Asset Price Fluctuations in Japan: 1980-2000

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Abstract

A notable feature of the Japanese economy in the last two decades is the large fluctuations in asset prices. We examine whether they can by accounted for by a stochastic growth model with habit persistence and costly capital adjustment. For the real estate price, people's expectations on the trend growth rate in the future plays a crucial role. In particular, our model with adaptive expectations about future productivity growth can reproduce the aggregate land price. However, even with habit persistence and costly capital adjustment, a substantial portion of the stock price fluctuations is left unexplained, and a puzzle remains. Our result suggests that the price of installed capital is close to zero or that people don't take into account the value of capital when trade shares.

Key words: Japanese economy; land prices; stock prices; adaptive expectations; productivity growth
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1 Introduction

A notable feature of the Japanese economy in the last two decades is the large fluctuations in asset prices. The asset price boom occurred in the late 1980s, followed by the collapse in the 1990s (Figures 1-2 below). The aggregate land price relative to output rose by a factor of 1.83 in the late 1980s and fell by the same factor in the 1990s. Stock prices fluctuated even more: TOPIX rose by a factor of 3.5 in the 1980s (again, relative to output), and fell by a similar amount in the 1990s. Many economists argue that the long-lasting output slump after 1991 in Japan has been caused by the collapse in asset prices and the cutback in lending associated with it (Blanchard, 1999, pp. 143). To fully understand this issue, we first need a general-equilibrium model which reproduces the observed amount of fluctuations in asset prices.

The aim of this paper is to make a step toward that direction. We examine whether a version of the neoclassical stochastic growth model can explain the asset price fluctuations in Japan in the last two decades. In particular, we take the productivity process as exogenously given, and see if it can generate the fluctuations in land and equity prices comparable to those in the data. Our paper is thus similar in spirit to Hayashi and Prescott (2002), who show that the output slump in Japan in the 1990s is consistent with the prediction of the neoclassical growth model with exogenous technology. Here, we conduct a similar exercise on asset prices over the period 1980-2000.

The model economy we consider is a one-sector growth model with habit persistence and costly capital adjustment. Habit persistence and adjustment costs of capital are known to help the model predict better the equity premium (Jermann, 1998; Boldrin, Christiano and Fisher, 2001; Christiano and Fisher, 1998). In particular, we obtain a large equity premium with large habit persistence and low elasticity of investment with respect to the price of installed capital. Our numerical results show, however, that adding these features in the model does not help much for our purpose.

For the land price, what matters most is the parameterization of the productivity process. In particular, expectations on the trend growth rate in the future plays a crucial role. We start with the case in which the productivity process follows a random walk with constant drift. With moderate values of habit persistence and investment-demand elasticity, the land-price-output ratio is predicted to be effectively constant throughout the two decades. With very high habit persistence and low investment-demand elasticity, the predicted land-price-output ratio is highest in 1980 and declines over time, which is inconsistent with the data. An intuition is that, given the constant drift in the productivity process, (i) with moderate values of habit persistence and investment-demand elasticity, short-run fluctuations in productivity do not matter for the land-price-output ratio, and it is determined by the trend growth rate, which is constant; (ii) with large habit persistence and low investment-demand elasticity, the fact that the initial state variables (both capital and consumption) are low affects the stochastic discount factor so much that the land-price-output ratio is highest in 1980.

We then consider the case in which the productivity process is a random walk with Markov-switching drift, in which the drift alternates stochastically between high and low values. Thus, the economy switches between the fast-growth and slow-growth regimes stochastically. By changing the average duration of each regime, we examine how much persistence is needed to generate fluctuations in the land price comparable to the actual ones. With the benchmark preference parameters, even if we assume that each regime lasts for one hundred years on average, the model fails to reproduce sufficient fluctuations in the land price.

We finally consider the case in which agents adjust their expectations about the future trend growth rate in an adaptive fashion. Specifically, in each period, the expected trend growth rate in the future is given by an exponential average of the past growth rates. Expectations are assumed to be adaptive in that agents don't take into account the fact that their expectations will be updated in the future. With this assumption, the model predicts both the paths of the land-price-output ratio and the capital-output ratio fairly well.

Given that the adaptive-expectations model can replicate the time series of the land price and capital stock, we examine the model's prediction on the equity price. The result is not satisfactory: a significant amount of the stock-price fluctuations is left unexplained, and a puzzle remains. The failure of the model is due to the behavior of the 'marginal Q' (the price of installed capital). With costly capital adjustment, the marginal Q moves, roughly speaking, in the opposite direction to the capital-output ratio. Since the capital-output ratio is lowest in 1980, our model predicts the price of installed capital is highest in the same year. Somewhat interestingly, if we remove the value of installed capital from the valuation of firms, the predicted path of the equity price becomes very close to the actual one. This might indicate that it is indeed the case that the marginal Q is extremely low in Japan. If so, the fact that the capital-output ratio has been increasing in the last thirty years means that firms' investment decision has been very inefficient. Alternatively, it might be the case that the marginal Q is similar to what the theory suggests, but traders of stocks simply ignore the value of capital. Exploring this issue is left for future research.

Barsky and De Long (1993) use an expectation formula similar to ours in a partial-equilibrium model and argue that it can explain the stock-price data in the U.S. A potentially important role of expectations to explain the asset price fluctuations in Japan has been noticed before. For example, see Ito and Iwaisako (1995). A contribution of our paper is to show, in a general equilibrium model, that the aggregate land price can be accounted for by adaptive expectations, but the stock price is not.

