Debt, liquidity and dynamics [1]

H. M. Polemarchakis [2]   Céline Rochon[3]

Working Paper No. 01-22

Department of Economics, Brown University

May, 2001

Abstract

Money, which provides liquidity, is distinct from debt. The introduction of a bank that issues money in exchange for debt and pays out its profit as dividend to shareholders modifies the model of overlapping generations. Monetary policy can set, alternatively, the nominal rate of interest or the circulation of real balances. The set of equilibrium paths, their dynamic properties, as well as the scope and effectiveness of monetary policy are significantly altered: (1) there is a continuum of Pareto comparable steady state paths, indexed by the nominal rate of interest; (2) for a set level of real balances, the associated steady state may be stable or unstable, but cyclical fluctuations do not arise; and, (3) though low rates of interest are associated with superior steady state allocations, they may account for unstable steady states or stable endogenous cycles.

Keywords: Overlapping Generations Models, Money, Liquidity Constraints, Debt, Dynamics.
JEL Classification: E30, E32, E50, E52.

1 Introduction

The model of overlapping generations of Allais [**1**] and Samuelson [**59**] abstracts from the role of money in the provision of liquidity, and it does not distinguish between debt and money; it may be inadequate for the study of monetary equilibria and the analysis of monetary policy.

The introduction of a monetary-fiscal authority that issues money in exchange for debt and pays out its profit as dividend to shareholders modifies the model of overlapping generations: it distinguishes money, which provides liquidity, from debt, which does not.

Monetary policy can target, alternatively, the nominal rate of interest or the supply of money.

A monetary authority lends money in exchange for interest bearing bonds; also, it accepts deposits and pays depositors the same rate of interest which they charge on loans; equivalently, there are interest bearing nominal assets that dominate money as a store of value, which is not the case in Grandmont and Laroque [**34**], [**35**].

The distinction between money, which provides liquidity services and is issued by a bank that distributes its profit as dividend, and debt, a store of value, modifies the set of equilibrium paths, their dynamic properties, as well as the scope and effectiveness of monetary policy:

1. there is a continuum of Pareto comparable steady state paths, indexed by the nominal rate of interest;

2. for a set level of real balances, the associated steady state may be stable or unstable, but cyclical fluctuations do not arise; and,

3. though low nominal interest rates are associated with superior steady state allocations, they may account for unstable steady states or stable endogenous cycles.

In a recent papers, Drèze and Polemarchakis [**26**] and Bloise, Drèze and Polemarchakis [**13**], [**14**] have put forward a simple extension of the model of general competitive equilibrium [1] that encompasses monetary economies.

All initial holdings of money are the counterpart of debts to the monetary authority. In the vocabulary of monetary economics, this is a model of inside money, as is appropriate for economies with properly functioning central banks that only issue money in exchange for offsetting nominal claims. Nevertheless, it allows for a well defined supply of money. The conundrum, pointed out by Hahn [**38**], according to which money cannot maintain a positive price at equilibrium, derives from a false distinction between inside and outside money. Once it is recognized that in an economy with an explicitly, if simply, modeled banking sector money is issued in exchange for bonds or other stores of value, the distinction between inside and outside money is a moot point.

---

[1] Arrow and Debreu [**4**], McKenzie [**47**].

Alternative formulations are possible. In Dubey and Geanakoplos [**27**], [**28**], individuals hold outside money, whose valuation absorbs the seignorage that accrues to the monetary authority. In Lucas [**43**], Lucas and Stokey [**44**] and Woodford [**68**], fiscal policy effects the transfer of seignorage; in Lerner [**41**], the accounts of the private sector and the monetary-fiscal authority are consolidated and outside balances are taxed at a terminal date, possibly at infinity.

Competitive equilibria exist, which ensures the consistency of the specification. Nevertheless, even though there is a well defined money market, competitive equilibrium allocations are indeterminate: they can be indexed by the nominal rates of interest, as pointed out in Sargent and Wallace [**60**], and they may be pareto comparable.

The optimality [2] and determinacy [3] of competitive equilibrium allocations, which obtain in an economy without an operative transactions technology, fail in a monetary economy. Similarly, the optimality [4] and determinacy [5] of steady state allocations, which characterize economies of overlapping generations, fail when, unlike debt, money provides liquidity services.

There exist alternative ways of modeling the cost and inconvenience of non-monetary transactions [6]. Most simply, they are captured by a cash-in-advance constraint of Clower [**18**], modified by a parameter that reflects the exogenous velocity of circulation of money balances; in Drèze and Polemarchakis [**26**], a versatile specification allows for generality and, in particular, as suggested by Baumol [**9**] and Tobin [**62**], an interest elastic velocity of circulation.

Economies that extend over an infinite horizon and are characterized by an operative cash-in-advance constraint have been extensively employed in theoretical macroeconomics and monetary theory — Grandmont and Younès [**36**],[**37**], Lucas and Stokey [**44**] or Woodford [**68**].

An important question in the theory of monetary policy concerns the appropriate targets and instruments for a monetary authority: an interest rate policy sets the nominal rate of interest and accommodates the demand for balances; alternatively, according to the real bills doctrine, the monetary authority controls the supply of balances, and the nominal rate of interest adjusts to maintain equilibrium. The question has been studied in reduced form models, Poole [**55**], Sargent and Wallace [**60**], that facilitate the application of control techniques, but are not explicit on individual optimization and market clearing; among others, do not allow for welfare comparisons. In a monetary economy of overlapping generations, the question can be analyzed in an equilibrium framework, and the results that obtain are of obvious relevance for the conduct of monetary policy.

The indeterminacy of equilibrium paths for a set path of real balances or, alternatively, of the rate of interest is most interesting under uncertainty, which, here, is absent. It depends, first on the distinction introduced in Woodford [**68**]: at a Ricardian policy, in Drèze and Polemarchakis [**26**] or Bloise, Drèze and

---

[2]Arrow [**2**], Debreu [**19**].

[3]Debreu [**21**].

[4]Cass [**15**], Balasko and Shell [**8**], Benveniste and Cass [**12**].

[5]Kehoe and Levine [**39**].

[6]Kiyotaki and Wright [**40**], Ostroy and Starr [**50**].

Polemarchakis [**13**], [**14**], the monetary-fiscal authority is restricted to satisfy a budget constraint at all date-events, while at a non-Ricardian policy, in Dubey and Geanakoplos [**27**], [**28**] or Woodford [**68**], the monetary-fiscal authority may violate the budget constraint and accumulate debt at terminal date-events, possibly at infinity. It also depends on the financial policy adopted by the monetary-fiscal authority, as analyzed in Nakajima and Polemarchakis [**49**] or the stationarity of monetary policy, as in Stokey and Lucas [**44**].

With a cash-in-advance constraint the nominal rate of interest is an ad valorem tax on purchases, while the distribution of seignorage in the form of a dividend acts as a lump sum transfer: indeed, the reduced form of the equilibrium conditions coincides with those in an economy with debt and fiscal policy that taxes the purchase of commodities but distributes tax revenue as lump sum transfers to individuals.

