Money and prices under uncertainty [1]

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Abstract

Monetary and fiscal policy do not determine the stochastic path of prices: in the absence of financial policy, there remains indeterminacy indexed by an arbitrary probability measure over the set of states of the world.

With an interest rate policy, and only if the asset market is complete, indeterminacy is nominal: it affects prices, but not the allocation of resources at equilibrium; with a money-supply policy, the indeterminacy is real.

Portfolio policy sets the portfolio of assets that the monetary authority employs in open market operations; it determines the equilibrium if the policy is non-Ricardian, but not otherwise; financial rate policy, which sets the rates of return of elementary securities, the contingent price of revenue at each date-event determines equilibrium allocations and prices.

long-lived nominally riskless assets can substitute for assets with contingent payoffs; financial policy, then, sets the maturity structure of debt or the yield curve.

Key words: monetary policy; fiscal policy; financial policy, uncertainty; indeterminacy.

JEL classification numbers: D50; D 52; E40; E50.

1 Introduction

Does monetary policy determine prices under uncertainty; and, if not, are there real effects of indeterminacy?

The appropriate framework is that of general competitive equilibrium. The conundrum of Hahn (1965), according to which there is an incompatibility between a general equilibrium specification and an operative money market can be overcome. In Dubey and Geanakoplos (1992) or Drèze and Polemarchakis (2000), the introduction of a monetary authority or central bank that issues balances in exchange for assets allows for a well specified general equilibrium model of a monetary economy; this is the case even if the horizon is finite.

There are different modes of conduct of monetary policy: interest rate policy sets the short term interest rate of and accommodates the demand for balances; money supply policy sets the supply of balances and the rate of interest adjusts.

In addition to monetary policy, the monetary - fiscal authority effects lump sum transfers to individuals — fiscal policy; and it trades in the market for assets — financial policy.

In the absence of financial policy, the monetary-fiscal authority accommodates the demand for assets, while the prices or rates of return of assets are free to adjust.

Policy is Ricardian if, as prices and rates of interest vary, it satisfies the budget constraint of the monetary - fiscal authority at all date-events; in particular, at terminal date-events or at infinity; it is non-Ricardian if it need not satisfy a budget constraint at terminal date-events or at infinity.

With a non-Ricardian policy, public debt need not vanish at the horizon; it does with a Ricardian policy.

In Bloise, Drèze and Polemarchakis (2000 a, b), seignorage is distributed contemporaneously as dividend to the private sector and policy is à fortiori Ricardian. In Dubey and Geanakoplos (2000), the monetary authority transfers seignorage to the private sector indirectly by honoring outside balances, and policy is non-Ricardian. In Woodford (1994), the transfer of seignorage is effected through fiscal transfers, and policy can be Ricardian or not.

In the absence of financial policy, monetary policy does not determine the stochastic path of prices and the allocation of resources. There remains indeterminacy indexed by an arbitrary probability measure over the set of states of the world.

With an interest rate policy, and if the asset market is complete, indeterminacy is nominal: it affects prices, but not the allocation of resources at equilibrium.

With a money-supply policy, the indeterminacy is real.

Under uncertainty, indeterminacy goes well beyond the indeterminacy of the overall price level in Sargent and Wallace (1975)

Financial rate policy, most strongly, sets the rates of return of elementary securities, the contingent price of revenue at each date-event; as such, it determines equilibrium allocations and prices. Alternatively, financial supply policy sets the portfolio of assets that the monetary authority employs in open market operations; it determines the equilibrium if the policy is non-Ricardian, but not otherwise.

Concerning financial policy, it is important that long lived nominally riskless assets or the yield curve can be substitute for by rates of return or supplies of assets with state-contingent payoffs.

In Lucas and Stokey (1987), determinacy obtains through a strong stationarity requirement: the growth of nominal balances varies only with the contemporaneous realization of shocks. Similarly, in Woodford (1994), in the absence of extrinsic uncertainty, and with restrictions on fundamentals, a constant supply of nominal balances guarantees determinacy. If, alternatively, the growth of the supply of nominal balances is allowed to follow a Markov process that parallels the process that governs the realization of shocks, indeterminacy persists.

2 Complete Markets

The economy extends over an infinite horizon, under uncertainty. There is a representative individual. A cash-in-advance constraint is operative, and the asset market is sequentially complete.

Dates are $t = 0, 1, 2, \ldots$.

A shock, $s_t \in \mathcal{S} = \{1, \ldots, S\}$, realizes at each date. A date-event, the history of shocks up through a date, is $\sigma_t = (s_0, s_1, \ldots, s_t)$; the event at the initial date is $s_0 = 0$. The distribution of shocks follows a Markov process with transition probabilities that are positive: $f(s'|s) > 0$.

The private sector consists of a representative individual.

Preferences are described by the intertemporal utility function

$$\mathrm{E}_0 \left\{ \sum_t \beta^t u\big[c(\sigma_t), l(\sigma_t)\big] \right\},$$

where $c(\sigma_t)$ is consumption and $l(\sigma_t)$ is leisure at the date-event σ_t.

The cardinal utility index, u, satisfies standard conditions of continuity, monotonicity, concavity and boundary behavior; the discount factor is $0 < \beta < 1$.

The endowment of leisure is positive: $y(\sigma_t) > 0$, and it can be transformed costlessly into consumption.

The real wage at each date-event is 1; the price level is $p(\sigma_t)$, and the rate of inflation is $p(\sigma_{t+1})/p(\sigma_t)$.

Money balances serve as medium of exchange for consumption.

Elementary securities effect transfers of revenue across date-events; the date-0 price of a claim that pays off a unit of revenue if σ_t occurs and only then is $\pi_0(\sigma_t)$, with $\pi_0(0) = 1$.

The nominal interest rate is $r(\sigma_t)$; it restricts elementary security prices through the no-arbitrage condition

$$\frac{1}{1 + r(\sigma_t)} = \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1}).$$

An arbitrary probability measure, μ, over the set, Σ_∞, of realizations of uncertainty, in conjunction with interest rates, determines the path of elementary security prices; if $\mu(\sigma_{t+1}|\sigma_t)$ is the probability of σ_{t+1} conditional on σ_t, then

$$\frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} = \frac{1}{1+r(\sigma_t)}\mu(\sigma_{t+1}|\sigma_t).$$

The budget and liquidity constraints for the individual are

$$m(\sigma_t) + \tfrac{1}{\pi_0(\sigma_t)} \textstyle\sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1}) b(\sigma_{t+1}) \leq w(\sigma_t) + p(\sigma_t)h(\sigma_t),$$

$$p(\sigma_t)(y(\sigma_t) - l(\sigma_t)) \leq m(\sigma_t),$$

$$w(\sigma_{t+1}) = m(\sigma_t) + b(\sigma_{t+1}) + p(\sigma_t)\big\{y(\sigma_t) - (c(\sigma_t) + l(\sigma_t))\big\},$$

where $m(\sigma_t)$ are money balances the individual carries over, $b(\sigma_{t+1})$ are holdings of the nominal claim that pays off one unit of revenue if σ_{t+1} occurs and only then, $w(\sigma_t)$ is the beginning-of-period wealth of the individual, and $h(\sigma_t)$ is an indexed fiscal transfer; at the initial date,

$$w(0) = \overline{m} + b(0) \geq 0,$$

with $\overline{m}$ initial holdings of balances or "outside money," and $b(0)$ initial holdings of public debt by the private sector; the budget constraint writes, alternatively, as

$$p(\sigma_t)(c(\sigma_t) + l(\sigma_t)) + \tfrac{r(\sigma_t)}{1+r(\sigma_t)} m(\sigma_t) + \tfrac{1}{\pi_0(\sigma_t)} \textstyle\sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1}) w(\sigma_{t+1})$$

$$\leq w(\sigma_t) + p(\sigma_t)y(\sigma_t) + p(\sigma_t)h(\sigma_t).$$

If

$$0 \leq \sum_t \sum_{\sigma_t} \pi_0(\sigma_t)p(\sigma_t)\left\{\frac{1}{1+r(\sigma_t)}y(\sigma_t) + h(\sigma_t)\right\} < \infty,$$

the borrowing constraints

$$w(\sigma_{t+1}) \geq$$

$$-\tfrac{1}{\pi_0(\sigma_{t+1})} \textstyle\sum_{j\geq 1} \sum_{\sigma_{t+j}|\sigma_{t+1}} \pi_0(\sigma_{t+j})p(\sigma_{t+j}) \left\{\tfrac{1}{1+r(\sigma_{t+j})}y(\sigma_{t+j}) + h(\sigma_{t+j})\right\}$$

eliminate Ponzi schemes.