The rest of the paper is organized as follows. In Section 2 we review some evidence. In Section 3 the model is described. Sections 4-6 give the numerical result for the case of constant drift, the case of Markov-switching drift, and the case of adaptive expectations, respectively. Section 7

2 Evidence

2.1 Asset Prices in Japan

In this subsection, we review the fluctuations in land and stock prices in Japan over the period 1980-2000. For the land price, we use the end-of-year value of nationwide land estimated in the Japanese National Income Accounts (NIA). The land-price-output ratio is the ratio of this aggregate price of land to nominal GDP.[1] Figure 1 displays the time series of the land-price-output ratio over the period 1980-2000. The figure also plots the corresponding ratio using the land price index for the six large city areas, estimated by the Japan Real Estate Institute (JREI). Its value for 1980 is normalized to three to make the two series comparable. The price index of JREI may reflect the market value of land better than the estimate of the NIA (Ito, 1992). The two series move in a similar way, in particular, prior to 1987. Real estate prices in Japan had a roughly constant proportion to GDP until 1985, and started grow faster than GDP from 1986.[2] The peak is reached in 1990, followed by a monotonic decline. After 1987, the JREI series show larger fluctuations than the NIA series, indicating significant regional heterogeneity. While it is an important problem, we shall ignore the regional difference in land price, and restrict our attention to the aggregate value in what follows.[3]

Figure 2 displays time series of stock prices over the same period. Three measures of stock prices are used: TOPIX (the stock price index constructed by the Tokyo Stock Exchange), the

[1] The National Income Accounts are adjusted using the procedure described in Hayashi and Prescott (2002).

[2] The JREI and NIA prices show very different paths before 1980. The JREI price index shows, roughly, that the land-price-output ratio declined monotonically from 1963 to 1980 for both the six large city areas and nationwide. But the land price data of NIA implies the opposite: the land-price-output ratio increased from 1963 to 1980. This may reflect the discrepancy between the market prices and the NIA estimates.

[3] JREI provides the price index for nationwide land. However, it is constructed as the simple (not weighted) average of the price indexes of 223 cities, and hence, not appropriate for our purpose.

total market value of the Tokyo Stock Exchange (first section), and the value of shares issued by private non-financial corporations estimated in the NIA. Each variable is divided by nominal GDP, and normalized so that its value in 1980 equals unity. The behaviors of the stock prices and the real estate prices are largely similar over the period in question. Both increased relative to GDP in the late 80s, and decreased in the 90s. However, there are some notable differences. First, the stock prices started to grow earlier than the land prices. The increase in the stock prices started as early as in 1983. Second, the stock prices collapsed earlier: all three measures of stock prices show a sharp decline in 1990, while the real estate prices started to decline in 1991.[4]

2.2 Macro Variables

The main purpose of this paper is to examine how much of those fluctuations in asset prices can be explained by a simple stochastic growth model. The key factor in the model will be stochastic productivity growth.

For each year t, the productivity index, A_t, is defined in the standard way. The production function is

$$Y_t = A_t^{1-\alpha_k} K_t^{\alpha_k} L H_t^{1-\alpha},$$

where $\alpha = \alpha_k + \alpha_l$, Y_t is aggregate output, K_t is aggregate stock of capital, L is aggregate stock of land, and H_t is human-capital-augmented labor supply.[5] Following Hall and Jones (1999) and Bils and Klenow (2000), we set

$$H(t) = e^{\phi(E_t)} N_t,$$

where E_t is the average number of years of schooling and N_t is hours worked. The function $\phi(E)$ reflects the efficiency of a unit of labor with E years of schooling relative to one with no schooling ($\phi(0) = 0$). We follow the convention to assume that $\phi(E)$ is piecewise linear and use the coefficients reported in Psacharopoulos and Patrinos (2002).

Figure 3 displays time series of the log productivity index, $\ln A_t$, over the period 1970-2000. The factor shares used are:[6] $\alpha_k = 0.269$ and $\alpha_l = 0.1$. According to this figure, we could divide

[4]It is noteworthy that in the Tokyo metropolitan area, the peak of the land price (relative to GDP) was reached in 1987-1988, and the decline started in 1988.

[5]Capital stock is private capital (including inventories). Unlike Hayashi and Prescott (2002), it does not include foreign capital.

[6]See Section 3.7 for how those values are chosen.

those three decades into three periods: two slow-growth periods (1970-1983, 1992-2000) and a fast-growth period (1984-1991). The average growth rates of productivity in the two slow-growth periods are 0.32 percent and 0.11 percent, respectively, and that in the fast-growth period is 3.27 percent. Note the correspondence between the asset price fluctuations in Figures 1-2 and the productivity growth. The stable asset prices in the early 1980s corresponds to the stagnant productivity growth in that period; the rise in asset prices in the middle and late 80s matches the fast productivity growth in that period; the decline in asset prices in the 1990s parallels the stagnant growth in the same period. We shall exploit this fact to explain the asset price fluctuations in the following sections.

Figure 4 plots the capital-output ratio, K_t/Y_t. The behavior of the capital-output-ratio also corresponds to the time path of the productivity index. The capital-output ratio rose in the slog-growth period of the 1980s; it fell in the fast-growth period; and went up again in the slow-growth period in the 1990s. We aim to reproduce this behavior of the capital-output ratio as well.

Figure 5 shows the human-capital-augmented labor supply per working-age (20-69) population. There are two offsetting effects on this variable: a downward trend in hours worked per person, and an upward trend in the average years of education. The second effect dominates in the 1980s, and the first one does in the 1990s. In the model, we abstract from the fluctuations in labor supply, and set it to a constant.

Figure 6 plots the shares of private consumption and private investment in GDP.[7] Over the period 1980-2000, there are a slight upward trend in consumption and a slight downward trend in investment, which suggests the economy in 1980 was below the steady state (or the balanced growth path). As expected from the theory, the share of consumption (investment) moves countercyclically (procyclically).

3 Model Economy

In this section, we describe the model economy, which is a stochastic growth model with habit persistence and costly capital adjustment. Related models have been used by Jermann (1998), Boldrin, Christiano and Fisher (2001), and Christiano and Fisher (1998). Time is discrete and indexed by $t = 0, 1, 2, \ldots$. One period in the model corresponds to a year in the data.

