tau - c_\tau) = 0 \tag{4}$$

and

$$E_t \sum_{\tau=t}^{n} (1+r)^{t-\tau}(c_\tau - y_\tau) \leq 0, \quad \text{for all values of } n = t, t+1, \ldots . \tag{5}$$

Condition (4) is an alternative solvency constraint. It says that, if the sovereign were to repudiate its accumulated debt, then the expected present value of present and future consumption would equal the expected present value of present and future income. The difference between Condition (4) and Condition (2) represents the temptation to repudiate the accumulated debt. If $\bar{b}$ were positive, then repudiation would permit the average level of consumption to be unambiguously higher.

Condition (5) represents the assumption that repudiation would preclude future borrowing. Condition (5) allows the sovereign to consume in excess of its income in some future period if and only if it has first accumulated some savings. The critical observation is that Condition (5) is identical to Condition (3). Specifically, Condition (5) would constrain the sovereign's ability to smooth future consumption exactly as does Condition (3).

Taken together, Conditions (2)–(5) imply that, if the debt ceiling $\bar{b}$ were positive, then once its accumulated debt had reached $\bar{b}$ the sovereign by repudiating its debt would increase its average future consumption without reducing its ability to smooth future consumption. Put another way, once it has reached its debt ceiling the sovereign gains nothing in the way of additional consumption smoothing by incurring the cost of continuing to service its debt. Accordingly, without a binding commitment to service debt and without collateralization, lenders cannot allow the sovereign a positive debt ceiling.

Chari and Kehoe (1993) derived this same result, albeit with more complicated mathematics, but Chari and Kehoe abstracted from the possibility of contingent debt servicing. We argue next that under the same assumption that a sovereign that repudiated its debt could not borrow again, but could continue to save and to dissave, contingent debt servicing can support a positive amount of sovereign debt.

3. Contingent debt servicing

Assume that the realization of income during any period τ, y_τ, can be either y_H, which is a high probability event, or y_L, which is a low probability event, or y_{LL}, which is a very low probability event, where $y_H > y_L > y_{LL}$. According to this assumption, the realization of income is large with high probability, small with low probability, and very small with very low probability. Assume further that, although the probabilities associated with these possible realizations of income can be history dependent, these probabilities are such that $y_{\mathrm{H}} > \bar{y}_\tau > y_{\mathrm{L}}$, where $\bar{y}_\tau$ denotes the expected realization of income in period τ conditional on all past realizations of income through period $\tau - 1$. In other words, we restrict the stochastic process generating income such that, although the conditional expected realization of income in the next period is not necessarily constant, it is always smaller than the large realization of income and larger than the small realization of income. The restriction implies that $\bar{y}_\tau$ is less variable than y_τ.

Let ρ denote the interest rate on one-period risk-free assets. Assume that prior to each period τ the sovereign issues an amount of one-period debt $\tilde{b}_\tau$, where $\tilde{b}_\tau = (\bar{y}_\tau - y_{\mathrm{LL}})/(1 + \rho)$ and that the sovereign invests the proceeds from this debt issue in one-period risk-free assets. Thus, at the end of period τ, when the sovereign has realized income y_τ, the sovereign also liquidates assets worth $(1 + \rho)\tilde{b}_\tau$.

Let s_τ denote the amount that the sovereign spends on servicing its debt at the end of period τ. Assume that the sovereign services its debt according to the state-contingent schedule

$$s_\tau = y_\tau - \bar{y}_\tau + (1 + \rho)\tilde{b}_\tau. \tag{6}$$

We can interpret Eq. (6) as follows. If the realization of income is high, a high probability event, then the sovereign services its debt in full. If the realization of income is low, a low probability event, then the sovereign defaults partially on its debts. If the realization of income is very low, a very low probability event, then the sovereign defaults fully on its debts.

Full debt servicing implies paying an interest rate, r, in excess of the risk-free interest rate. Specifically, if y_τ equals y_{H}, then

$$1 + r = \frac{s_\tau}{\tilde{b}_\tau} = \frac{y_{\mathrm{H}} - y_{\mathrm{LL}}}{\bar{y}_\tau - y_{\mathrm{LL}}}(1 + \rho).$$

Eq. (6) also implies that the expected return to the lenders equals the risk-free interest rate. In addition, Eq. (6) implies that s_τ is non-negative in all states, including the worst state, in which y_τ equals y_{LL}.