The simple model of overlapping generations, in which individuals are active over two dates, poses a problem of interpretation when short run matters are concerned: if a date is understood as half of the economic life-span of a generation, cash holdings should correspond to a negligible fraction of consumption expenditures at a date. The point here, is, however, a simple observation: the distinction between debt, a store of value, and balances, a medium of exchange, alters established intuitions concerning the dynamics of monetary economies and optimal policy; it should be the subject of subsequent work to establish the empirical relevance of this observation. An alternative justification of the approach would rely on the derivation of the model of overlapping generations as the reduced form of a model with longer life-spans but operative finance constraints, as suggested in Woodford [**67**]; the generality and robustness of this argument are, however, problematic.

Economies of overlapping generations have provided an appropriate framework for the study of endogenous cycles, Benhabib and Day [**11**] and Grandmont [**33**]. The point made is twofold: stationary economies may have cyclical equilibrium paths and, as a structural parameter, such as the elasticity of savings with respect to the real rate of interest varies, the economy may switch from an economy in which the steady state is stable in the equilibrium dynamics to one where a cycle of arbitrary order is attractive.

When money provides liquidity and the nominal rate of interest along an equilibrium path is indeterminate, it is possible to study the dynamic properties of equilibrium paths for different levels of real balances or, alternatively, of the nominal rate of interest: structural change is the outcome of a policy decision, such as the targeted nominal rate of interest or the money supply.

If monetary policy targets the supply of real balances, the stability of the steady state cannot be guaranteed; nevertheless, cyclical fluctuations do not arise.

If, alternatively, monetary policy targets the nominal rate of interest, a trade-off emerges: though low rates of interest are associated with higher levels of welfare at the steady state [7], they are associated with troublesome dynamic

---

[7] Friedman [**30**].

properties of the equilibrium path: either the steady state is unstable or an attractive two-period cycle emerges as the nominal interest rate tends to zero.

2 The Economy

Discrete time, t, extends into the infinite future as well as, possibly, the infinite past. One, perishable commodity is exchanged and consumed at each date. The price of the commodity is $p_t > 0$. The rate of inflation is

$$\pi_{t+1} = \frac{p_{t+1}}{p_t} - 1.$$

The economy is stationary: individuals differ only on the dates at which they are active, and the rate of growth of population, n, is a constant. An individual, t, is active at dates t and $t+1$. A generation, t, consists of

$$n_t = (1 + n)^t$$

identical individuals.

The preferences of an individual are described by an intertemporally separable utility function

$$u(x_t) + v(y_{t+1}),$$

where x_t is the consumption of an individual at the first date that he is active and y_{t+1} the consumption at the second. The cardinal utility indices, u and v, are smooth, strictly monotonically increasing and concave functions that satisfy the boundary conditions $\lim_{x \to 0} u'(x) = \infty$ and $\lim_{y \to 0} v'(y) = \infty$.

The endowment of an individual is $\bar{x} > 0$, at the first date that he is active, while it is $\bar{y} > 0$ at the second date that he is active.

Debt held by individuals at their first date of activity, b_t^1, is only used to finance the first period money holdings, m_t^1, and bears nominal interest $r_t \geq 0$. Debt held by individuals at their second date of activity, b_{t+1}^2, finances the second period money holdings, m_{t+1}^2, and serves as a store of value between dates t and $t+1$. It bears nominal interest $r_{t+1} \geq 0$. The real rate of interest is

$$\rho_{t+1} = \frac{1 + r_{t+1}}{1 + \pi_{t+1}} - 1,$$

and indexed debts are

$$\beta_t^1 = \frac{b_t^1}{p_t}, \quad \text{and} \quad \beta_{t+1}^2 = \frac{b_{t+1}^2}{p_{t+1}}$$

respectively.

Money provides liquidity services and serves as a store of value that bears no interest; balances, m_t^1 and m_{t+1}^2, are issued at dates $t-1$ and t and provide liquidity at dates t and $t+1$, respectively. Real balances are

$$\mu_t^1 = \frac{m_t^1}{p_t}, \quad \text{and} \quad \mu_{t+1}^2 = \frac{m_{t+1}^2}{p_{t+1}}.$$

The budget constraints of an individual, t, are

$$m_t^1 + b_t^1 = 0,$$

$$p_t x_t + m_{t+1}^2 + b_{t+1}^2 = p_t \overline{x} + (1 + r_t)b_t^1 + m_t^1,$$

$$p_{t+1} y_{t+1} = p_{t+1} \overline{y} + (1 + r_{t+1})b_{t+1}^2 + m_{t+1}^2 + T_{t+1},$$

where T_{t+1} is the profit of the monetary authority redistributed as a lump sum transfer to the individual at his second date of activity (the precise expression will be given below). These constraints reduce to the intertemporal budget constraint

$$x_t + \frac{y_{t+1}}{1 + \rho_{t+1}} + \frac{r_{t+1}}{1 + \rho_{t+1}}\mu_{t+1}^2 + r_t \mu_t^1 = \overline{x} + \frac{\overline{y}}{1 + \rho_{t+1}} + \frac{\tau_{t+1}}{1 + \rho_{t+1}},$$

where $\tau_{t+1} = \frac{T_{t+1}}{p_{t+1}}$.

Liquidity constraints are operative in both periods that an individual is active; they are described by

$$\gamma p_t x_t \leq m_t^1, \quad \text{and} \quad \delta p_{t+1} y_{t+1} \leq m_{t+1}^2,$$

or, equivalently,

$$\gamma x_t \leq \mu_t^1, \quad \text{and} \quad \delta y_{t+1} \leq \mu_{t+1}^2$$

where $1 \geq \gamma, \delta > 0$ are the reciprocal of the velocity of circulation of balances. Unless otherwise mentioned, $\gamma > 0$ and $\delta > 0$. For $\gamma = 0$ and $\delta = 0$ the liquidity constraints vanish. As long as the nominal rate of interest is positive, $r_t > 0$, the liquidity constraints are binding.

Per-capita aggregate real balances at t are

$$\mu_t = \kappa\left(\mu_t^1 + \frac{1}{1+n}\mu_t^2\right),$$

where

$$\mu_t^2 = \left(\overline{x} + \frac{\overline{y}}{1+n} - \frac{\mu_t^1}{\gamma}\right)\delta(1+n), \quad \text{and} \quad \kappa = \frac{1+n}{2+n}.$$

Whenever $\delta = \gamma$, the path of aggregate real balances is constant through time: at all dates,

$$\mu_t = \delta\kappa\left(\overline{x} + \frac{\overline{y}}{1+n}\right).$$

To avoid such a degenerate situation, we assume $\delta \neq \gamma$. Though this poses problems if the parameters γ and δ are to be interpreted as the reciprocal of the velocity of circulation of balances, it is natural that individuals at different stages of their life cycle face different liquidity constraints.