[7] For each variable, the share in GDP is obtained by dividing its nominal value by nominal GDP.

3.1 Households

There is an infinitely-lived, representative household who owns capital and labor. There is no population growth. The household has preference given by

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t, C_{t-1})\right],$$

where $0<\beta<1$, E_0 is the conditional expectation operator, C_t is consumption in period t, and the instantaneous utility function, $U(C_t, C_{t-1})$, takes the form:

$$U(C_t, C_{t-1}) = \frac{1}{1-\sigma}\big[(C_t - bC_{t-1})^{1-\sigma} - 1\big],$$

with $\sigma \geq 0$, and $b \geq 0$. When $b>0$, there is habit persistence.

Let K_t and L_t be the stock of capital and the amount of land owned by the household at the beginning of period t. It rents the capital and land to firms and earn $R_t^k K_t + R_t^l L_t$, where R_t^k and R_t^l are the rental rates of capital and land, respectively. The household also supplies one unit of labor inelastically. The rental income net of depreciation is subject to taxes; let τ be the (constant) tax rate. The lump-sum taxes are given by T_t. The flow budget constraint for the household is

$$\begin{aligned} C_t + P_t^k K_{t+1} + P_t^l L_{t+1} = W_t &+ \big[P_t^l + (1-\tau)R_t^l\big]L_t \\ &+ \big[\tilde{P}_t^k + (1-\tau)(R_t^k - \delta \tilde{P}_t^k)\big]K_t - T_t, \end{aligned} \tag{1}$$

where $\delta \in (0,1)$ is the depreciation rate of capital, P_t^l is the price of land, $\tilde{P}_t^k$ is the beginning-of-period price of capital, and P_t^k is the end-of-period price of capital. Here, $\tilde{P}_t^k$ and P_t^k are different because of costly capital adjustment. Without such costs, $P_t^k = \tilde{P}_t^k = 1$ as in the standard real business cycle model.

Let Λ_t be the multiplier associated with the flow budget constraint. Then, under the natural debt limit (Ljungqvist and Sargent, 2000), the first-order conditions for utility maximization are the flow budget constraint (1),[8]

$$U_1(C_t, C_{t-1}) + \beta E_t\big[U_2(C_{t+1}, C_t)\big] = \Lambda_t, \tag{2}$$

$$P_t^k = E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\Big\{\tilde{P}_{t+1}^k + (1-\tau)(R_{t+1}^k - \delta\tilde{P}_{t+1}^k)\Big\}\right], \tag{3}$$

$$P_t^l = E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\Big\{P_{t+1}^l + (1-\tau)R_{t+1}^l\Big\}\right]. \tag{4}$$

Together with the transversality condition, those first-order conditions determines the optimal contingent plan for the household.

[8]Throughout this paper, $f_i(x)$ denotes the derivative of $f(x)$ with respect to its i-th argument.

3.2 Production of Capital

As in Christiano and Fisher (1998), we introduce capital adjustment costs by assuming that the end-of-period capital is produced by competitive firms using the technology:

$$K_{t+1} = Q(\tilde{K}_t, I_t), \tag{5}$$

where $\tilde{K}_t \equiv (1-\delta)K_t$ is the net-of-depreciation stock of previously installed capital, and I_t is the quantity of new investment goods. The capital-adjustment technology, $Q(\tilde{K}, I)$, has the constant-elasticity form:

$$Q(\tilde{K}, I) = \left[a_1 \tilde{K}^{\frac{\psi-1}{\psi}} + a_2 I^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}},$$

where $\psi \geq 0$ is the elasticity of substitution, and the share parameters, $a_i > 0$, are set to guarantee $Q_1 = Q_2 = 1$ at the non-stochastic steady state (balanced growth path).[9] The conventional evolution of capital, $K_{t+1} = I_t + (1-\delta)K_t$, corresponds to the case of $\psi = \infty$, in which the marginal rate of transformation between C_t and K_{t+1} is constant (unity). When $\psi < \infty$, it is not constant, and, as Jermann (1998) shows, this feature helps explain the equity premium (with habit persistence preferences).

The representative capital-producing firm purchases investment goods, I_t, and previously installed capital, $\tilde{K}_t$, and sells new stock of installed capital, K_{t+1}, to households. Thus, its profit maximization problem is given by

$$P_t^k Q(\tilde{K}_t, I_t) - \tilde{P}_t^k \tilde{K}_t - I_t.$$

The first-order conditions are

$$P_t^k Q_1\big[(1-\delta)K_t, I_t\big] = \tilde{P}_t^k, \tag{6}$$

$$P_t^k Q_2\big[(1-\delta)K_t, I_t\big] = 1. \tag{7}$$

These equations are understood in the context of the q-theory (Hayashi, 1982). For example, when the price of installed capital, P_t^k, rises, investment goes up, because $Q_{22} < 0$.

3.3 Production of Consumption-Investment Goods

Consumption and investment goods are produced by competitive firms with the technology:

$$Y_t = A_t^{1-\alpha_k} K_t^{\alpha_k} L_t^{\alpha_l} H_t^{1-\alpha}, \tag{8}$$

[9] As equations (6)-(7) show, this implies $P^k = \tilde{P}^k = 1$ at the non-stochastic steady state.

where $\alpha = \alpha_k + \alpha_l$, Y_t is output in period t, A_t is the productivity index, L_t is the input of land, and H_t is the labor input. We normalize the aggregate amount of land and labor to unity: $L_t = H_t = 1$, all t.

The market clearing for consumption-investment goods is

$$Y_t = C_t + I_t + G_t, \tag{9}$$

where G_t is government purchases in period t. Profit maximization leads to

$$W_t = (1 - \alpha)Y_t, \tag{10}$$

$$R_t^k = \alpha_k \frac{Y_t}{K_t}, \tag{11}$$

$$R_t^l = \alpha_l Y_t, \tag{12}$$

where we have used the condition that $L_t = H_t = 1$.