The sequence of budget and cash-in-advance constraints, together with the borrowing constraints, reduce to the intertemporal budget constraint

$$\textstyle\sum_t \sum_{\sigma_t} \pi_0(\sigma_t)p(\sigma_t)\left\{c(\sigma_t) + \tfrac{1}{1+r(\sigma_t)}l(\sigma_t)\right\} \leq$$

$$w_0 + \textstyle\sum_t \sum_{\sigma_t} \pi_0(\sigma_t)p(\sigma_t)\left\{\tfrac{1}{1+r(\sigma_t)}y(\sigma_t) + h(\sigma_t)\right\}.$$

Under standard assumptions, the solution to the optimization problem of the representative individual is characterized by the first-order conditions

$$\frac{u_1\left[c(\sigma_t),l(\sigma_t)\right]}{u_2\left[c(\sigma_t),l(\sigma_t)\right]} = 1 + r(\sigma_t),$$

$$\frac{\beta u_1\left[c(\sigma_{t+1}),l(\sigma_{t+1})\right] f(s_{t+1}|s_t)}{u_1\left[c(\sigma_t),l(\sigma_t)\right]} = \frac{\pi_0(\sigma_{t+1})p(\sigma_{t+1})}{\pi_0(\sigma_t)p(\sigma_t)},$$

and the intertemporal budget constraint satisfied with equality, which implies that the transversality condition,

$$\lim_{j\to\infty} \sum_{\sigma_{t+j}|\sigma_t} \pi_0(\sigma_{t+j})w(\sigma_{t+j}) = 0,$$

holds at all date-events

Of the first-order conditions, the first refers to the optimization of the individual between the cash and the credit commodity at a date-event, while the second refers to intertemporal optimization.

2.1 Policy

A public authority conducts monetary, fiscal and financial policy.

Monetary policy sets nominal interest rates, $r(\sigma_t)$, and it accommodates the demand for balances, $M(\sigma_t)$ — this is interest rate policy; alternatively, it sets the supply of balances — this is money supply policy; in addition, it trades in assets, $B^m(\sigma_t)$, and transfers dividends, $D(\sigma_t)$, subject to the budget constraints

$$M(\sigma_t) + \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})B^m(\sigma_{t+1}) = M(\sigma_{t-1}) + B^m(\sigma_t) + D(\sigma_t).$$

Fiscal policy raises taxes or distributes transfers, $H(\sigma_t)$; in addition, it trades in assets, $B^f(\sigma_t)$, subject to the budget constraints

$$D(\sigma_t) + \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})B^f(\sigma_{t+1}) = B^f(\sigma_t) + p(\sigma_t)H(\sigma_t).$$

The unified budget constraints for the public sector are

$$M(\sigma_t) - M(\sigma_{t-1}) + \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})B(\sigma_{t+1}) = B(\sigma_t) + p(\sigma_t)H(\sigma_t),$$

where

$$B(\sigma_t) \equiv B^m(\sigma_t) + B^f(\sigma_t)$$

are asset holdings; equivalently,

$$\frac{r(\sigma_t)}{1+r(\sigma_t)}M(\sigma_t) + \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})W(\sigma_{t+1}) = W(\sigma_t) + p(\sigma_t)H(\sigma_t),$$

or

$$W(\sigma_t) = \sum_{j=0}^{\infty} \sum_{\sigma_{t+j}|\sigma_t} \frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)} \left\{ \frac{r(\sigma_{t+j})}{1+r(\sigma_{t+j})} \frac{M(\sigma_{t+j})}{p(\sigma_{t+j})} - H(\sigma_{t+j}) \right\}$$

$$+ \lim_{j\to\infty} \sum_{\sigma_{t+j}|\sigma_t} \frac{\pi_0(\sigma_{t+j})}{\pi_0(\sigma_t)} W(\sigma_{t+j}),$$

where the first term is summable for the optimization of the private sector to be well defined, and

$$W(\sigma_t) = B(\sigma_t) + M(\sigma_{t-1})$$

are the total liabilities of the public sector at each date-event; the initial claims of the private sector against the public sector,

$$W(0) = \overline{M} + B(0),$$

consist of "outside money" and outstanding government debt.

Financial policy sets the portfolio of the public sector, $W(\sigma_t)$ — this is strong portfolio policy; weak portfolio policy sets $\overline{W}(\sigma_t)$, the composition of the portfolio of the public sector, while $d(\sigma_{t-1})$ determine the scale of the portfolio, $W(\sigma_t) = d(\sigma_{t-1})\overline{W}(\sigma_t)$; alternatively, financial policy sets the rates of return of elementary securities, $\pi_0(\sigma_t)$ — this is financial rate policy; in the absence of financial policy, the portfolio of the public sector is not specified and can accommodate the demand of the private sector, while the rates of return on assets are free to adjust.

As in Woodford (1994), policy is Ricardian if, as commodity and asset prices or rates of interest vary, fiscal transfers and trades in assets are set to satisfy the transversality condition,

$$\lim_{j\to\infty} \sum_{\sigma_{t+j}|\sigma_t} \pi_0(\sigma_{t+j})W(\sigma_{t+j}) = 0,$$

at all date-events; otherwise, policy is non-Ricardian.

With weak portfolio policy, the budget constraints takes the form

$$\frac{r(\sigma_t)}{1+r(\sigma_t)} M(\sigma_t) + \frac{1}{\pi_0(\sigma_t)} \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})d(\sigma_t)\overline{W}(\sigma_{t+1}) =$$

$$d(\sigma_{t-1})\overline{W}(\sigma_t) + p(\sigma_t)H(\sigma_t).$$

A particular non-Ricardian policy sets arbitrarily fiscal transfers, $\{H(\sigma_t)\}$ and the composition of the portfolio $\{\overline{W}(\sigma_t)\}$, while the scale of the portfolio, $\{d(\sigma_t)\}$ is set to satisfy the budget constraint at each date-event; evidently, this does not guarantee that the transversality condition is met.

In Dubey and Geanakoplos (2000), there are no fiscal transfers, $H(\sigma_t) = 0$, and policy is à fortiori non-Ricardian; financial policy sets the portfolio of the public sector to consist of riskless bonds; the initial claims of the private sector consist of "outside money," but this is not essential.

A particular Ricardian policy, in Bloise, Drèze and Polemarchakis (2001 b), sets $H(\sigma_t) = (r(\sigma_t)/(1 + r(\sigma_t)))(M(\sigma_t)/p(\sigma_t))$ and $W(\sigma_t) = 0$.

2.2 Equilibria

Market-clearing in the markets for consumption requires that

$$y(\sigma_t) = c(\sigma_t) + l(\sigma_t).$$

In the money market, equilibrium requires that

$$M(\sigma_t) = m(\sigma_t) = p(\sigma_t)(y(\sigma_t) - l(\sigma_t)),$$

and in the asset market that

$$W(\sigma_t) = d(\sigma_t)\overline{W}(\sigma_t) = w(\sigma_t);$$

evidently,

$$H(\sigma_t) = h(\sigma_t),$$

by definition.

Proposition 1 *If policy is Ricardian, given interest rate policy, $\{r(\sigma_t)\}$, strong portfolio policy, $\{W(\sigma_t)\}$, a probability measure, μ, over the set Σ_∞ of realizations of uncertainty, and $p(0)$, the initial price level, a competitive equilibrium exists and is unique.*

Competitive equilibrium allocations and rates of inflation associated with different financial policies, $\{W(\sigma_t)\}$, coincide.