At dates $t-1$ and t, a monetary authority issues per-capita balances, $\overline{m}_t^1 \geq 0$ and $\overline{m}_{t+1}^2 \geq 0$, that provide liquidity at dates t and $t+1$ respectively. The rate of growth of per capita balances is

$$\sigma_{t+1} = \frac{\overline{m}_{t+1}^1}{\overline{m}_t^1} - 1 = \frac{\overline{m}_{t+1}^2}{\overline{m}_t^2} - 1.$$

5

The monetary authority supplies balances in exchange for debt through open market operations; part of the debt issued by the old individuals, $\tilde{b}^2_{t+1}$, is the counterpart of second period balances:

$$\overline{m}^2_{t+1} + \tilde{b}^2_{t+1} = 0.$$

The total amount of debt issued by individuals at their first date of activity is the counterpart of first period balances: $\tilde{b}^1_t = b^1_t$ and $\overline{m}^1_t + b^1_t = 0$. The profit of the monetary authority out of open market operations at date t that are concluded at date $t + 1$ is seignorage,

$$r_{t+1}(\overline{m}^1_{t+1}(1 + n) + \overline{m}^2_{t+1}),$$

which is distributed as dividend to individuals at their second date of activity at date $t + 1$. Hence

$$T_{t+1} = r_{t+1}(\overline{m}^1_{t+1}(1 + n) + \overline{m}^2_{t+1}).$$

We denote the supplies of real balances per-capita by

$$\overline{\mu}^1_t = \frac{\overline{m}^1_t}{p_t}, \qquad \overline{\mu}^2_{t+1} = \frac{\overline{m}^2_{t+1}}{p_{t+1}},$$

and the real per-capita dividends by

$$r_t \overline{\mu}^1_t, \qquad r_{t+1} \overline{\mu}^2_{t+1}.$$

The budget constraint of the fiscal authority that manages the debt is

$$(\beta^2_{t+1} + \overline{\mu}^2_{t+1})(1 + r_{t+1}) = \frac{1 + \rho_{t+1}}{1 + n}(\beta^2_t + \overline{\mu}^2_t)(1 + r_t).$$

2.1 Equilibrium

A competitive equilibrium is a path,

$$((x_t, y_t, \mu^1_t, \mu^2_t, \beta^1_t, \beta^2_t, \overline{\mu}^1_t, \overline{\mu}^2_t, \rho_t, r_t) : t = \dots, 0, 1, \dots),$$

such that

1. $(x_t, y_{t+1}, \mu^1_t, \mu^2_{t+1}, \beta^1_t, \beta^2_{t+1})$ is a solution to the optimization problem of individual t at real rate of interest ρ_{t+1}, nominal rate of interest r_{t+1}, and real dividends $r_t \overline{\mu}^1_t$, and $r_{t+1} \overline{\mu}^2_{t+1}$,

2. $x_t + \frac{y_t}{1+n} = \overline{x} + \frac{\overline{y}}{1+n}$, and

3. $\mu^1_t = \overline{\mu}^1_t$, and $\mu^2_t = \overline{\mu}^2_t$.

6

The debt market clears as a residual.

A steady state is a competitive equilibrium that is stationary: at all dates,

$$x_t = x, \quad y_t = y.$$

Competitive equilibria are fully described by paths of nominal rates of interest and per-capita aggregate supplies of real balances. Equilibria are of two kinds: with debt, or autarkic.

Paths along which debt held by the old individuals exceeds bonds holdings that finance the provision of liquidity:

$$0 < x_t < \frac{\overline{x}}{1 + \gamma r_t}, \quad \text{or else} \quad 0 < y_t - x_t \gamma r_t (1 + n) < \overline{y},$$

are equilibria with debt.

Paths along which all bonds holdings finance the provision of liquidity:

$$x_t (1 + \gamma r_t) = \overline{x}, \quad \text{or else} \quad y_t - x_t \gamma r_t (1 + n) = \overline{y},$$

are autarkic.

Lemma 1 *A path of nominal rates of interest and per-capita aggregate supplies of real balances determines a competitive equilibrium path with debt if and only if*

$$\frac{u'(x_t)}{v'(y_{t+1})} = (1 + \rho_{t+1}) \frac{(1 + \gamma r_t)}{(1 + \delta r_{t+1})},$$

where

$$x_t = \frac{\mu_t^1}{\gamma} = \frac{\mu_t - \delta \kappa (\overline{x} + \frac{\overline{y}}{1+n})}{\kappa (\gamma - \delta)},$$

$$y_{t+1} = \frac{\mu_{t+1}^2}{\delta},$$

$$\beta_t^1 = -\mu_t^1,$$

$$\beta_{t+1}^2 = \frac{\frac{\mu_{t+1}^2}{\delta}(1 - \delta - \delta r_{t+1}) - \overline{y} - r_{t+1} \mu_{t+1}^1 (1 + n)}{1 + r_{t+1}},$$

$$1 + \rho_{t+1} = (1 + n) \frac{\overline{x} - \frac{\mu_{t+1}^1}{\gamma}(1 + \gamma r_{t+1})}{\overline{x} - \frac{\mu_t^1}{\gamma}(1 + \gamma r_t)}$$

$$= (1 + n) \frac{\frac{\mu_{t+1}^2}{\delta} - r_{t+1} \mu_{t+1}^1 (1 + n) - \overline{y}}{\frac{\mu_t^2}{\delta} - r_t \mu_t^1 (1 + n) - \overline{y}};$$

7

equivalently,

$$\frac{u'\left(\frac{\mu_t - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}\right)}{v'\left((\overline{x} + \frac{\overline{y}}{1+n} - \frac{\mu_{t+1} - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)})(1+n)\right)} =$$

$$\frac{(1+\gamma r_t)}{(1+\delta r_{t+1})}(1+n)\left(\frac{\overline{x} - \frac{\mu_{t+1} - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}(1+\gamma r_{t+1})}{\overline{x} - \frac{\mu_t - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}(1+\gamma r_t)}\right).$$

Along an equilibrium path with debt, the rate of growth of real balances is

$$1 + \sigma_{t+1} = \frac{1+r_{t+1}}{1+n}\frac{\overline{x} - \frac{\mu_t^1}{\gamma}(1+\gamma r_t)}{\overline{x} - \frac{\mu_{t+1}^1}{\gamma}(1+\gamma r_{t+1})}\frac{\mu_{t+1}^1}{\mu_t^1},$$

and the rate of inflation is

$$1 + \pi_{t+1} = \frac{1+r_{t+1}}{1+n}\frac{\overline{x} - \frac{\mu_t^1}{\gamma}(1+\gamma r_t)}{\overline{x} - \frac{\mu_{t+1}^1}{\gamma}(1+\gamma r_{t+1})}.$$

2.2   Stationary states

A constant path of nominal rates of interest and supplies of real balances,

$$(r, \mu),$$

determines a steady state with debt if and only if

$$u'\left(\frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}\right) =$$

$$v'\left((\overline{x} + \frac{\overline{y}}{1+n} - \frac{\mu_- \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)})(1+n)\right)\frac{1+\gamma r}{1+\delta r}(1+n).$$

At a steady state with debt, the real rate of interest coincides with the rate of growth of population:

$$\rho = n;$$

the rate of inflation coincides with the rate of growth of per capita balances:

$$\sigma = \pi.$$

Alternatively, a constant path of nominal rates of interest determines an autarkic steady state with

$$x_t = \frac{\overline{x}}{1+\gamma r}, \quad y_{t+1} = \overline{y} + \frac{\overline{x}\gamma r}{1+\gamma r}(1+n), \quad \mu_t = \kappa\left(\overline{x}\gamma\frac{1+\delta r}{1+\gamma r} + \frac{\overline{y}\delta}{1+n}\right)$$

8

if and only if

$$u'\left(\frac{\overline{x}}{1+\gamma r}\right) = v'\left(\overline{y} + \frac{\overline{x}\gamma r}{1+\gamma r}(1+n)\right)\frac{1+\gamma r}{1+\delta r}(1+\rho).$$

The real rate of interest indexes the one-dimensional continuum of autarkic steady states.