3.4 Government

The government consumes a constant fraction, s_g, of output, and has balanced budget in each period:

$$G_t = s_g Y_t = T_t + \tau(R_t^k - \delta \tilde{P}_t^k)K_t + \tau R_t^l.$$

3.5 Productivity Growth

The productivity index, A_t, is a random variable, which is the only stochastic shock in our economy. We assume that (the household believes that) it follows a random walk with drift:

$$\ln A_{t+1} = \ln A_t + z_{t+1},$$

with

$$z_{t+1} = \zeta_{t+1} + \epsilon_{t+1},$$

where ϵ_{t+1} is an i.i.d. random variable with distribution $N(0, \sigma_\epsilon^2)$. The drift term, ζ_{t+1}, may or may not be time varying. We consider three different cases: (i) ζ_t is a constant; (ii) ζ_t follows a Markov switching process; (iii) ζ_t is an exponential average of the past growth rates, z_{t-j}, $j = 1, 2, \ldots$.

3.6 Balanced Growth Path

Before examining the stochastic equilibrium, let us first look at the balanced growth equilibrium when the productivity index grows at a constant rate, $\exp(\zeta)-1$. The balanced growth equilibrium is obtained as the steady state of the economy with adequately transformed variables and with no productivity growth. Let lower-case letters denote those transformed variables:

$$x_t = \frac{X_t}{A_t}, \qquad \text{for } X = Y, C, I, W, R^l, P^l,$$
$$k_{t+1} = \frac{K_{t+1}}{A_t}, \quad \lambda_t = A_t^{\sigma}\Lambda_t, \quad p_t^k = P_t^k, \quad \tilde{p}_t^k = \tilde{P}_t^k, \quad r_t^k = R_t^k.$$

Since $p_t^k = \tilde{p}_t^k = 1$ along the balanced growth path, the Euler equations for the household's problem (3)-(4) imply that

$$\frac{K_t}{Y_t} = \frac{(1-\tau)\alpha_k}{e^{\sigma\zeta}/\beta - 1 + (1-\tau)\delta}, \tag{13}$$
$$\frac{P_t^l}{Y_t} = \frac{(1-\tau)\alpha_l}{e^{(\sigma-1)\zeta}/\beta - 1}. \tag{14}$$

Here, K_t/Y_t is the capital-output ratio based on the replacement cost, corresponding to the data shown in Figure 4. A higher tax rate, τ, reduces both ratios. Note that the real interest rate in the balanced growth equilibrium is $e^{\sigma\zeta}/\beta$, which moves in the same direction as the growth rate, ζ. A higher growth rate, ζ, reduces the capital-output ratio, K/Y, because it raises the interest rate. Faster growth has two opposing effects on the land-price-output ratio: first, it increases the growth rate of the rental price of land, R_t^l, which tends to increase the land price relative to output; but it also increases the interest rate, which tends to decrease the land price. The overall effect depends on σ, which determines the elasticity of the steady-state interest rate with respect to the productivity growth rate: when $\sigma < 1$, faster growth increases the land-price-output ratio, and vice versa. When $\sigma = 1$ (the logarithmic case), the steady-state land-price-output ratio is not affected by ζ.

3.7 Benchmark Parameter Values

Aside from those associated with the productivity process, the parameters to be estimated are: β, b, σ, τ, δ, s_g, α, α_k, α_l, δ, a_1, a_2, and ψ. For τ, s_g, α, and δ, we use the 1980-2000 average values of the corresponding figures in the data.[10] Given α, we simply set $\alpha_l = .1$ and $\alpha_k = \alpha - \alpha_l$.

[10] To estimate τ_t, del_t, and α_t in the data, I followed the procedures used in Hayashi and Prescott (2002). To abstract for foreign capital, nets exports are included in government purchases.

The share parameters, a_i, in the capital-adjustment function are set so that $Q_1 = Q_2 = 1$ along the non-stochastic balanced growth path. The discount factor, β, is set so that the steady-state land-price-output ratio for $\zeta = 0$ is 3.2 (note that if $\zeta = 0$, the steady-state land-price-output ratio is independent of σ). Based on the quarterly data, Jermann (1998) sets $b = .82$ and Boldrin, Christiano and Fisher (2001) uses $b = .73$. As it turns out, however, the habit persistence is not very helpful in explaining the asset price fluctuations in Japan. Indeed, we shall set $b = 0$ in the adaptive-expectations model in Section 6. But, for now, we set the benchmark value of b to be relatively high at .8. The benchmark values of the remaining parameters, σ, and ψ, are set to make the adaptive-expectations model fit the data 'best' (the procedure is described in Section 6). These values are summarized in Table 1.

4 Random Walk with a Constant Drift

In this section, we examine quantitative properties of the economy when the productivity process follows a random walk with constant drift:

$$z_{t+1} = \zeta + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, \sigma_\epsilon^2).$$

We have estimated ζ and σ_ϵ using the data over the period 1980-2000, and the estimates are in Table 2.

Figure 7 plots the land-price-output ratios, capital-output ratios, and prices of installed capital predicted by the model economy with the benchmark parameter values.[11] The predicted land-price-output ratio slightly increases over time (from 3.52 in 1980 to 3.78 in 2000), but it is effectively constant and not comparable to its fluctuations in the data. With the benchmark parameter values and $\zeta = .0116$, the capital-output ratio, K_t/Y_t, at the non-stochastic steady-state is 1.88. It starts with the value 1.71 in 1980, and the model predicts the economy reaches the steady state by the middle of the 1990s. While the model predicts the capital-output ratio well for the slow-growth periods (1930-1983, 1992-2000), it underpredicts the decline in the capital-output ratio in the fast-growth periods (1984-1991). In Section 6, we shall see that the model with adaptive expectations does better in this respect as well. As the bottom panel of Figure 7 shows, the price of installed capital and the capital-output ratio move in the opposite directions. This is because the marginal value of capital is higher when capital is scarce (see equation (7)).