Given this debt-servicing schedule, the sovereign's net cash flow in period τ is

$$y_\tau + (1 + \rho)\tilde{b}_\tau - s_\tau = \bar{y}_\tau, \quad \text{for any realization of } y_\tau. \tag{7}$$

In this simple example with contingent debt servicing net cash flow in period τ equals $\bar{y}_\tau$ rather than y_τ. The amount of debt $\tilde{b}_\tau$ is just large enough so that even in the worst state, in which y_τ equals y_{LL}, the sovereign, by defaulting on its debts, would be able to avoid having its net cash flow be less than $\bar{y}_\tau$. Moreover, because in this example the lenders do not require a return in excess of the risk-free interest rate, this smoothing of net cash flow is costless.

Let U_t denote the present value of present and expected future utility from consumption, given the realization of y_τ in period $\tau = t$. Assume that

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau), \tag{8}$$

where the utility function, $u(c_\tau)$, is increasing and concave, and the discount factor, β, is a positive fraction. If in period $\tau = t$ the sovereign services its debt according to Eq. (6) and expects to continue to service its debt according to Eq. (6) in future periods, then U_t equals U_t^*, where

$$U_t^* = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau^*), \tag{9}$$

and where, assuming for simplicity that the sovereign has not accumulated any savings prior to period t, c_τ^* is the value of c_τ that in each period would be consistent with maximizing U_t subject to

$$c_t - \bar{y}_t \leq 0 \tag{10}$$

and

$$c_t - \bar{y}_t + E_t \sum_{\tau=t+1}^{n} (1+\rho)^{t-\tau}(c_\tau - \bar{y}_\tau) \leq 0,$$

$$\text{for all values of } n = t+1, t+2, \ldots, \infty. \tag{11}$$

Conditions (10) and (11) say that, if the sovereign services its debt according to Eq. (6) and expects to continue to service its debt according to Eq. (6) in future periods, then over any horizon the expected present value of present and future consumption can be as large as the expected present value of present and future net cash flow, where in each period net cash flow equals the conditional expected realization of income. According to Conditions (10) and (11) in either the present period or any future period the sovereign can consume as much as the conditional expected realization of its income. Further, in any future period the sovereign can consume in excess of the conditional expected realization of its income if and only if it has first accumulated some savings.

In the special case in which the probabilities associated with the possible realizations of income are constant, $\bar{y}_\tau$ would equal a constant, denoted by $\bar{y}$. In this case, by issuing a constant amount of debt, denoted by $\tilde{b}$, where $\tilde{b} = (\bar{y} - y_{\mathrm{LL}})/(1+\rho)$, and by servicing debt according to Eq. (6) the sovereign would achieve constant net cash flow. Thus, in this case contingent debt servicing could yield complete consumption smoothing. Specifically, we could have

$$U_t^* = u(\bar{y}) + \frac{\beta}{1-\beta}u(\bar{y}).$$

Assume further that, if in period t the sovereign were to provide less debt servicing than the amount required by Eq. (6), then it would be unable to issue debt in future periods. Accordingly, if in period t the sovereign were to succumb to the temptation to repudiate its debt, then, U_t would equal U_t^o, where

$$U_t^o = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau^o), \tag{12}$$

and where, setting s_t equal to zero, and again assuming for simplicity that the sovereign has not accumulated any savings prior to period t, c_τ^o is the value of c_τ that in each period would be consistent with maximizing U_t subject to

$$c_t - [y_t + (1+\rho)\tilde{b}_t] \leq 0 \tag{13}$$

and

$$c_t - [y_t + (1+\rho)\tilde{b}_t] + E_t \sum_{\tau=t+1}^{n} (1+\rho)^{t-\tau}(c_\tau - y_\tau) \leq 0,$$
$$\text{for all values of } n = t+1, t+2, \ldots, \infty. \tag{14}$$

Conditions (13) and (14) say that, if the sovereign repudiates its debt, then over any horizon the expected present value of present and future consumption can be as large as the sum of present liquid assets and the expected present value of present and future income. According to Conditions (13) and (14) in the present period the sovereign can consume as much as its present income plus its liquid assets, and in any future period the sovereign can consume as much as its current income. Further, in any future period the sovereign can consume in excess of its current income if and only if it has first accumulated some savings.