Proposition 1 *There exists a one-dimensional continuum of distinct steady states with debt indexed by the per-capita aggregate supply of real balances or the nominal rate of interest, and a one-dimensional continuum of autarkic steady states indexed by the nominal rate of interest. A steady state with debt is pareto superior to a steady state with debt with a higher nominal rate of interest; in particular, the steady state with debt with nominal rate of interest $r = 0$ is pareto superior to all other steady states.*

Proof At a steady state with debt, for any given aggregate supply of real balances, μ, the corresponding nominal rate of interest is

$$r = \frac{v'(\overline{y} + (1+n)(\overline{x} - x))(1+n) - u'(x)}{\delta u'(x) - \gamma v'(\overline{y} + (1+n)(\overline{x} - x))(1+n)},$$

where

$$x = \frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}.$$

From the boundary behavior and the concavity of the cardinal utility indices and since $\delta \neq \gamma$, it follows that there exists an interval of non-negative rates of interest, $[0, \overline{r})$, in which there is a one-to-one relation between the nominal rate of interest and the allocation of resources and money balances at the associated steady state.

Alternatively, for a given nominal rate of interest, r, the corresponding supply of real balances is given implicitly by

$$u'(x) = v'(\overline{y} + (1+n)(\overline{x} - x))\frac{1+\gamma r}{1+\delta r}(1+n).$$

At the autarkic steady state, the real rate of interest is given by

$$(1+\rho) = \frac{u'\left(\frac{\overline{x}}{1+\gamma r}\right)\frac{1+\delta r}{1+\gamma r}}{v'\left(\overline{y} + \frac{\overline{x}\gamma r}{1+\gamma r}(1+n)\right)}.$$

For the optimization problem

$$\max_r u(x(r)) + v(y(r)),$$

where $x(r)$ and $y(r)$ are the steady state values of consumption with debt, the derivative of the objective function is

$$u'(x(r))\left(\frac{\mu'(r)}{\kappa(\gamma - \delta)}\right) + v'(y(r))\left(\frac{-\mu'(r)}{\kappa(\gamma - \delta)}\right)(1+n),$$

9

which reduces to

$$\mu'(r)v'(y(r))\frac{(1+n)}{\kappa}\frac{r}{1+\delta r}.$$

Since $u'(x(r)) = (1+n)[(1+\gamma r)/(1+\delta r)]v'(y(r))$, one finds that

$$\mu'(r) = \frac{(1+n)\kappa\frac{(\gamma-\delta)^2}{(1+\gamma r)^2}v'(y(r))}{u''(x(r)) + \frac{(1+\gamma r)}{(1+\delta r)}(1+n)^2 v''(y(r))} < 0.$$

Consequently, for any positive rate of interest, the derivative of the objective function is negative, and the unique solution to the maximization problem is $r = 0$. As $\rho = n$ at any steady state with debt, the corresponding optimal level of inflation is $-n/(1+n)$. Moreover, since the derivative of the objective function is negative, the lower the nominal rate of interest the higher the welfare at the steady state.

$\square$

The allocation of resources at autarkic steady states with different nominal rates of interest are distinct as well.

The nominal interest rate r for which $\rho = n$ at the associated autarkic steady state is implicitly given by

$$u'\left(\frac{\overline{x}}{1+\gamma r}\right)\frac{1+\delta r}{1+\gamma r} = v'\left(\overline{y} + \frac{\overline{x}\gamma r}{1+\gamma r}(1+n)\right)(1+n).$$

It corresponds to a limit point of the set of steady states with debt. Indeed, a steady state with debt is characterized by aggregate real balances and nominal rates of interest, (μ, r), for which debt is not only the counterpart of money: $\beta^2 \neq -\mu^2$, while, in an autarkic steady state, debt is exactly the counterpart of money: $\beta^2 = -\mu^2$.

Since, at a steady state with debt, the rate of inflation coincides with the rate of growth of the supply of balances while the real rate of interest coincides with the rate of growth of population,

$$1 + r = (1 + \sigma)(1 + n),$$

the rate of growth of per capita balances indexes the one-dimensional continuum of steady states with debt.

3   Monetary policy and dynamics

Monetary policy can be conducted to control the aggregate supply of real balances, or alternatively, the nominal rate of interest.

The argument considers only equilibria with debt.

3.1 Controlling the circulation of real balances

The monetary authority sets the aggregate supply of real balances, $\mu > 0$, constant over time, and the nominal rate of interest adjusts to maintain equilibrium.

The path of the nominal interest rate $(r_t : t = \dots, 0, 1, \dots)$ along an equilibrium with debt is described by the initial rate, r_0, and the first order, non-linear equation

$$\frac{u'\left(\frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}\right)}{v'\left((\overline{x} + \frac{\overline{y}}{1+n} - \frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)})(1 + n)\right)} =$$

$$\frac{(1 + \gamma r_t)}{(1 + \delta r_{t+1})}(1 + n)\left(\frac{\overline{x} - \frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}(1 + \gamma r_{t+1})}{\overline{x} - \frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}(1 + \gamma r_t)}\right).$$

A first order approximation at a steady state (μ, r) yields the path of linear deviations $(\tilde{r}_t)$ from the steady state nominal rate of interest; it is described by the equation

$$\tilde{r}_{t+1} = A\tilde{r}_t,$$

where

$$A = \left(\frac{1 + \delta r}{1 + \gamma r}\right)\frac{\gamma\overline{x}}{\delta\overline{x} + x(\gamma - \delta)}, \quad \text{and} \quad x = \frac{\mu - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}.$$

For $\gamma = 0$, and with real balances constant over time, the nominal rate of interest is necessarily constant as well; for $\gamma \neq 0$, the dynamics in the nominal interest rate is not trivial. The coefficient A is always positive, since, with debt, $\overline{x} > x$. The system is stable for $A < 1$, but unstable for $A > 1$; cyclical behavior cannot arise.

3.2 Interest rate policy

The monetary authority sets the rate of interest, $r > 0$, constant over time, and accommodates the demand for real balances; prices adjust and markets clear.

Equivalently, the monetary authority sets the rate of growth of the per capita supply of balances, σ, constant over time; this is the case, since, along an equilibrium path with debt, the rate of interest is determined by the rate of growth of the supply of balances and vice-versa.