Different probability measures, μ, are associated with different rates of inflation, but not different allocations of resources.

Proof Given interest rates $\{r(\sigma_t)\}$, the first-order condition

$$\frac{u_1\big[c(\sigma_t), y(\sigma_t) - c(\sigma_t)\big]}{u_2\big[c(\sigma_t), y(\sigma_t) - c(\sigma_t)\big]} = 1 + r(\sigma_t)$$

determines the allocation of resources at each date-event: it suffices to assume that the marginal rate of substitution is monotonically decreasing in c and to restrict the interest factor, $(1+r(\sigma_t))$ to the interval $(1, \sup_{c\to 0} u_1\big[c(\sigma_t), y(\sigma_t) - c(\sigma_t)\big]/u_2\big[c(\sigma_t), y(\sigma_t) - c(\sigma_t)\big])$; the relative prices of consumption across date-events is then determined by the intertemporal first-order condition

$$\frac{\beta u_1\big[c(\sigma_{t+1}), l(\sigma_{t+1})\big] f(s_{t+1}|s_t)}{u_1\big[c(\sigma_t), l(\sigma_t)\big]} = \frac{\pi_0(\sigma_{t+1})p(\sigma_{t+1})}{\pi_0(\sigma_t)p(\sigma_t)}.$$

This is the extent to which, without further restrictions, interest rate policy determines the path of prices and consumption allocations at equilibrium. Importantly, however, interest rate policy does not determine $\pi(\sigma_t)$ and $p(\sigma_t)$ separately.

The initial price level, $p(0)$, otherwise indeterminate, is set exogenously.

The arbitrary probability measure μ over the set Σ_∞ of realizations of uncertainty, in conjunction with interest rates, determines elementary security prices,

$\pi(\sigma_{t+1}$, at each date-event; if $\mu(\sigma_{t+1}|\sigma_t)$ is the probability of σ_{t+1} conditional on σ_t, then

$$\frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} = \frac{1}{1+r(\sigma_t)}\mu(\sigma_{t+1}|\sigma_t).$$

The rates of return on elementary securities, in conjunction with the first order conditions, determine the price level at each date-event.

Different portfolio policies are offset by fiscal transfers determined as residuals and associated adjustments of the portfolio holdings of the private sector, and do not affect the equilibrium otherwise; this is an instance of Modigliani and Miller (1958). □

It is important that, even with a representative consumer, the measure μ need not coincide with the objective distribution induced by the transition probabilities $f(s'|s)$.

Proposition 2 *If policy is non-Ricardian, given interest rate policy, $\{r(\sigma_t)\}$, and a weak portfolio policy, $\{\overline{W}(\sigma_t)\}$, and fiscal policy, $\{H(\sigma_t)\}$, that satisfy a consistency condition, a competitive equilibrium exists and is unique; this includes the initial price level, $p(0)$, since, in particular, the initial claims of the private sector do not vanish.*

Different financial policies, $\{\overline{W}(\sigma_t)\}$, or different fiscal policies, $\{H(\sigma_t)\}$, are associated with different rates of inflation, but not different allocations of resources.

Proof As in the proof of proposition 1, interest rate policy determines the allocation of resources.

At equilibrium, fiscal transfers and trades in assets satisfy the transversality condition,

$$\lim_{j\to\infty} \sum_{\sigma_{t+j}|\sigma_t} \pi_0(\sigma_{t+j})W(\sigma_{t+j}) = 0;$$

this follows from market clearing and the transversality condition of the private sector, and it implies that

$$W(\sigma_t) = \sum_{j=0}^{\infty} \sum_{\sigma_{t+j}|\sigma_t} \frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)} \left\{ \frac{r(\sigma_{t+j})}{1+r(\sigma_{t+j})} \frac{M(\sigma_{t+j})}{p(\sigma_{t+j})} - H(\sigma_{t+j}) \right\}.$$

From the budget constraint

$$\tfrac{r(\sigma_t)}{1+r(\sigma_t)}M(\sigma_t) + \tfrac{1}{\pi_0(\sigma_t)} \textstyle\sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1})d(\sigma_t)\overline{W}(\sigma_{t+1}) =$$

$$d(\sigma_{t-1})\overline{W}(\sigma_t) + p(\sigma_t)H(\sigma_t),$$

at each date-event, it follows that weak portfolio policy, $\{\overline{W}(\sigma_t)\}$, in conjunction with fiscal policy, $\{H(\sigma_t)\}$, determine the scale of the portfolio of the public sector, and, hence, $\{W(\sigma_t)\}$.

If

$$\frac{W(\sigma_t)}{\sum_{j=0}^{\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)p(\sigma_t)}\left\{\frac{r(\sigma_{t+j})}{1+r(\sigma_{t+j})}c(\sigma_{t+j})-H(\sigma_{t+j})\right\}}>0,$$

the consistency condition, then

$$\frac{W(\sigma_t)}{p(\sigma_t)}=\sum_{j=0}^{\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)p(\sigma_t)}\left\{\frac{r(\sigma_{t+j})}{1+r(\sigma_{t+j})}c(\sigma_{t+j})-H(\sigma_{t+j})\right\},$$

determines the price level, $p(\sigma_t)$, at every date-event, since $W(\sigma_t)=M(\sigma_{t-1})+B(\sigma_t)$, is determined prior to σ_t, while $(\pi_0(\sigma_{t+j})p(\sigma_{t+j}))(\pi_0(\sigma_t)p(\sigma_t))$ is determined by the intertemporal optimization of the private sector. In particular, since $W(0)\neq 0$ is given, the initial price level, $p(0)$, is determined.

For the consistency condition, it is sufficient that $W(0)>0$, while $H(\sigma_t)\leq 0$, and $\overline{W}(\sigma_t)>0$; if the condition fails, an equilibrium does not exist. □

If the initial claims of the private sector vanish, $W(0)=0$, an equilibrium exists only exceptionally; if it does, the initial price level, $p(0)$, is indeterminate and can be set exogenously.

Financial policy fixes the composition of the portfolio held by the public authority at every date-event, with its scale determined by the budget constraint. The public authority can thus control the price level at each date-event. In the absence of financial policy, the initial price level remains determinate, if initial holdings do not vanish, but not the path of prices of commodities or elementary securities.

Corollary 1 *If policy is non-Ricardian, in the absence of financial policy, a probability measure, μ, over the set, Σ_∞, of realizations of uncertainty, determines a unique equilibrium.*

Different probability measures, μ, are associated with different rates of inflation, but not different allocations of resources.

If fiscal transfers are set in nominal terms, $\{\tilde{H}(\sigma_t)=p(\sigma_t)H(\sigma_t)\}$, the distinction between Ricardian and non-Ricardian policies remains; as does the role of financial policy. Nevertheless, explicit computations for a general specification of fiscal policy are not possible.

2.3 Maturity structure

Assets are of one-period maturity. Implicit in the prices of short-term assets are prices of long-term assets. Controlling the prices of long-term bonds is an alternative available to the monetary authority for the control of the prices of elementary securities beyond short-term rates.

For simplicity, the shock takes two values, $s \in \{s_1, s_2\}$. If $r_2(\sigma_t)$ is the two-period rate at the date-event σ_t, then

$$\tfrac{1}{1+r(\sigma_t)} = \textstyle\sum_{\sigma_{t+1}|\sigma_t} \tfrac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} = \textstyle\sum_{s_{t+1}} \tfrac{\mu(\sigma_t, s_{t+1})}{1+r(\sigma_t)},$$

$$\left(\tfrac{1}{1+r_2(\sigma_t)}\right)^2 = \textstyle\sum_{\sigma_{t+2}|\sigma_t} \tfrac{\pi_0(\sigma_{t+2})}{\pi_0(\sigma_t)} = \tfrac{1}{1+r(\sigma_t)} \textstyle\sum_{s_{t+1}} \tfrac{\mu(\sigma_t, s_{t+1})}{1+r(\sigma_t, s_{t+1})}.$$

As long as $r(\sigma_{t+1})$ is not independent of the realization of the shock s_{t+1}, setting the two-period rate, $r_2(\sigma_t)$, in addition to the one-period rate, $r(\sigma_t)$, determines unambiguously the price level at every date-event at $t+1$.