Having issued the amount of one-period debt $\tilde{b}_t$, the sovereign would succumb to the temptation to repudiate this debt if and only if U_t^o is larger than U_t^*. To compare U_t^* with U_t^o observe first that, if the realization of y_t is larger than y_{LL}, so that s_t as given by Eq. (6) is positive, then $y_t + (1+\rho)\tilde{b}_t$ is larger than $\bar{y}_t$. Hence, comparing Conditions (10) and (11) with Conditions (12) and (13), we see that, if the realization of y_t is larger than y_{LL}, then on average c_τ^o is larger than c_τ^*. (In doing this comparison, note that the average value of $\bar{y}_\tau$ equals the average value of y_τ.) Thus, given any realization of income larger than the smallest possible realization of income, debt repudiation would permit larger average consumption.

These observations imply that, unless the realization of income in period t equals the smallest possible realization of income, the sovereign would be tempted to repudiate its debt. Furthermore, the difference between $y_t + (1+r)\tilde{b}_t$ and $\bar{y}_t$ is an increasing function of y_t. Thus, the larger realization of income in period t, the greater is the temptation to repudiate the debt.

Observe next that, because $\bar{y}_\tau$ is less variable that y_τ, Conditions (10) and (11) permit more consumption smoothing than Conditions (12) and (13). (As we have noted, in the special case in which the probabilities associated with the possible realizations of income are constant, Conditions (10) and (11) would permit complete consumption smoothing.) Given diminishing marginal utility, the attraction of being able to smooth consumption in the future counters the temptation to repudiate the debt. In fact, if the utility function were sufficiently concave, then U_t^* would be as large as U_t^o for all possible combinations of $\tilde{b}_t$ and realizations of y_t. In this case, the sovereign would be able to issue in any period τ an amount of debt equal to $\tilde{b}_\tau$, and contingent debt servicing would equate its net cash flow in each period with the conditional expected realization of its income.

More generally, even if U_t^* is not as large as U_t^o for all possible combinations of $\tilde{b}_t$ and realizations of y_t, contingent debt servicing would support the issuance of a smaller, but positive amount of debt. In this event, even in the special case in which the probabilities associated with the possible realizations of income are constant, the sovereign would not be able to smooth consumption completely. Nevertheless, as long as the sovereign can issue a positive amount of debt, contingent debt servicing would permit more consumption smoothing than would saving and dissaving alone.

The essential observation is that, because there is a finite limit to accumulated savings, as there is a finite limit to accumulated debt, the possibility of contingent saving, like the possibility of contingent debt issuance, would not permit complete smoothing of consumption. Thus, even if the sovereign could save and dissave, it would still be valuable to be able to issue debt with contingent servicing in order to achieve more complete consumption smoothing. Accordingly, the sovereign will resist the temptation to repudiate its debt as long as the amount of debt outstanding is not too large.

4. Summary

What is the critical difference between consumption smoothing by means of contingent debt servicing and consumption smoothing by means of contingent debt issuance that makes a positive amount of sovereign debt possible with contingent debt servicing but not with contingent debt issuance? The answer is that with contingent debt issuance once the sovereign reaches any positive debt ceiling repudiation would not reduce the possibilities for future consumption smoothing, whereas with contingent debt servicing repudiation always would reduce the possibilities for future consumption smoothing.

Grossman and Van Huyck (1988) motivated their model of contingent debt servicing by arguing that viewing sovereign debt as a contingent claim introduced an important element of realism into the analysis of sovereign debt. They argued that their contingent-claim model was useful in explaining why actual defaults are associated with identifiably bad states of the world, why defaults are usually partial rather than complete, and why sovereigns often are able to borrow again soon after default. The present note suggests further that state-contingent debt servicing also is a critical element in understanding the very existence of sovereign debt. Specifically, we have shown that without contingent debt servicing, a sovereign could not use uncollateralized debt to smooth consumption.

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