[11] The initial capital stock is chosen so that the predicted level of the capital-output ratio for 1980 coincides the actual level.

Figures 8-10 show how the predictions of the model change for different values of σ, b, and ψ. Figure 8 shows the result for $\sigma = 0.5, 1, 2$ (the other parameters are set to their benchmark values). The prediction of the model is affected by σ mainly through its effect on the steady-state interest rate: given $\zeta > 0$, a higher σ raises the steady-state interest rate, and hence, lowers the steady-state capital-output ratio (equation (13)). Thus, other things being equal, a higher σ leads to a lower land-price-output ratio. Now, look at the case of $\sigma = 2$. In this case, the prediction on the land-price-output ratio is made worse: the land-price-output ratio declines over time. This reflects the fact that the steady-state capital-output ratio is lower than its 1980 level. On the other hand, the predicted path of the capital-output ratio matches well the hump-shaped pattern observed in the 1980s. However, it does not replicate the increase in the capital-output ratio in the 1990s. The case of $\sigma = 0.5$ shows that lowering σ does not help much: it makes the model predict a monotonically increasing path of the land-price-output ratio, and the prediction on the capital-output ratio becomes worse.

It has been argued that a large habit persistence and a low elasticity of substitution in the capital-adjustment technology help the model to generate a large equity premium (Jermann, 1998; Boldrin, Christiano and Fisher, 2001; Christiano and Fisher, 1998). This is because a greater habit persistence makes the stochastic discount factor more volatile, and because a lower elasticity of substitution in the capital-adjustment technology makes investment less elastic with respect to the price of installed capital (Tobin's Q). To illustrate the latter, log-linearize the investment demand equation (7) around the steady state to obtain

$$\hat{i}_t = \frac{\psi}{1-\delta}\hat{p}_t^k + \hat{k}_t - \epsilon_t,$$

where a hat over a variable denotes the log-deviation from the steady-state value, and we have used the fact that $Q_1 = Q_2 = 1$ at the steady state. Thus, the elasticity of investment with respect to Tobin's Q is $\psi/(1-\delta)$. Thus, choosing a smaller value for ψ makes investment less elastic.

Here, we see how those parameters affect the prediction of our model. Figure 9 shows the sensitivity analysis for the habit persistence parameter, b. The cases of $b = 0$ and $b = 0.5$ are indistinguishable. When b is raised to as high as 0.9, the prediction on the land-price-output ratio becomes slightly better in that the predicted path of the land-price-output ratio exhibits a hump-shaped pattern as in the data (the predicted values for 1980, 1990, 2000 are 3.64, 4.10, 3.76, respectively). But, it is too small to be comparable to the actual fluctuations. Figure 10 exhibits the result for different values of the elasticity of substitution in the capital-adjustment technology, ψ. The cases of $\psi = 1.5$ and $\psi = 5$ appear identical. When $\psi = 0.25$, the price of installed capital becomes considerably more volatile and exhibits a large increase in the fast-growth periods and decline in the following periods, which is consistent with the stock market boom and bust during those periods. However, the predicted price of capital is highest in 1980, which is discouraging to use a low value of ψ in order to account for the equity price behavior over the whole period. The middle panel shows that lowering ψ helps the model predict the capital-output ratio somewhat better: it reproduces both the increase in the slow-growth periods (1980-1983, 1992-2000) and the decrease in the fast-growth periods (1984-1991). However, as far as the land-price-output ratio is concerned, lowering the elasticity of investment demand worsens the performance of the model: it predicts that the land-price-output ratio is highest in 1980, and declines over time.

From these numerical exercises, we conclude that assuming high habit persistence and low elasticity of investment demand does not help account for the asset price fluctuations in Japan in the last twenty years. In the next section, we examine how persistence in the productivity growth rate affects the prediction of the model.

5 Random Walk with a Markov-Switching Drift

To illustrate the effect of persistence in the productivity growth rate, let us assume that the productivity growth rate, $z_t = \ln A_t - \ln A_{t-1}$, follows a regime switching process. In each period, the economy is in one of the two regimes, the fast-growth or slow-growth regimes. Let $s_t \in \{1, 2\}$ denote the regime in period t, and the economy is in the fast-growth regime when $s_t = 1$ and in the slow-growth regime when $s_t = 2$. Regime s_t follows a Markov chain with the transition matrix Π:

$$\Pi = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix},$$

where $p_{ii} \in (0, 1)$ denotes the probability that the regime in the next period is i given that the current regime is i. The productivity growth rate follows the process:

$$z_t = \zeta_{s_t} + \epsilon_t,$$

where ζ_i, $i = 1, 2$, is the expected growth rate in regime i with $\zeta_1 > \zeta_2$, and $\epsilon_t \sim \text{i.i.d.}N(0, \sigma_\epsilon^2)$. Thus, $\ln A_t$ follows a random walk with a Markov-switching drift, ζ_{s_t}.