If the cardinality of the set of realizations of the shock is larger than two, setting multi-period long-term rates suffices to determine the distribution of the price level, subject to the non-degeneracy condition on short-term rates.

Continuing with the special case of two possible realizations for the shock,

$$\frac{1+r(\sigma_t)}{\big[1+r_2(\sigma_t)\big]^2} = \frac{\mu(\sigma_t, s_1)}{1+r(\sigma_t, s_1)} + \frac{1-\mu(\sigma_t, s_1)}{1+r(\sigma_t, s_2)},$$

and

$$\mu(\sigma_t, s_1) = \frac{[1+r(\sigma_t, s_1)][1+r(\sigma_t, s_2)]}{r(\sigma_t, s_2) - r(\sigma_t, s_1)} \left\{ \frac{1+r(\sigma_t)}{\big[1+r_2(\sigma_t)\big]^2} - \frac{1}{1+r(\sigma_t, s_2)} \right\}$$

If $r(\sigma_t, s_1) > r(\sigma_t, s_2)$, then an increase in the two-period rate, $\Delta r_2(\sigma_t) > 0$, leads to a higher level of $\mu(\sigma_t, s_1)$ and as a consequence, to a lower price level or rate of inflation at s_1, $\Delta p(\sigma_t, s_1) < 0$, but a higher level at s_2, $\Delta p(\sigma_t, s_2) > 0$. It follows that the two-period rate, $r_2(\sigma_t)$, is inversely related to the difference in inflation rates one-period ahead: $\Delta r_2(\sigma_t) > 0$ implies $\Delta\big[p(\sigma_t, \sigma_1)/p(\sigma_t) - p(\sigma_t, \sigma_2)/p(\sigma_t)\big] < 0$.

A reference point for the term structure is given by the level associated with the expectations theory, $r_2^e(\sigma_t)$, defined by

$$\left(\frac{1}{1+r_2^e(\sigma_t)}\right)^2 \equiv \frac{1}{1+r(\sigma_t)} \sum_{s_{t+1}} \frac{1}{1+r(\sigma_t, s_{t+1})} f(s_{t+1}|s_t).$$

If resources do not vary with the realization of the shock, $y(s_1) = y(s_2)$, then the expectations theory implies that

$$\tfrac{p(\sigma_t, s_1)}{p(\sigma_t)} - \tfrac{p(\sigma_t, s_2)}{p(\sigma_t)} =$$

$$\tfrac{\beta u_1\left[c(\sigma_t, s_1), l(\sigma_t, s_1)\right]}{u_1\left[c(\sigma_t), l(\sigma_t)\right]} - \tfrac{\beta u_1\left[c(\sigma_t, s_2), l(\sigma_t, s_2)\right]}{u_1\left[c(\sigma_t), l(\sigma_t)\right]} > 0.$$

Higher interest rates are associated with higher rates of inflation. The two-period rate that induces a stable inflation rate, invariant across realizations of the shock, exceeds the rate of the expectations theory.

2.4 Controlling the money supply

Alternatively, monetary policy controls the money supply at every date-event, $\{M(\sigma_t)\}$.

Proposition 3 *If policy is Ricardian, given money supply policy,* $\{M(\sigma_t)\}$, *strong portfolio policy* $\{W(\sigma_t)\}$, *a probability measure,* μ *over the set* Σ_∞ *of realizations of uncertainty, and* $p(0)$, *the initial price level, a competitive equilibrium exists and is unique.*

Competitive equilibrium allocations and rates of inflation associated with different financial policies, $\{W(\sigma_t)\}$, *coincide.*

Different initial price levels, $p(0)$, *or different probability measures,* μ, *are associated with different rates of inflation, as well as different allocations of resources.*

Proof At the initial period, $c(0)$ and $r(0)$ are determined by

$$c(0) = \min\{\frac{M(0)}{p(0)}, c^*(0)\} \qquad 1 + r(0) = \frac{u_1\big[c(0), y(0) - c(0)\big]}{u_2\big[c(0), y(0) - c(0)\big]},$$

where

$$c_1^*(\sigma_t) = \arg\max u(c, y(\sigma_t) - c));$$

at subsequent date-events, $c(\sigma_{t+1})$, $p(\sigma_{t+1})$ and $r(\sigma_{t+1})$ are determined by

$$c(\sigma_{t+1}) = \min\{\tfrac{M(\sigma_{t+1})}{p(\sigma_{t+1})}, , c^*(\sigma_{t+1})\}$$

$$p(\sigma_{t+1}) = p(\sigma_t)\tfrac{1+r(\sigma_t)}{\mu(\sigma_{t+1}|\sigma_t)}\tfrac{\beta u_1\big[c(\sigma_{t+1}), y(\sigma_{t+1}) - c(\sigma_{t+1})\big] f(s_{t+1}|s_t)}{u_1\big[c(\sigma_t), y(\sigma_t) - c(\sigma_t)\big]},$$

$$1 + r(\sigma_{t+1}) = \tfrac{u_1\big[c(\sigma_{t+1}), y(\sigma_{t+1}) - c(\sigma_{t+1})\big]}{u_2\big[c(\sigma_{t+1}), y(\sigma_{t+1}) - c(\sigma_{t+1})\big]}.$$

Standard conditions on the utility index $u(c, l)$ guarantee that a solution to the system of equations at each date-event exists and is unique, and, as a consequence, the allocation of resources at each date-event is well defined; in particular, it suffices that

$$\lim_{c \to 0} cu_1(c, y(\sigma_t) - c) = 0,$$

while the function $cu_1(c, y(\sigma_t) - c)$ is monotonically increasing in the interval $(0, c^*(\sigma_t))$.

That variations in the initial price level or the measure μ have real effects follows from the construction. □

If policy is non-Ricardian , the argument parallels the case of interest rate policy with fiscal transfers set in nominal terms.

2.5 Stationarity

The economy is stationary if fundamentals, here endowments, are determined by the shock at each date, $y(s)$.

A consumption allocation is stationary if it is determined by the shock at each date, $(c(s), l(s))$.

At a stationary equilibrium, in addition to consumption, the rate of growth of the money supply is determined by the shock at each date,

$$g(s) = M(\sigma_t)/M(\sigma_{t-1});$$

evidently, the rate of inflation is genuinely Markovian, $\phi(s, s_-) = p(\sigma_t)/p(\sigma_{t-1})$.

At a Markovian equilibrium, consumption is stationary, as is the interest rate, $r(s)$, but the rate of growth of the money supply is allowed to be genuinely Markovian,

$$g(s, s_-) = M(\sigma_t)/M(\sigma_{t-1}).$$

Proposition 4 *At a Markovian equilibrium of a stationary economy, if policy is Ricardian, interest rate policy does not determine the path of inflation; it does, under the additional restriction of a stationary money supply.*

Proof A Markovian equilibrium is characterized by

$$\frac{u_1\big[c(s), y(s)-c(s)\big]}{u_2\big[c(s), y(s)-l(s)\big]} = 1 + r(s),$$

$$g(s|s_-) = \big(1 + r(s_-)\big)\frac{\beta u_1\big[c(s), y(s)-c(s)\big]}{u_1\big[c(s_-), y(s_-)-c(s_-)\big]}\frac{c(s)}{c(s_-)}\frac{f(s|s_-)}{\mu(s|s_-)},$$

where the arbitrary probability distribution μ over sequences of shocks is generated by the Markov transition probabilities $\mu(s|s_-)$.

Interest-rate policy does not determine the path of prices if the money supply process is allowed to be Markovian.