The path of the aggregate supply of real balances $(\mu_t : t = \dots, 0, 1, \dots)$ along an equilibrium with debt is described by the initial supply, μ_0, and the first order, non-linear equation

$$\frac{u'\left(\frac{\mu_t - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)}\right)}{v'\left((\overline{x} + \frac{\overline{y}}{1+n} - \frac{\mu_{t+1} - \delta\kappa(\overline{x} + \frac{\overline{y}}{1+n})}{\kappa(\gamma - \delta)})(1 + n)\right)} =$$

$$\frac{(1+\gamma r)}{(1+\delta r)}(1+n)\left(\frac{\overline{x}-\frac{\mu_{t+1}-\delta\kappa(\overline{x}+\frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}(1+\gamma r)}{\overline{x}-\frac{\mu_t-\delta\kappa(\overline{x}+\frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}(1+\gamma r)}\right).$$

A first order approximation at a steady state (μ, r) yields the path of linear deviations $(\tilde{\mu}_t)$ from the steady state aggregate supply of real balances; it is described by the equation

$$\tilde{\mu}_{t+1} = A\tilde{\mu}_t,$$

where

$$A = \frac{1 + \frac{\overline{U}}{x}\frac{\overline{x}-x(1+\gamma r)}{1+\gamma r}}{1 - \frac{\overline{V}}{y}\frac{(1+n)(\overline{x}-x(1+\gamma r))}{1+\gamma r}},$$

with

$$x = \frac{\mu(r) - \delta\kappa(\overline{x}+\frac{\overline{y}}{1+n})}{\kappa(\gamma-\delta)}, \quad \frac{y}{1+n} = \overline{x} + \frac{\overline{y}}{1+n} - x.$$

For computational reasons, one restricts attention to the case $\gamma = 0$ and $\overline{y} = 0$, under which

$$A = \frac{1 + \overline{U}\frac{\overline{x}-x}{x}}{1 - \overline{V}}, \quad \text{and} \quad x = \overline{x} - \frac{\mu}{\delta\kappa},$$

and a parametric specification of the preferences of the individuals.

Proposition 2 *If the intertemporal utility function is*

$$u(x_t) + v(y_{t+1}) = \frac{1}{\alpha}x_t^\alpha + \frac{\nu}{\alpha}y_{t+1}^\alpha, \quad \alpha < 1, 0 < \nu,$$

then there exists a value of the nominal rate of interest,

$$\overline{r} = \frac{1}{\delta}\left(\nu(1+n)^\alpha\left(-\frac{1-\alpha}{1+\alpha}\right)^{1-\alpha} - 1\right),$$

that determines the dynamic behavior of real balances along an equilibrium path with debt:

1. *the associated path of linear deviations from the steady state supply of real balances is unstable if and only if $r < \overline{r}$ or $\alpha > -1$: the elasticity of the marginal utility of consumption is less than 2, and*

2. *in the linear case, there is no equilibrium path of supplies of real balances with $\mu_0 \neq \mu(r)$ whenever $r < \overline{r}$.*

Proof A first order approximation at a steady state, (r, μ), yields the path of linear deviations $(\tilde{\mu}_t : t = \ldots, 0, 1, \ldots)$ from the steady state supply of real

12

balances; it is described by the initial deviation, $\tilde{\mu}_0$, and the first order, linear equation

$$\tilde{\mu}_{t+1} = A\tilde{\mu}_t,$$

where

$$A = \frac{1 + (1 - \alpha)\frac{\mu}{\overline{x}\delta\kappa - \mu}}{\alpha},$$

and the steady state value of real balances is

$$\mu = \mu(r) = \delta \frac{\left(\frac{\nu(1+n)^\alpha}{1+\delta r}\right)^{\frac{1}{1-\alpha}} \overline{x}\kappa}{1 + \left(\frac{\nu(1+n)^\alpha}{1+\delta r}\right)^{\frac{1}{1-\alpha}}}.$$

The sign and magnitude of the coefficient A determine the dynamic behavior of deviations or, equivalently, the local dynamic behavior of the supply of real balances.

The path of linear deviations from the steady state supply of real balances is unstable if and only if $|A| > 1$; by direct computation, this is the case if and only if $\alpha > -1$ or $r < \overline{r}$.

$\square$

Proposition 2 reports only the linear case as the nonlinear dynamics associated with this specification is not well defined over an unrestricted domain.

The minimal nominal rate of interest that prevents instability of the steady state increases with the rate of time preference, ν; and it decreases with the velocity of circulation of δ. In particular, any nominal rate of interest allows for stability if

$$\left(-\frac{1-\alpha}{1+\alpha}\right)^{1-\alpha}\nu(1+n)^\alpha \leq 1,$$

while, if the last inequality is reversed, rates of interest sufficiently close to the first best rate lead to instability.

In the case of no growth, $n = 0$, and for $\alpha \in (-100, -1)$, rates of interest sufficiently close to 0 lead to instability as long as $\nu > .13$.

Figure 1 below shows the relationship between α and $\overline{r}$, the minimal rate of interest that eliminates instability, for $\delta = 0.5$, $n = 0.02$ and $\nu = 0.98$.

With a policy of low nominal rates, any supply of real balances other than the one corresponding to the steady state may eventually lead to a failure of feasibility.
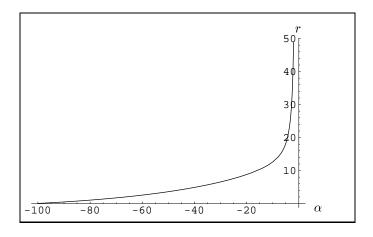
13

Figure 1
For any value of the nominal rate of interest below the curve
or for $\alpha > -1$, the steady state is unstable.

More generally, the elasticity of the marginal utility of consumption differs in the two dates of activity of the individual and the elasticity of intertemporal substitution is not constant.

The intertemporal utility function is

$$u(x_t) + v(y_{t+1}) = \frac{1}{\alpha}x_t^\alpha + \frac{\nu}{\theta}y_{t+1}^\theta, \quad \alpha < 1, \theta < 1, 0 < \nu.$$

It is necessary to assign values to the parameters. Let $\alpha = -1$ and $\theta = -3$. If the intertemporal utility function is

$$u(x_t) + v(y_{t+1}) = -x_t^{-1} - \frac{\nu}{3}y_{t+1}^{-3}, \quad \nu > 0,$$

then the steady state value of real balances is

$$\mu(r) = \frac{\delta\kappa}{1+n}\left(\overline{x}\left(\frac{\nu(1+n)}{1+\delta r}\right)^{\frac{1}{2}} + \frac{\nu}{4(1+\delta r)(1+n)}\right)^{\frac{1}{2}} - \frac{1}{2}\left(\frac{\nu}{(1+\delta r)(1+n)}\right)^{\frac{1}{2}}.$$

The associated path of linear deviations from the steady state supply of real balances is unstable if and only if $r < \overline{r}$, where

$$\overline{r} = \frac{1}{\delta}\left(\frac{\nu}{(1+n)^3}\frac{4}{\overline{x}^2} - 1\right).$$

If $\overline{x} > 2(\nu/(1+n)^3)^{\frac{1}{2}}$, no choice of the nominal rate of interest leads to instability.

Low endowments $\overline{x}$ are compatible with asymptotic stability in the deviations of the supply of real balances for high values of the nominal rate of interest only.

14

Alternatively, preferences display constant elasticity of the marginal utility of consumption at the first day an individual is active but constant partial elasticity of the marginal utility of consumption at the second date.