The Markov chain s_t has the invariant distribution:

$$\xi = \begin{bmatrix} \frac{1-p_{22}}{2-p_{11}-p_{22}} \\ \frac{1-p_{11}}{2-p_{11}-p_{22}} \end{bmatrix}.$$

Thus, the unconditional expected rate of productivity growth is

$$\zeta = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}\zeta_1 + \frac{1 - p_{11}}{2 - p_{11} - p_{22}}\zeta_2. \tag{15}$$

The transition matrix Π has two eigenvalues, 1 and $p_{11} + p_{22} - 1$. The persistence of the Markov chain is measured by the latter. In this section, we examine how the persistence of s_t affects the prediction of the model. In particular, we are interested in looking at how much persistence is needed to generate the observed quantity of the difference in the land-price-output ratio between the slow-growth and fast-growth periods. The parameters for the productivity growth process, $(\zeta_1, \zeta_2, p_{11}, p_{22}, \sigma_\epsilon)$, are set simply by assuming that the economy is in the slow growth regime between 1971 and 1983 and between 1992 and 2000. Thus, ζ_1 is the average growth rate over the period 1984-1991, ζ_2 is the average growth rate over the period 1971-1983 and 1992-2000. For simplicity, we assume that $p_{11} = p_{22}$. Thus, the persistence of the trend growth rate, ζ_{s_t}, is measured by p_{ii}. The parameter values for the productivity process used in this section are listed in the middle panel of Table 2. For simplicity, we assume that agents in our economy observe s_t.

Figures 8-10 display how the persistence of the trend growth rate, p_{ii}, affects the prediction of the model for $\sigma = 0.781, 0.5, 0$. For each σ, we consider three cases: $p_{ii} = .9, .95, .99$. To make the predictions with different values of σ comparable, for each value of σ, we set β so that the land-price-output ratio equals 4 along a non-stochastic balanced growth path with the constant growth rate ζ defined in (15). The habit persistence parameter is set to $b = .8$ as in the previous section, but, as we have seen in the previous section, it does not affect the result much. As the middle and bottom panels of those figures show, given σ, changing the persistence parameter, p_{ii}, does not alter the prediction on the capital-output ratio, or the price of installed capital.

As shown in Figure 8, for the benchmark value of σ, when the persistence is relatively low, $p_{ii} = .9, .95$, the land-price-output ratio is lower in the fast-growth periods. This is because when the trend growth is relatively less persistent the short-run effect of faster growth on the stochastic discount factor is stronger than its effect on dividend growth. The figure demonstrates that even when $p_{ii} = .99$, that is, when the trend growth rate is so persistent that each regime lasts for a hundred years on average, the predicted difference in the land-price-output ratio between the fast-growth and slow-growth periods is far smaller than the actual difference. Note, however, that when σ is at the benchmark value (0.781), the predicted path of the capital-output ratio does match the actual one, at least qualitatively. It reproduces the rise in the slow-growth periods and the fall in the fast-growth periods.

Figures 9-10 show that, as σ lowers, the predicted difference in the land-price-output ratio between the two regimes gets larger. However, even when we assume linear utility ($\sigma = 0$), we still need to assume an extremely persistent trend growth rate. Also, notice that lowering σ makes the capital-output ratio fluctuate less. Thus, lowering σ tends to improve the prediction of the model regarding the land-price-output ratio, but worsen it with respect to the capital-output ratio.

6 Adaptive Expectations for Future Growth

The discussion in the previous section clarifies the importance of expectations on future growth rate in accounting for the behavior of the Japanese land-price-output ratio in the past twenty years. In this section, we consider a particular kind of the model with such a feature.[12]

In each period t, given the past growth rates, z_{t-k}, $k = 0, 1, \ldots$, the representative agent expects that future productivity growth rates, z_{t+j}, $j = 1, 2, \ldots$, follow the distribution given by

$$z_{t+j} = \zeta_t + \epsilon_{t+j}, \tag{16}$$

where $\epsilon_{t+j} \sim \text{i.i.d.}N(0, \sigma_\epsilon^2)$, and ζ_t is the trend growth rate expected in period t defined by

$$\begin{aligned} \zeta_t &\equiv (1-\rho)\sum_{k=0}^{\infty} \rho^k z_{t-k}, \\ &= (1-\rho) z_t + \rho \zeta_{t-1}, \end{aligned} \tag{17}$$

for some $\rho \in [0, 1)$. The (subjectively) expected growth rate is

$$\tilde{E}_t[z_{t+j}] = \zeta_t, \qquad j = 1, 2, \ldots\,, \tag{18}$$

where $\tilde{E}_t$ is the (subjective) conditional expectation operator.

As discussed in Barsky and De Long (1993), we can obtain a productivity-growth process similar to (16)-(18) even in the rational-expectations framework. Assume that the growth rate, z_t, follows the random walk with $MA(1)$ disturbance:

$$z_{t+1} = z_t + \epsilon_{t+1} - \rho \epsilon_t. \tag{19}$$

Define ζ_t as in (17). Then, (19) implies that $\epsilon_t = z_t - \zeta_{t-1}$. Substituting for ϵ_t from this equation into (19) yields

$$z_{t+1} = \zeta_t + \epsilon_{t+1}, \tag{20}$$

[12] Barsky and De Long (1993) use a similar approach to explain the U.S. stock data in a partial-equilibrium model.

and for $j \geq 2$,

$$z_{t+j} = \zeta_t + \epsilon_{t+j} + (1-\rho)\sum_{k=1}^{j-1} \epsilon_{t+k}. \tag{21}$$

The conditional expectations of z_{t+j}, $j \geq 1$, are

$$E_t[z_{t+j}] = \zeta_t. \tag{22}$$

Thus, the stochastic process given by (19) shares some key properties with the one given by (16)-(18). We choose the adaptive-expectations specification for a technical reason. If we assume the stochastic process (19) in the rational-expectation framework, then the agent in our model knows that the future trend growth rates, ζ_{t+j}, are unbounded, which makes the lifetime utility infinite as long as $\sigma < 1$. With adaptive expectations, this problem does not arise. As long as ζ_t in the current period is not too large, the model has a solution, since the agent does not take into account the possibility that ζ_{t+j} becomes too large in the future.