If the money-supply process is required to be stationary,

$$g(s) = \big(1 + r(s_-)\big)\frac{\beta u_1\big[c(s), y(s) - c(s)\big]}{u_1\big[c(s_-), y(s_-) - c(s_-)\big]}\frac{c(s)}{c(s_-)}\frac{f(s|s_-)}{\mu(s|s_-)}.$$

A stationary equilibrium exists and is unique; the argument is as follows: If

$$A = \mathrm{diag}(\ldots, \frac{u_1\big[c(s_-), y(s_-) - c(s_-)\big]c(s_-)}{1 + r(s_-)}, \ldots),$$

and

$$B = \mathrm{diag}(\ldots, \beta u_1\big[c(s), y(s) - c(s)\big]c(s), \ldots),$$

while the matrix of transition probabilities, $F = (f(s|s_-))$ is invertible, then

$$\gamma = (FB)^{-1}A\mathbf{1}_S \quad \text{and} \quad M = A^{-1}FB\Gamma,$$

where

$$\gamma = (\ldots, \frac{1}{g(s)}, \ldots)' \quad \text{and} \quad \Gamma = \text{diag}(\ldots, \frac{1}{g(s)}, \ldots),$$

while $M = (\mu(s|s_-))$ is the matrix of transition probabilities that generate the measure μ over sequences of shocks. $\square$

Corollary 2 *At a Markovian equilibrium of a stationary economy, if policy is non-Ricardian, interest rate policy does not determine the path of inflation in the absence of financial policy; it does, under the additional restriction of a stationary money supply.*

The determinacy of stationary equilibria is independent of the conduct of monetary policy: it obtains whether monetary authority follows an interest-rate policy or a money-supply policy.

If policy is non-Ricardian, with weak portfolio policy, the determinacy result of proposition 2 applies.

2.6 An open economy

There are two countries.

Each country consists of a representative individual and a monetary - fiscal authority.

A superscript $*$ designates the foreign country, when necessary.

There is perfect mobility of currencies and assets. The rate of exchange of the foreign for the domestic currency is $e(\sigma_t)$ and, with a complete set of elementary securities, interest rate parity requires that

$$\frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} = \frac{e(\sigma_t)}{e(\sigma_{t+1})} \frac{\pi_0^*(\sigma_{t+1})}{\pi_0^*(\sigma_t)},$$

and, as a consequence,

$$\frac{1}{1 + r^*(\sigma_t)} = \sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1}) \frac{e(\sigma_{t+1})}{e(\sigma_t)}.$$

Consumption is tradable across countries, but labor is not.Purchasing power parity requires that

$$p(\sigma_t) = e(\sigma_t) p^*(\sigma_t).$$

For the home country the budget and liquidity constraints of the representative individual are

$$p(\sigma_t)(c(\sigma_t) + l(\sigma_t)) + \tfrac{r(\sigma_t)}{1+r(\sigma_t))} m(\sigma_t) + \tfrac{1}{\pi_0(\sigma_t)} \textstyle\sum_{\sigma_{t+1}|\sigma_t} \pi_0(\sigma_{t+1}) w(\sigma_{t+1})$$

$$\leq w(\sigma_t) + p(\sigma_t) y(\sigma_t) + p(\sigma_t) h(\sigma_t),$$

$$p(\sigma_t)(y(\sigma_t) - l(\sigma_t)) \leq m(\sigma_t),$$

while, for the foreign country, they are

$$\frac{p(\sigma_t)}{e(\sigma_t)}(c^*(\sigma_t)+l^*(\sigma_t))+\frac{r^*(\sigma_t)}{1+r^*(\sigma_t))}m^*(\sigma_t)\ +$$

$$\frac{1}{\pi_0(\sigma_t)e(\sigma_t)}\sum_{\sigma_{t+1}|\sigma_t}\pi_0(\sigma_{t+1})e(\sigma_{t+1})w^*(\sigma_{t+1})$$

$$\leq w^*(\sigma_t)+\frac{p(\sigma_t)}{e(\sigma_t)}y^*(\sigma_t)+\frac{p(\sigma_t)}{e(\sigma_t)}h^*(\sigma_t),$$

$$\frac{p(\sigma_t)}{e(\sigma_t)}(y^*(\sigma_t)-l^*(\sigma_t))\leq m^*(\sigma_t).$$

The intertemporal budget constraint for the home country is

$$\sum_t\sum_{\sigma_t}\pi_0(\sigma_t)p(\sigma_t)\left\{c(\sigma_t)+\frac{1}{1+r(\sigma_t)}l(\sigma_t)\right\}\leq$$

$$w_0+\sum_t\sum_{\sigma_t}\pi_0(\sigma_t)p(\sigma_t)\left\{\frac{1}{1+r(\sigma_t)}y(\sigma_t)+h(\sigma_t)\right\},$$

while, for the foreign country, it is

$$\sum_t\sum_{\sigma_t}\pi_0(\sigma_t)p(\sigma_t)\left\{c(\sigma_t)+\frac{1}{1+r^*(\sigma_t)}l(\sigma_t)\right\}\leq$$

$$w_0^*+\sum_t\sum_{\sigma_t}\pi_0(\sigma_t)p(\sigma_t)\left\{\frac{1}{1+r^*(\sigma_t)}y^*(\sigma_t)+h^*(\sigma_t)\right\}.$$

The budget constraints for the public sector in the home country are

$$W(\sigma_t)=$$

$$\sum_{j=0}^{\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)}\left\{\frac{r(\sigma_{t+j})}{1+r(\sigma_{t+j})}\frac{M(\sigma_{t+j})}{p(\sigma_{t+j})}-H(\sigma_{t+j})\right\}$$

$$+\lim_{j\to\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})}{\pi_0(\sigma_t)}W(\sigma_{t+j}),$$

where

$$W(\sigma_t)=B(\sigma_t)+M(\sigma_{t-1}),$$

and

$$W(0)=\overline{M}+B(0);$$

for the foreign country,

$$e(\sigma_t)W^*(\sigma_t)=$$

$$\sum_{j=0}^{\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})p(\sigma_{t+j})}{\pi_0(\sigma_t)}\left\{\frac{r^*(\sigma_{t+j})}{1+r^*(\sigma_{t+j})}\frac{e(\sigma_{t+j})M^*(\sigma_{t+j})}{p(\sigma_{t+j})}-H^*(\sigma_{t+j})\right\}$$

$$+\lim_{j\to\infty}\sum_{\sigma_{t+j}|\sigma_t}\frac{\pi_0(\sigma_{t+j})}{\pi_0(\sigma_t)}e(\sigma_{t+j})W^*(\sigma_{t+j}),$$

where

$$W^*(\sigma_t) = B^*(\sigma_t) + M^*(\sigma_{t-1}),$$

and

$$W^*(0) = \overline{M}^* + B^*(0).$$

Market-clearing in the markets for consumption requires that

$$y(\sigma_t) + y^*(\sigma_t) = c(\sigma_t) + c^*(\sigma_t) + l(\sigma_t) + l^*(\sigma_t).$$

In the money market, equilibrium requires that

$$M(\sigma_t) = m(\sigma_t) = p(\sigma_t)(y(\sigma_t) - l(\sigma_t)),$$

and

$$M^*(\sigma_t) = m^*(\sigma_t) = \frac{p(\sigma_t)}{e(\sigma_t)}(y^*(\sigma_t) - l^*(\sigma_t)).$$

In the asset market, equilibrium requires that

$$B(\sigma_t) + e(\sigma_t)B^*(\sigma_t) = b(\sigma_t) + e(\sigma_t)b^*(\sigma_t).$$

Evidently,

$$W(\sigma_t) + e(\sigma_t)W^*(\sigma_t) = w(\sigma_t) + e(\sigma_t)w^*(\sigma_t),$$

while, by definition,

$$H(\sigma_t) = h(\sigma_t), \quad \text{and} \quad H^*(\sigma_t) = h^*(\sigma_t).$$

At equilibrium, the trade balance of the home country is

$$p(\sigma_t)[c(\sigma_t) - (y(\sigma_t) - l(\sigma_t))],$$

while the capital account is

$$w(\sigma_t) - \sum_{\sigma_{t+1}|\sigma_t} \frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} w(\sigma_{t+1}) - [W(\sigma_t) - \sum_{\sigma_{t+1}|\sigma_t} \frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} W(\sigma_{t+1})]$$

or

$$w(\sigma_t) - \sum_{\sigma_{t+1}|\sigma_t} \frac{\pi_0(\sigma_{t+1})}{\pi_0(\sigma_t)} w(\sigma_{t+1}) - [\frac{r(\sigma_t)}{1 + r(\sigma_t)} M(\sigma_t) - p(\sigma_t)H(\sigma_t)];$$

the current account is balanced at every date - event.