Proposition 3 *If the intertemporal utility function is*

$$u(x_t) + v(y_{t+1}) = -\frac{1}{\alpha}\exp(-\alpha x_t) + \frac{\nu}{\theta}y_{t+1}^{\theta}, \quad 0 < \alpha, \theta < 1, 0 < \nu,$$

then there exists a value of the nominal rate of interest,

$$\overline{r} = \frac{1}{\delta}\left(\nu(1+n)^{\theta}\left(-\frac{1+\theta}{\alpha}\right)^{\theta-1}\exp(\alpha\overline{x}+1+\theta)-1\right),$$

that determines the dynamic behavior of real balances along an equilibrium path with debt:

1. *the associated path of linear deviations from the steady state supply of real balances is unstable if and only if $\theta > -1$ or $r < \overline{r}$, and*

2. *according to simulations for $\theta \ll -1$, in the dynamical system in the supply of real balances, if $r < \overline{r}$ the steady state is unstable: if $\mu_0 \neq \mu(r)$, then the equilibrium path with debt of the supply of real balances converges to a cycle of period two.*

Proof By direct computation, the dynamical system $(\mu_t : t = \ldots, 0, 1, \ldots)$ is described by the equation
$$\mu_{t+1} = \psi(\mu_t; r),$$
where
$$\psi(\mu_t; r) = \left(\frac{\nu(1+n)}{1+\delta r}\left(\frac{\delta\kappa}{1+n}\right)^{1-\theta}\frac{\exp\left(\alpha\left(\overline{x}-\frac{\mu_t}{\delta\kappa}\right)\right)}{\mu_t}\right)^{-\frac{1}{\theta}}.$$

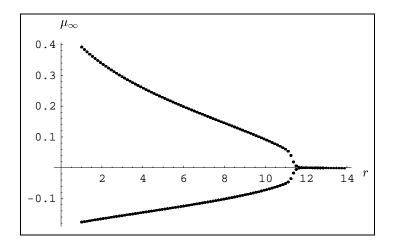Similarly, the steady state value of real balances is implicitly given by the equation

$$\frac{\alpha}{\delta\kappa}\mu(r) - \alpha\overline{x} = \ln\left(\frac{\nu(1+n)}{1+\delta r}\left(\frac{\delta\kappa}{1+n}\right)^{1-\theta}\mu(r)^{\theta-1}\right).$$

As the steady state value of μ cannot be computed explicitly, it is convenient to translate ψ by the steady state value of μ for which a bifurcation occurs, as this value of μ is independent of r.

The path of linear deviations from the steady state supply of real balances is unstable if and only if
$$|\psi'(\mu(r))| > 1;$$

15

by direct computation, this is the case if and only if $\theta > -1$ or $r < \overline{r}$.

Figure 2

The bifurcation diagram of ψ.
μ_∞ are the stable fixed points.

Figure 2 shows the bifurcation diagram of ψ, for $n = 0.02$, $\nu = 0.98$, $\alpha = -\theta = 10$ and $\delta = 0.5$, after 2000 iterations.

For values of the nominal rate of interest below the bifurcation point $\overline{r}$, there is a stable cycle of period two.

$\square$

To show the uniqueness of a stable cycle, one mainly needs to show that the Schwarzian derivative associated to the underlying dynamical system is negative. We have not performed such computations here for the map ψ but the result is observed through simulations.

Values of the nominal rate of interest that do not allow for instability and, possibly cycles, are moderate as long as $\alpha = -\theta$ and α is high. Variations in $\overline{x}$ do matter.

Figure 3 below shows the relationship between α and the $\overline{r}$, the minimal rate of interest that eliminates instability, for $\theta = -\alpha$, $\delta = 0.5$, $\overline{x} = 1$, $n = 0.02$, and $\nu = 0.98$.

The speed of convergence to the steady state of the non-linear path increases with the nominal rate of interest.

With a policy of low nominal rates, any supply of real balances other than the one corresponding to the steady state may lead to endogenous equilibrium fluctuations.
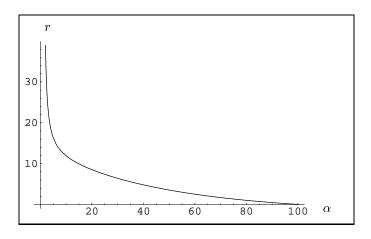
Figure 3

Below the curve or for values of $\theta > -1$, the steady state is unstable.

4 Remarks

Aspects of the specification of the model play an important role, which should be brought forward.

The liquidity constraint   With the consumption of individuals different from their net trades at the date that the transactions constraint is operative, it is possible to impose the cash-in-advance requirement on transactions:

$$\delta p_{t+1} \max\{0, (y_{t+1} - \overline{y})\} \leq m_{t+1}^2,$$

or on consumption:

$$\delta p_{t+1} y_{t+1} \leq m_{t+1}^2.$$

The constraint on transactions is more convincing; it introduces computational complications which may not be without interest. The constraint on consumption can be rationalized by arguing that the endowment of individuals is labor which is employed to produce the consumption good with a linear technology.

Transfers   The nominal rate of interest acts as a consumption tax, while the distribution of seignorage to individuals acts as a lump sum transfer that alleviates the burden of the tax.

We now present an economy with debt and fiscal policy that taxes the purchase of commodities but distributes tax revenue as lump sum transfers to individuals.

Debt, d_{t+1}, serves as a store of value and bears nominal interest, r_{t+1}. The real rate of interest is ρ_{t+1} and indexed debt is β_{t+1}.

17

No liquidity constraints are operative and balances that provide liquidity are absent.

At dates t and $t+1$, a fiscal authority imposes tax rates, z_t^1 and z_{t+1}^2, on the purchases of the consumption good. It distributes the revenue as lump sum per capita transfers, w_t^1 and w_{t+1}^2, to individuals in the corresponding date of activity; the real per-capita transfers are τ_t^1 and τ_{t+1}^2.

The budget constraints of an individual, t, are

$$(1 + z_t^1)p_t x_t + \frac{1}{1+r_{t+1}}d_{t+1} = p_t \overline{x} + w_t^1,$$

$$(1 + z_{t+1}^2)p_{t+1}y_{t+1} = p_{t+1}\overline{y} + d_{t+1} + w_{t+1}^2,$$

which reduce to the intertemporal budget constraint

$$(1 + z_t^1)x_t + \frac{1 + z_{t+1}^2}{1 + \rho_{t+1}}y_{t+1} = \overline{x} + \tau_t^1 + \frac{\overline{y} + \tau_{t+1}^2}{1 + \rho_{t+1}}.$$

The budget constraints of the fiscal authority are

$$z_t^1 x_t = \tau_t^1, \qquad z_{t+1}^2 y_{t+1} = \tau_{t+1}^2.$$

A competitive equilibrium is a path

$$((x_t, y_t, \beta_t, \tau_t^1 \tau_t^2, \rho_t, r_t, z_t^1, z_t^2) : t = \dots, 0, 1, \dots),$$

such that

1. $(x_t, y_{t+1}, \beta_{t+1})$ is a solution to the optimization problem of individual t at real rate of interest ρ_{t+1}, nominal rate of interest r_{t+1}, consumption taxes z_t^1 and z_{t+1}^2 and transfers τ_t^1 and τ_{t+1}^2,

2. $x_t + \frac{1}{1+n}y_t = \overline{x}$, and

3. $\tau_t^1 = z_t^1 x_t$ and $\tau_{t+1}^2 = z_{t+1}^2 y_{t+1}$.