To simulate the model, we need to assign values to ρ, σ_ϵ, and the initial value of ζ, $\zeta_1 980 \equiv \zeta_0$. We have estimated the process given in (19) using the maximum likelihood method, and obtained $\rho = .6564$. Given this, we set $\rho = 0.7$. Given ρ, the initial value ζ_0 is constructed using z_t, $t = 1961, \ldots, 1980$, in the data and assuming $\zeta_{1960} = 0$. The value of ζ_0 is not affected by using different (sensible) values of ζ_{1960}. The variance of ϵ_t is set to the sample variance of $z_t - \zeta_t$. These values are in the bottom panel of Table 2.

Here, we also choose b, σ, and ψ to maximize the model's ability to fit the land-price-output ratio in the data. Let x_t be the land-price-output ratio in year $t = 1980, \ldots, 2000$ in the data. Let $\hat{x}_t(b, \sigma, \psi)$ be the corresponding value predicted by the model. We choose b, σ, and ψ to minimize the average difference between x and $\hat{x}$:

$$\min_{b,\sigma,\psi} \sum_{t=1980}^{2000} \omega_t \big(\hat{x}_t(b,\sigma,\psi) - x_t\big)^2,$$

where ω_t are the weights. They are chosen so that $\omega_t = 1$ for $t = 1981, 1982, 1999, 2000$, $\omega_t = 1.5$ for $t = 1990, 1991$, and $\omega_t = 0$ for the rest of years. The solution to this minimization problem is $b = 0$, $\sigma = .781$, and $\psi = 3.7$ (see Table 1).

Figure 14 shows the paths of the subjective trend growth rate, ζ_t, for different values of ρ. As ρ gets larger, the time path of ζ_t becomes smoother, and its difference between the slow-growth periods and the fast-growth periods becomes smaller. Indeed, when $\rho = 0.9$, the trend growth rate in the early 80s is almost as high as that in the late 80s, which would make the model difficult to match the land-price-output ratio in the data. On the other hand, when $\rho = 0.4$, ζ_t is much more volatile, and the difference between the slow-growth and high-growth periods is large. In this case, however, the model tends to predict the land-price-output ratio fluctuates more than in the data. Figure 15 displays the prediction of the model for $\rho = 0.7$. The predicted paths of the land-price-output ratio and the capital-output ratio match the actual ones fairly well. The model generates considerably higher values of the land-price-output ratio for 1984 and 1985, which reflects the relatively high growth rates in those years.

7 Stock Prices

In the previous section, we have seen that our adaptive-expectations model reproduces the actual land-price-output ratio to a large extent. In this section we examine how well it accounts for the behavior of the stock price.

Consider a firm that holds capital, K^a_{t+1}, land, L^a_{t+1}, and net financial asset, W^a_{t+1}, at the end of period t. The end-of-period value of the firm, S^a_t, is then

$$S^a_t = P^k_t K^a_{t+1} + P^l_t(t) L^a_{t+1} + W^a_{t+1}.$$

We use the simulated series of the stock of capital and the prices of installed capital and land to compute the series of stock prices predicted by the model. We consider the value of the private, non-financial corporation (NFC) sector. The Japanese NIA (SNA93) reports the assets and liabilities of the private NFC sector for the years after 1990. According to it, the fractions of capital and land held by the private NFC sector in the aggregate stocks of capital and land are both fairly constant in spite of the large change in the economic environment in the last decade. The average share of capital held by the private NFC sector, K^{nfc}/K, is 0.546, and that of land, L^{nfc}/L, is 0.267. The model does not determine W^{nfc}, so we use the estimated net financial asset of the private NFC sector. Again, the NIA (SNA93) shows that the shares of financial asset and liabilities (excluding shares) of the private non-financial firms in the total (private and public) NFC sector are roughly constant over the period 1990-2000. For the years prior to 1990, the NIA reports only the assets and liabilities of the total NFC sector. We use the 1990 shares of financial assets and liabilities of private firms in the NFC sector to extrapolate their financial assets and liabilities prior to 1990. This provides the estimated series of the net financial assets of the private NFC sector, $\hat{W}^{\text{nfc}}_{t+1}$. Let $\{\hat{K}_{t+1}, \hat{P}^k_t, \hat{P}^l_t\}$ be the prediction of the model with adaptive expectations described in the previous section. Then, the predicted market value of the private NFC sector at the end of period t, $\hat{S}_t^{\mathrm{nfc}}$, is given by

$$\hat{S}_t^{\mathrm{nfc}} = 0.546 \hat{P}_t^k \hat{K}_{t+1} + 0.267 \hat{P}_t^l + \hat{W}_{t+1}^{\mathrm{nfc}}. \tag{23}$$

Figure 16 shows the predicted path of the stock price (legend 'predicted') and the actual one (TOPIX). Each price is divided by output, and normalized so that its 1980 value equals unity. The figure shows the fluctuations in the stock price predicted by the model are too small compared to its actual fluctuations. The failure is due to the predicted path of the price of installed capital. As Figure 15 shows, it takes on the highest value in 1980 and declines in the 1980s, which is a natural consequence of costly capital adjustment: with the adjustment cost of capital, the price of capital is higher when the capital-output ratio is lower relative to the steady-state value. The data does suggest that the capital-output ratio relative to the steady-state value is lowest in 1980, and all the numerical exercises consistent with the decline in the capital-output ratio in the late 80s predict that the price of capital is highest in 1980.