Proposition 5 *If policy is Ricardian, given interest rate policies, $\{r(\sigma_t)\}$ and $\{r^*(\sigma_t)\}$, strong portfolio policies, $\{W(\sigma_t)\}$ and $\{W^*(\sigma_t)\}$, a probability measure, μ, over the set Σ_∞ of realizations of uncertainty, exchange rates $\{e(\sigma_t)\}$, that satisfy the non - arbitrage condition*

$$\sum_{\sigma_{t+1}|\sigma_t} \mu(\sigma_{t+1}|\sigma_t) \frac{e(\sigma_{t+1})}{e(\sigma_t)} = \frac{1 + r(\sigma_t)}{1 + r^*(\sigma_t)},$$

and $p(0)$, the initial price level, a competitive equilibrium exists and is unique.

Different probability measures, μ, or rates of appreciation or depreciation of the exchange rate, $\{(e(\sigma_{t+1}/e(\sigma_t))\}$, are associated with different rates of inflation, but not different allocations of resources; the initial price level, $p(0)$, and initial exchange rate, $e(0)$, may have distributional effects.

If policy is non-Ricardian, given interest rate policies, $\{r(\sigma_t)\}$ and $\{r^*(\sigma_t)\}$, weak portfolio policies, $\{\overline{W}(\sigma_t)\}$ and $\{\overline{W}^*(\sigma_t)\}$,and fiscal policies, $\{H(\sigma_t)\}$ and $\{H^*(\sigma_t)\}$, that satisfy consistency conditions, a competitive equilibrium exists, but is not unique: it determines a probability measure, μ, over the set of realizations of uncertainty, but exchange rates only up to the non - arbitrage condition.

Of interest here is the indeterminacy of the rates of appreciation or depreciation of the exchange rate under uncertainty.

3 Incomplete Markets

Dates are $t = 0, 1$.

States of the world, $s \in \mathcal{S} = \{1, \ldots, S\}$. All uncertainty is resolved at date 1, according to the probability measure which is strictly positive: $f(s) > 0$.

Consumption and labor are exchanged in spot markets at every date-event; a consumption-leisure plan is $x(0) = (c(0), l(0))$, at date 0, and $x(1) = \{c(s), l(s)\}$, at date 1.

Consumption is produced costlessly from labor, and the real wage is 1.

Money serves as a medium of exchange for consumption and numéraire at every date-event. Balances are $m(0)$, at date 0, and $m(1) = \{m(s)\}$, at date 1.

Assets, $a \in \mathcal{A} = \{1, \ldots, A\}$, are traded at date 0 and pay off at date 1. The payoff of an asset, a, at a state of the world, s, is $v_a(s)$; it is denominated in money, the medium of exchange; across events,the payoffs of the asset are $v_a = (\ldots, v_a(s), \ldots)'$, while, across assets, payoffs at a state of the world are $V_s = (\ldots, v_a(s), \ldots)$; the matrix of payoffs of assets is

$$V = (\ldots v_a, \ldots) = (\ldots, V(s), \ldots)'.$$

The matrix V is in general position: every submatrix of dimension A is invertible. In particular, $\dim[V] = A$, which eliminates redundant assets.

The asset market is complete if $A = S$, and incomplete otherwise.

Asset prices are $q = (\ldots q_a, \ldots)$.

A nominally riskless bond is available: $\sum_a v_a(s) = 1$, at every state of the world. The risk-free interest rate is, implicitly, $\sum_a q_a = 1/(1 + r(0))$.

A portfolio of assets is $\theta = (\ldots, \theta_a, \ldots)$.

At each state of the world at date 1, a bond serves to transfer revenue to a terminal date that serves for accounting purposes; the price of the bond is $q(s) = 1/(1 + r(s))$, where $r(1) = \{r(s)\}$ are interest rates.

Holdings of bonds are $b = \{b(s)\}$.

The price of consumption is $p(0)$, at date 0 , and $p(1) = \{p(s)\}$, at date 1.

Agents in the economy are individual consumer-investors, the private sector, and a monetary-fiscal authority, the public sector.

Individuals are $i \in \mathcal{I} = \{1, \ldots, I\}$.

An individual is described by the intertemporal utility function,

$$u^i(c(0), l(0)) + \beta \ \mathrm{E}_0 u^i(c(s), l(s)),$$

the endowment, $y^i(0)$, at date 0, and $y^i(1) = \{y^i(s)\}$, at date 1, and initial claims against the monetary-fiscal authority, balances or outside money, $\overline{m}^i$, and debt $b^i(0)$.

The cardinal utility index, u^i, satisfies standard conditions of continuity, monotonicity, concavity, boundary behavior and, when required, smoothness; the discount factor is $0 < \beta^i < 1$.

The endowment of leisure is positive: $y^i(0) > 0$, and $y^i(s) > 0$, while initial claims against the public sector are non-negative: $w^i(0) = \overline{m}^i + b^i(0) \geq 0$.

An economy is identified by the endowments of individuals. For an open set of economies, a property holds generically if it holds for an open set of full Lebesgue measure.

The economy is heterogeneous: $I > A \geq 1$.

Indexed transfers to the individual from the monetary-fiscal authority are $h^i(0)$, at date 0, and $h^i(s) = \{h^i(s)\}$, at date 1.

Across individuals, the aggregate endowment is $y(0)$, at date 0, and $y(1) = \{y(s)\}$, at date 1, and initial claims against the monetary-fiscal authority are holdings of balances or outside money, $\overline{m}$, and debt, $b(0)$.

Transfers from the monetary-fiscal authority to the private sector are $H(0)$, at date 0 , and $H(s) = \{H(s)\}$, at date 1.

An individual selects his consumption-leisure plan, $x^i(0) = (c^i(0), l^i(0))$, at date 0 and $x^i(1) = \{x^i(s) = (c^i(s), l^i(s))\}$, at date 1, his holdings of balances, $m^i(0)$, and $m^i(1) = \{m^i(s)\}$, and his trades in assets, $\theta^i(0)$, at date 0 and $b^i(1) = \{b^i(s)\}$, at date 1.

The budget constraints for an individual are

$$\tfrac{r(0)}{1+r(0)} m(0) + q\theta \leq p(0)(y^i(0) - c(0) - l(0)) + w^i(0) + p(0)h^i(0),$$

$$\tfrac{r(s)}{1+r(s)} m(s) + q(s)\hat{w}^i(s) \leq p(s)(y^i(s) - c(s) - l(s)) + w^i(s) + p(s)h^i(s),$$

$$0 \leq \hat{w}^i(s)$$

and the liquidity constraints are

$$p(0)(y^i(0) - l^i(0)) \leq m(0),$$

$$p(s)(y^i(s) - l^i(s)) \leq m(s),$$

where $w^i(0) = \overline{m}^i + b^i(0) \geq 0$ are initial claims against the public sector, $w^i(s) = V(s)\theta$ is wealth at date 1 contingent on the state of the world, s, and

$\hat{w}^i(s)$ is terminal wealth of the individual; evidently, at at a solution to the optimization problem, $\hat{w}^i(s) = 0$.

Monetary policy sets interest rates, $r(0)$, at date 0, and $r(1) = \{r(s)\}$, at date 1, and it supplies balances, $M(0)$ and $M(1) = \{M(s)\}$, to accommodate demand — this is interest rate policy; alternatively, the monetary policy sets the supply of balances and interest rates adjust — this is money supply policy.