A path of tax rates and indexed per capita debt,

$$((z_t^1, z_t^2, \beta_t) : t = \dots, 0, 1, \dots),$$

such that
$$\beta_t < (1 + n)\overline{x},$$

determines a non-autarkic competitive equilibrium path if and only if

$$u'(\overline{x} - \frac{1}{1 + n}\beta_t) = v'(\overline{y} + \beta_{t+1})(1 + \rho_{t+1})\left(\frac{1 + z_t^1}{1 + z_{t+1}^2}\right),$$

where
$$1 + \rho_{t+1} = \frac{\beta_{t+1}}{\beta_t}(1 + n).$$

18

We observe that the equilibrium paths for the monetary economy with operative liquidity constraints and for the economy with active fiscal policy coincide whenever

$$z_t^1 = \gamma r_t, \qquad z_{t+1}^2 = \delta r_{t+1}.$$

The monetary authority   In the spirit of Wilson [**66**], one could imagine that at the beginning of time, a bank, whose value represents the discounted stream of liquidity services, is owned by an individual with continuous preferences over streams of consumption and a finite or infinite horizon of economic activity.

Shares to the bank constitute stores of value and absence of arbitrage between shares and debt is a necessary condition for equilibrium.

The inherent asymmetry of the construction may well prevent the existence of steady states. The question of interest is whether the optimization of the initial shareholders of the bank suffices to guarantee the optimality of competitive allocations.

5   Extensions

The argument extends to economies with capital. In Rochon [**58**], the simplest extension of the Diamond [**23**] model with bonds, money and a liquidity constraint is studied: capital and debt (bonds) are substitute assets that dominate money which provides liquidity services; this builds on a distinction insisted upon in Tobin [**61**]. There are two one-dimensional families of steady states indexed by the nominal rate of interest or by the growth rate of the money supply; in one family the real rate of interest is equal to the rate of growth of population, while in the other debt is the counterpart of money. Policy parameters, such as the nominal rate of interest or the rate of growth of the money supply, characterize the steady states and the dynamics.

The literature on the effectiveness of monetary policy has focused, following Lucas [**42**], on the information revealed by prices: in a monetary economy with asymmetrically informed individuals, generic full revelation as in Radner [**57**] fails, and, monetary policy, which affects the information content of prices, is effective — Polemarchakis and Seccia [**52**], Polemarchakis and Siconolfi [**53**], Weiss [**65**].

Asymmetric information and the incentive compatibility constraints required to overcome the adverse selection and moral hazard that ensue account both for the viability of money and the effectiveness of monetary policy — Dutta and Kapur [**29**].

The model of general competitive equilibrium encompasses economies under uncertainty either with a complete market in elementary securities, as in Arrow [**3**], or a complete market in contingent commodities, as in Debreu [**20**], or with an incomplete asset market, as in Radner [**56**]. When the asset market is incomplete, the role of money as a unit of account renders monetary policy effective through its impact on the attainable reallocations of revenue across date-events;

conditions necessary for the neutrality of money, recognized by Chamley and Polemarchakis [17] along the lines of Modigliani and Miller [48], fail, which confirms the argument in Tobin [63]. The argument has been developed extensively in the literature on economies with an incomplete asset market — Balasko and Cass [7], Cass [16], Geanakoplos and Mas-Colell [31], and Polemarchakis [51], with inside money, Magill and Quinzii [45], with outside money or Polemarchakis and Siconolfi [54], with operative transactions constraints that reflect, possibly, information asymmetries; in particular, Detêmple, Gottardi and Polemarchakis [22] developed the argument for economies of overlapping generations under uncertainty, but without the distinction between debt and money.

Following the study of self-fulfilling prophecies of Azariadis [5], Azariadis and Guesnerie [6] observed that two-cycles are limiting sunspot equilibria. In particular, stationary sunspot equilibria exist in the neighborhood of a periodic orbit. It would be worth studying the possible existence of sunspots when money provides liquidity services.

Wealth effects and price rigidities or structural constraints on price adjustments, as in Benassy [10] and Drèze [24] also make for effective monetary policy. If, at the onset, individuals have nominal positions, debts or credits denominated in money, variations in the purchasing power of units of account redistribute purchasing power across individuals and, if the latter are heterogeneous, have real effects. Similarly, if prices are not free to absorb monetary shocks, standard arguments for the neutrality of monetary policy fail, as argued in Drèze [25].

The foundations of Keynesian analysis, as for example in Malinvaud and Younès [46], with price rigidities and rationing, or in Geanakoplos and Polemarchakis [32], with indeterminate equilibrium paths in economies of overlapping generations, are enriched by the introduction of a bona fide medium of exchange.

A proper study of inflation would require an explicit consideration of behavior out of equilibrium. The indeterminacy of equilibrium leaves room for a process of price adjustment, in particular with price setting firms, that would absorb some of the degrees of freedom traditionally assigned to the monetary authority.

The considerations above are absent from the argument here, but well within the scope of extensions.

References

[1] M. Allais, *Economie et intérêt,* Imprimerie Nationale, France, (1947).

[2] K. J. Arrow, An extension of the basic theorems of classical welfare economics, in J. Neyman (ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,* University of California Press (1951), 507-532.

[3] K. J. Arrow, Le rôle des valeurs boursières pour la répartition la meilleure des risques, *Econométrie,* Edition du CNRS **11**, (1953), 41-48.

[4] K. J. Arrow and G. Debreu, Existence of equilibria for a competitive economy, *Econometrica* **22** (1954), 265-290.

[5] C. Azariadis, Self-fulfilling prophecies, *Journal of Economic Theory* **25** (1981), 380-396.

[6] C. Azariadis and R. Guesnerie, Sunspots and cycles, *Review of Economic Studies* **53** (1986), 725-737.

[7] Y. Balasko and D. Cass, The structure of financial equilibrium, I: exogenous yields and unrestricted participation, *Econometrica* **57** (1989), 135-162.

[8] Y. Balasko and K. Shell, The overlapping generations model, II: the case of pure exchange with money, *Journal of Economic Theory* **24** (1981), 112-142.

[9] W. J. Baumol, The transactions demand for cash: an inventory theoretic approach, *Quarterly Journal of Economics* **66** (1952), 545-556.

[10] J.-P. Bénassy, Neo - keynesian disequilibrium theory in a monetary economy, *Review of Economic Studies* **42** (1975), 503-523.

[11] J. Benhabib and R. Day, A characterization of erratic dynamics in the overlapping generations model, *Journal of Economic Dynamics and Control* **4** (1982), 37-55.

[12] L. Benveniste and D. Cass, An extension of Cass' characterization of infinite efficient production programs, *Journal of Economic Theory* **10** (1975), 229-238.

[13] G. Bloise, J. H. Drèze and H. M. Polemarchakis, Monetary equilibria over a finite horizon, mimeo (2000).

[14] G. Bloise, J. H. Drèze and H. M. Polemarchakis, Monetary equilibria over an infinite horizon, mimeo (2000).

[15] D. Cass, On capital overaccumulation in the aggregative, neoclassical model of economic growth: a complete characterization, *Journal of Economic Theory* **4** (1972), 200-223.

[16] D. Cass, On the 'number' of equilibrium allocations with financial markets, Working Paper No. 8500, CARESS, University of Pennsylvania, 1985.

[17] C. Chamley and H. M. Polemarchakis, Assets, general equilibrium and the neutrality of money, *Review of Economic Studies* **51** (1984), 129-138.

[18] R. Clower, A reconsideration of the microfoundations of monetary theory, *Western Economic Journal* **6** (1967), 1-9.

[19] G. Debreu, The coefficient of resource utilization, *Econometrica* **19** (1951), 271-292.

[20] G. Debreu, Une économie de l'incertain, *Economie Appliquée* **13** (1960), 111-116.

[21] G. Debreu, Economies with a finite set of equilibria, *Econometrica* **38** (1970), 387-392.

[22] J. Detêmple, P. Gottardi and H. M. Polemarchakis, The relevance of financial policy, *European Economic Review* **39** (1995).

[23] P. Diamond, National debt in a neoclassical growth model, *The American Economic Review* **55** (1965), 1125-1150.

[24] J. H. Drèze, Existence of an exchange equilibrium under price rigidities, *International Economic Review* **16** (1975), 301-320.

[25] J. H. Drèze, *Money and Uncertainty,* Edizioni del Elefante, (1994).

[26] J. H. Drèze, J. H. and H. M. Polemarchakis, "Intertemporal equilibrium and monetary policy,"in A Leijonhufvud (ed.), *Monetary Theory and Monetary Policy,* Macmillan (2000), xxx-xxx.

[27] P. Dubey and J. D. Geanakoplos, The value of money in a finite horizon economy: a role for banks, in P. S. Dasgupta, D. Gale, O. D. Hart and E. Maskin (eds.) *Economic Analysis of Markets and Games, Essays in Honor of Frank Hahn,* M.I.T. Press (1992), 407-444.

[28] Dubey, P. and J. D. Geanakoplos, "Inside-outside money, gains to trade and IS-LM," mimeo. (2000).

[29] J. Dutta and S. Kapur, Liquidity preference and financial intermediation, *Review of Economic Studies* **65** (1998),xxx-xxx.

[30] M. Friedman, The quantity theory of money: a restatement, in M. Friedman (ed.), *Studies in the Quantity Theory of Money,* University of Chicago Press (1956), 3-21.

[31] J. D. Geanakoplos and A. Mas-Colell, Real indeterminacy with financial assets, *Journal of Economic Theory* **47** (1989), 22-38.

[32] J. D. Geanakoplos and H. M. Polemarchakis, Walrasian indeterminacy and keynesian macroeconomics, *Review of Economic Studies* **53** (1986), 755-779.

[33] J.-M. Grandmont, On endogenous competitive business cycles, *Econometrica* **53** (1985), 995-1045.

[34] J.-M. Grandmont and G. Laroque, On and banking, *Review of Economic Studies* **42** (1975), 207-236.

[35] J.-M. Grandmont and G. Laroque, The liquidity trap, *Econometrica* **44** (1976), 129-135.

[36] J.-M. Grandmont and Y. Younès, On the role of money and the existence of a monetary equilibrium, *Review of Economic Studies* **39** (1972), 355-372.

[37] J.-M. Grandmont and Y. Younès, On the efficiency of monetary equilibrium, *Review of Economic Studies* **40** (1973), 149-165.

[38] F. H. Hahn, On some problems in proving the existence of an equilibrium in a monetary economy, in F. H. Hahn and F. P. R. Brechling (eds), *The Theory of Interest Rates*, Macmillan (1965), 126-135.

[39] T. Kehoe and D. Levine, Regularity in overlapping generations exchange economies, *Journal of Mathematical Economics* **13** (1984), 69-93.

[40] N. Kyotaki and R. Wright, On money as a medium of exchange, *Journal of Political Economy* **97** (1989), 927-954.

[41] A. Lerner, Money as a creature of the state, *American Economic Review* **37** (1947), 312-317.

[42] R. E. Lucas, Expectations and the neutrality of money, *Journal of Economic Theory* **4** (1972), 103-124.

[43] R. E. Lucas, Equilibrium in a pure currency economy, *Economic Inquiry* **xx**(1980), xxx-xxx.

[44] R. E. Lucas and N. L. Stokey, Money and rates of interest in a cash - in - advance economy, *Econometrica* **55** (1987), 491-513.

[45] M. J. P. Magill and M. Quinzii, Real effects of money with nominal assets, *Journal of Mathematical Economics* **21** (1992), 301-342.

[46] E. Malinvaud and Y. Younès, Une nouvelle formulation générale pour l'étude des fondements microéconomiques de la macro - économie, *Cahiers du Séminaire d'Economérie,* **xx** (1977), xxx-xxx.

[47] L. McKenzie, Equilibrium in Graham's model of world trade and other competitive systems, *Econometrica* **22** (1954), 147-161.

[48] F. Modigliani and M. Miller, The cost of capital, corporation finance and the theory of investment, *American Economic Review* **48** (1958), 261-297.

[49] T. Nakajima and H. Polemarchakis, Financial policy and the price level, mimeo (2001).

[50] J. M. Ostroy and R. M. Starr, Money and the decentralization of exchange, *Econometrica* **42** (1974), 1093-1113.

[51] H. M. Polemarchakis, Incomplete markets, exchange rates and indeterminacy, *Journal of Economic Theory* **46** (1988), xxx-xxx.

[52] H. M. Polemarchakis and G. Seccia, A role for monetary policy when prices reveal information: an example, *Journal of Economic Theory* **xx** (2001), xxx -xxx.

[53] H. M. Polemarchakis and P. Siconolfi, Asset markets and the information revealed by prices, *Economic Theory* **3** (1993), 645-661.

[54] H. M. Polemarchakis and P. Siconolfi, Prices, asset markets and indeterminacy, *Journal of Economic Theory* **82** (1998), 46-64.

[55] W. Poole, Optimal choice of monetary policy instruments in a simple stochastic macro model, *Quarterly Journal of Economics* **84** (1970), 197-216.

[56] R. Radner, Existence of equilibrium of plans, prices and price expectations, *Econometrica* **40** (1972), 289-303.

[57] R. Radner, Rational expectations equilibria: generic existence and the information revealed by prices, *Econometrica* **47** (1979), 665-678.

[58] C. Rochon, Liquidity constraints and cycles, mimeo, (1999).

[59] P. Samuelson, An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy* **66** (1958), 467-482.

[60] T. Sargent and N. Wallace, Rational expectations, the optimal monetary instrument and the optimal money supply rule, *Journal of Political Economy* **83** (1975), 241-254.

[61] J. Tobin, A dynamic aggregative model, *Journal of Political Economy* **63** (1955), 103-115.

[62] J. Tobin, The interest - elasticity of transactions demand for cash, *Review of Economics and Statistics* **38** (1956), 241-247.

[63] J. Tobin, Liquidity preference as behaviour towards risk, *Review of Economic Studies* **25** (1958), 65-86.

[64] J. Tobin, *Asset Accumulation and Economic Activity,* Basil Blackwell, (1987).

[65] L. Weiss, The role of monetary policy with rational expectations, *Journal of Political Economy* **88** (1980), 221-233.

[66] C. Wilson, Equilibrium in dynamic models with an infinity of agents, *Journal of Economic Theory* **24** (1981), 95-111.

[67] M. Woodford, Imperfect financial intermediation and complex dynamics, in W. A. Barnett, J. Geweke and K. Shell (eds.), *Economic Complexity,* Cambridge University Press, (1989), xxx-xxx.

[68] M. Woodford, Monetary policy and price level determinacy in a cash-in-advance economy, *Economic Theory* **4** (1994), 345-380.