Fiscal policy effects transfers to the private sector, $H(0)$ and $H(1) = \{H(s)\}$; the distribution of transfers across individuals is described by the distribution scheme $\{\delta^i\}$, with the share of an individual equal to his share in initial claims:

$$\delta^i = \frac{w^i(0)}{\sum_j w^j(0)} \geq 0;$$

this eliminates distributional effects associated with variations in the over-all price level; more elaborate distribution allows the share of an individual to vary with his initial holdings, the prices of commodities or the date-event.

Financial policy sets trades in assets, Θ, at date 0, and $\hat{W}(1) = \{\hat{W}(s)\}$, at date 1 — this is strong portfolio policy; weak portfolio policy sets only the composition of the portfolio, $\overline{\Theta}$, at date 0, but not its scale, d; alternatively it sets the prices of assets; q, at date 0 or, equivalently, their rates of return; this is financial rate policy; evidently, financial rate policy and interest rate policy cannot be set independently: non-arbitrage requires that

$$\frac{1}{1 + r(0)} = \sum_a q_a;$$

in the absence of financial policy, the public sector accommodates the demand for assets.

The budget constraints of the monetary-fiscal authority are

$$W(0) + p(0)H(0) = \tfrac{r(0)}{1+r(0)} M(0) + q\Theta,$$

$$W(s) + p(s)H(s) = \tfrac{r(s)}{1+r(s)} M(s) + q(s)\hat{W}(s),$$

where $W(0) = \overline{M} + B(0)$ are initial liabilities, $W(s) = V(s)\Theta$ are liabilities at date 1 contingent on the state of the world, s, and $\hat{W}(s)$ are terminal liabilities of the public sector.

In the case of weak portfolio policy, which sets the composition of the portfolio of the public sector, the budget constraint at date 0 takes the form

$$W(0) + p(0)H(0) = \frac{r(0)}{1 + r(0)} M(0) + qd\overline{\Theta}.$$

With Ricardian policy, $\hat{W}(s) = 0$, which corresponds to the transversality condition with an infinite-horizon. With non-Ricardian policy, $\hat{W}(s)$ is unrestricted and, as a consequence, the monetary-fiscal authority faces a single constraint, at $t = 0$.

In Bloise, Drèze and Polemarchakis (2000 a), individuals have no initial holdings, $\overline{M} = B(0) = 0$ while aggregate transfers coincide with seignorage at every date-event, $H(0) = (r(0)/(1 + r(0)))M(0)$, and $H(s) = (r(s)/(1 + r(s)))M(s)$; this makes for Ricardian policy, with no trades by the monetary-fiscal authority in the market for assets, $\Theta = 0$.

In Dubey and Geanakoplos (2000), individuals have initial holdings of balances, outside money, $\overline{M} > 0$, but they receive no further transfers, $H(0) = 0$ and $H(s) = 0$; trades in assets are set to satisfy the budget constraint at date 0, and the policy is non-Ricardian since the budget constraint at date 1 fails.

In Woodford (1994), the monetary-fiscal authority effects transfers that need not coincide with revenue from seignorage and policy can be Ricardian or not.

Prices of assets do not allow for arbitrage if and only if

$$q = \pi V,$$

where $\pi = \{\pi(s)\}$, and

$$\pi(s) = \frac{\mu(s)}{1 + r(0)},$$

for an arbitrary, strictly positive probability measure, $\mu = \{\mu(s)\}$.

The sequence of budget and cash-in-advance constraints for an individual reduce to the intertemporal budget constraint

$$p(0)c^i(0) + \tfrac{p(0)}{1+r(0)}l^i(0) + \textstyle\sum_s(\tilde{p}(s)c^i(s) + \tfrac{\tilde{p}(s)}{1+r(s)}l^i(s)) \leq$$

$$\tfrac{p(0)}{1+r(0)}y^i(0) + \textstyle\sum_s(\tfrac{\tilde{p}(s)}{1+r(s)}y^i(s)) + \overline{m}^i + b^i(0) + p(0)h^i(0) + \textstyle\sum_s \tilde{p}(s)h^i(s),$$

and the attainability constraint

$$\left(\begin{array}{c} \vdots \\ \\ \tilde{p}(s)c^i(s) + \tfrac{\tilde{p}(s)}{1+r(s)}l^i(s) - \tfrac{\tilde{p}(s)}{1+r(s)}y^i(s) - \tilde{p}(s)h^i(s) \\ \\ \vdots \end{array}\right) \in [\tilde{V}],$$

where

$$\tilde{V}(s) = \pi(s)V(s), \quad \tilde{p}(s) = \pi(s)p(s), \quad \tilde{m}(s) = \pi(s)m(s);$$

and the cash-in-advance constraints are

$$p(0)(y^i(0) - l(0)) \leq m(0), \quad \tilde{p}(s)(y^i(s) - l(s)) \leq \tilde{m}(s).$$

Evidently, if the asset market is complete, the attainability constraint does not bind.

Commodity markets clear if

$$\sum_i (c^i(0) + l^i(0)) = y(0), \quad \text{and} \quad \sum_i (c^i(s) + l^i(s)) = y(s);$$

similarly, money markets clear if

$$\sum_i m_1^i(0) = M(0), \quad \text{and} \sum_i m_1^i(s) = M(s),$$

and the asset market clears if

$$\sum_i \theta^i = d\overline{\Theta} = \Theta, \quad \text{and} \quad \sum_i \hat{w}^i(s) = \hat{W}(s).$$

At a competitive equilibrium, all markets clear.

Proposition 6 *If monetary-fiscal policy is Ricardian, given interest rate policy, $r(0)$ and $r(1) = \{r(s)\}$, strong portfolio policy Θ, a probability measure, $\mu = \{\mu(s)\}$, over states of the world, and $p(0)$, the initial price level,competitive equilibria exist; generically, competitive equilibria are locally unique.*

Competitive equilibrium allocations and rates of inflation associated with different financial policies, Θ, coincide.

If the asset market is complete, different probability measures, μ, are associated with different distributions of inflation, but not different allocations of resources; if the asset market is incomplete, generically, the associated allocations of resources are distinct.

Proof The domain of discounted prices $(\tilde{p}(0), \dots, \tilde{p}(s), \dots)$ is the simplex of dimension S; discounted balances, $(\tilde{M}(0), \dots, \tilde{M}(s), \dots)$ are restricted to the domain $0 \leq \tilde{M}(0) \leq y(0), \dots, 0 \leq \tilde{M}(s) \leq y(s), \dots$.

Asset prices are

$$q = \sum_s \frac{\mu(s)}{1 + r(0)} V(s);$$

the budget constraint at each date-event then determines fiscal transfers, $H(0), \dots, H(s), \dots$.

As in Cass (1985) or Balsko and Cass (1989), individuals $i = 2, \dots$ express demand for commodities and balances subject to the intertemporal budget constraint

$$\tilde{p}(0)c^i(0) + \tfrac{\tilde{p}(0)}{1+r(0)} l^i(0) + \textstyle\sum_s (\tilde{p}(s)c^i(s) + \tfrac{\tilde{p}(s)}{1+r(s)} l^i(s)) \leq$$

$$\tfrac{\tilde{p}(0)}{1+r(0)} y^i(0) + \textstyle\sum_s (\tfrac{\tilde{p}(s)}{1+r(s)} y^i(s)) + \tfrac{\tilde{p}(0)}{p(0)}(\overline{m}^i + b^i(0)) + \tilde{p}(0)h^i(0) + \textstyle\sum_s \tilde{p}(s)h^i(s),$$

where

$$\tfrac{\tilde{p}(0)}{p(0)}(\overline{m}^i + b^i(0)) + \tilde{p}(0)h^i(0) + \textstyle\sum_s \tilde{p}(s)h^i(s) =$$

$$\delta^i(\tfrac{r(0)}{1+r(0)} \tilde{M}(0) + \textstyle\sum_s \tfrac{r(s)}{1+r(s)} \tilde{M}(s)) \geq 0,$$

as well as the attainability constraint imposed by the asset structure; individual 1 is exempt from the attainabilty constraint.

A standard fixed point argument yields prices of commodities, $(\tilde{p}^*(0), \ldots, \tilde{p}^*(s), \ldots)$ and money supplies, $(\tilde{M}^*(0), \ldots, \tilde{M}^*(s), \ldots)$, such that commodity and money markets clear.

The construction of an equilibrium is completed by setting

$$M^*(0) = \frac{p(0)}{\tilde{p}^*(0)} \tilde{M}^*(0), \quad M^*(s) = \frac{p(0)(1 + r(0))}{\tilde{p}^*(0)\mu(s)} \tilde{M}^*(s),$$

and

$$p^*(s) = \frac{p(0)(1 + r(0))}{\tilde{p}^*(0)\mu(s)} \tilde{p}^*(s).$$

Both the allocation of resources and the path of prices at equilibrium are determined without reference to the financial policy, Θ, which is residual.

Since

$$\frac{p^*(s)}{p(0)} = \frac{1 + r(0)}{\tilde{p}^*(0)\mu(s)} \tilde{p}^*(s),$$

the distribution of the rate of inflation across states of the world varies with different probability measures, μ; the interest rate determines the "average" rate of inflation.

The generic local uniqueness of equilibria for smooth economies follows from an argument as in Debreu (1970).

As in Balasko and Cass (1989) or Geanakoplos and Mas - Colell (1989), different probability measures, μ, are associated with different equilibrium allocations, if the asset market is incomplete. The consumption of the individual exempt from the attainability constraint in the existence argument determines the discounted prices of commodities, $(\tilde{p}^*(0), \ldots, \tilde{p}^*(s), \ldots)$; if the matrix of asset payoffs is in general position, the column span $[\tilde{V}]$ changes as the probability measure μ changes. Generically, the expenditures of individuals span A dimensions, and cannot coincide for different payoff spans. □

Alternative specifications of Ricardian policies do not affect the results.

The indeterminacy associated with the probability measure, μ, corresponds to the $(S-1)$ degrees of indeterminacy of the allocation in Geanakoplos and Mas-Colell (1989), with an incomplete markets and inside assets. In Balasko and Cass (1989) or Cass (1985), the $(S-1)$ degrees of indeterminacy reduces to $(S-A)$, since the prices or yields of assets are set exogenously; importantly, with inside assets and à fortiori Ricardian policy, financial policy does not impact either the allocation of commodities or the path of prices at equilibrium.

Proposition 7 *If monetary-fiscal policy is non-Ricardian, given interest rate policy,* $r(0)$ *and* $r(1) = \{r(s)\}$, *and a weak portfolio policy,* $\overline{\Theta}$, *and fiscal policy,* $H(0)$ *and* $H(1) = \{H(s)\}$, *that satisfy a consistency condition, competitive equilibria exist; generically, competitive equilibria are locally unique, including the initial price level, as long as initial claims of the private sector do not vanish.*

If the asset market is complete, different financial policies, $\overline{\Theta}$ *or alternatively, different fiscal policies,* $H(0)$ *and* $H(1) = \{H(s)\}$, *are associated with* *different rates of inflation, but not different allocations of resources; if the asset market is incomplete, generically, the associated allocations of resources are distinct.*

Proof The domain of discounted prices, $(\tilde{p}(0), \ldots, \tilde{p}(s), \ldots)$, is the simplex of dimension S; discounted balances, $(\tilde{M}(0), \ldots, \tilde{M}(s), \ldots)$, are restricted to the domain $0 \leq \tilde{M}(0) \leq y(0), \ldots, 0 \leq \tilde{M}(s) \leq y(s), \ldots$; the domain of probability measures over the set of states of the world, $(\ldots, \mu(s), \ldots)$, is the simplex of dimension $(S-1)$.

Prices of assets are

$$q = \sum_s \frac{\mu(s)}{1+r(0)} V(s).$$

As in Cass (1985) or Balsko and Cass (1989), individuals $i = 2, \ldots$ express demand for commodities and balances subject to the intertemporal budget constraint

$$\tilde{p}(0)c^i(0) + \tfrac{\tilde{p}(0)}{1+r(0)} l^i(0) + \textstyle\sum_s (\tilde{p}(s)c^i(s) + \tfrac{\tilde{p}(s)}{1+r(s)} l^i(s)) \leq$$

$$\tfrac{\tilde{p}(0)}{1+r(0)} y^i(0) + \textstyle\sum_s (\tfrac{\tilde{p}(s)}{1+r(s)} y^i(s)) + \delta^i (\tfrac{r(0)}{1+r(0)} \tilde{M}(0) + \textstyle\sum_s \tfrac{r(s)}{1+r(s)} \tilde{M}(s)),$$

as well as the attainability constraint imposed by the asset structure; individual 1 is exempt from the attainabilty constraint.

A standard fixed point argument yields prices of commodities, $(\tilde{p}^*(0), \ldots, \tilde{p}^*(s), \ldots)$, money supplies, $(\tilde{M}^*(0), \ldots, \tilde{M}^*(s), \ldots)$ and a measure $(\ldots, \mu^*(s), \ldots)$, such that commodity and money markets clear, while the budget constraint of the public sector is satisfied at all date-events other than the initial date; in particular,

$$\tilde{W}(s) + \tilde{p}^*(s)H(s) = \frac{r(s)}{1+r(s)} \tilde{M}^*(s),$$

where

$$\tilde{W}(s) = \tfrac{\mu(s)}{1+r(0)} \tilde{M}^*(0) + \tilde{V}(s)\tilde{d}\overline{\Theta},$$

$$\tilde{d} = \tfrac{\tilde{W}(0) + \tilde{p}^*(0)H(0) - \tilde{M}^*(0)}{q^*\overline{\Theta}},$$

and

$$\tilde{W}(0) = \frac{r(0)}{1+r(0)} \tilde{M}^*(0) - \tilde{p}^*(0)H(0) + \sum_s \frac{r(s)}{1+r(s)} \tilde{M}^*(s) - \tilde{p}^*(s)H(s).$$

The construction of an equilibrium is completed by setting

$$p^*(0) = \tilde{p}^*(0) \frac{W(0)}{\tilde{W}(0)},$$

and, subsequently,

$$M^*(0) = \frac{p^*(0)}{\tilde{p}^*(0)} \tilde{M}^*(0), \quad M^*(s) = \frac{p^*(0)(1+r(0))}{\tilde{p}^*(0)\mu^*(s)} \tilde{M}^*(s),$$

and

$$p^*(s) = \frac{p^*(0)(1 + r(0))}{\tilde{p}^*(0)\mu^*(s)}\tilde{p}^*(s).$$

If

$$\frac{W(0)}{\frac{r(0)}{1+r(0)}\tilde{M}^*(0) - \tilde{p}^*(0)H(0) + \sum_s \frac{r(s)}{1+r(s)}\tilde{M}^*(s) - \tilde{p}^*(s)H(s)} > 0,$$

and

$$\frac{W(s)}{\frac{r(s)}{1+r(s)}\frac{\tilde{M}^*(s)}{\tilde{p}^*(s)} - H(s)} > 0,$$

the consistency condition, then $p^*(0) > 0$, and $p^*(s) > 0$.

For the consistency condition, it is sufficient that $W(0) > 0$, while $H(0) \leq 0$, $H(s) \leq 0$, and $\overline{W}(s) = V(s)\overline{\Theta} > 0$; if the condition fails, an equilibrium does not exist.

The remainder follows as in the proof of proposition 6. □

In Dubey and Geanakoplos (2000) and Woodford (1994), the monetary-fiscal authority trades only in riskless bonds: $\overline{\Theta} = (\ldots, 1, \ldots)$.

Alternative specifications of Ricardian policies do affect the results: if, at non-terminal date-events, policy sets the prices of assets, q, but not the portfolio, $\overline{B}$. There remain, then, $S - A$ degrees indeterminacy, with real effects if the asset market is incomplete; this corresponds to the specification in Balasko and Cass (1989) or Cass (1985), where the rate of return on assets, as opposed to payoffs, is exogenous.

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