Thus, introducing costly capital adjustment may not be a good idea to explain the stock-price fluctuations in Japan. For a comparison, we have computed $\hat{S}_t$ in (23) under two alternative assumptions on P_t^k: one with $P_t^k \equiv 1$ and the other with $P_t^k \equiv 0$ using the same series for $\hat{K}_{t+1}$, $\hat{P}_t^l$ and $\hat{W}_{t+1}^{\mathrm{nfc}}$. The predicted paths of $\hat{S}_t$ for these cases are plotted in Figure 16 under the legends of 'predicted ($p^k = 1$)' and 'predicted ($p^k = 0$),' respectively. The case of $P_t^k \equiv 1$ improves the prediction of the model only slightly, but there is a dramatic improvement when we assume $P_t^k \equiv 0$.[13] We are not certain about how to interpret this result. One interpretation would be that it is indeed the case that the price of installed capital, P_t^k, is extremely low in Japan. If so, the fact that the capital-output ratio has been increasing in the last thirty years means that firms' investment decision has been very irrational. Another interpretation would be that the price of capital, P_t^k, might be similar to what the theory suggests but the value of capital, $P_t^k K_{t+1}$, is not considered when shares of firms are traded. In that case, the market value of a firm would equal the value of land and net financial asset held by it. The difference in liquidity between capital and other assets might justify it. Exploring this issue is left for future research.

[13] This is related to the finding of Kiyotaki and West (1996) that the estimated Tobin's Q is negative for many years in Japan. Also, see Ando, Christelis and Miyagawa for a related discussion.

8 Conclusion

In this paper, we have examined whether a version of the neoclassical stochastic growth model can explain the asset price fluctuations in Japan over the period 1980-2000. The key factor to explain the behavior of the aggregate land price is the people's expectations on future productivity growth. We have used a model of adaptive expectations in which the expected trend growth rate in the future is given by the weighted average of the past growth rates, and showed that it can reproduces the actual time path of the land-price-output ratio. However, the model's prediction on the stock price is not satisfactory, and a puzzle remains. Our result suggests that the price of installed capital is close to zero or that people don't take into account the value of capital when trade shares.

Throughout this paper, we have treated productivity growth as an exogenous variable. An important question is to explain why the productivity index followed the path as plotted in Figure 3. In this respect, it would be interesting to explore the possibility that asset prices affect productivity growth, say, through credit constraint.

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Table 1: Benchmark Parameter Values

Parameter	Values	Description
β	.983	discount factor
b	.8, 0	habit persistence
σ	.781	curvature of instantaneous utility
s_g	.163	share of government purchases
τ	.442	tax rate on rental income
δ	.95	Depreciation rate
α_k	.269	share of capital
α_l	.1	share of land
ψ	3.7	elasticity of substitution in capital adjustment

Table 2: Productivity Processes

Constant drift				
	ζ	σ_ϵ		
	.0116	.0221		
Markov switching				
	ζ_1	ζ_2	p_{ii}	σ_ϵ
	.0322	-.0011	.9, .95, .99	.0154
Adaptive expectations				
	ζ_0	ρ	σ_ϵ	
	.0022	.7	.0157	

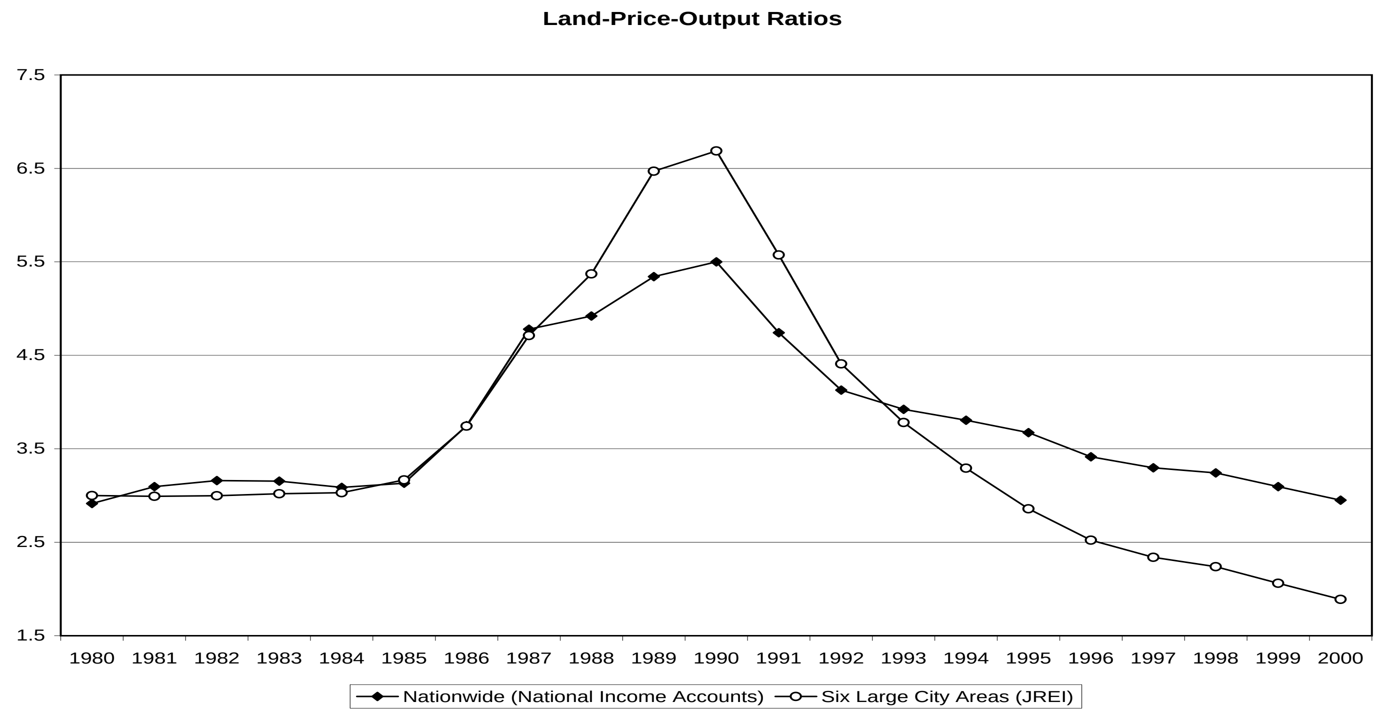

Figure 1: Land-price-output ratios: 1980-2000

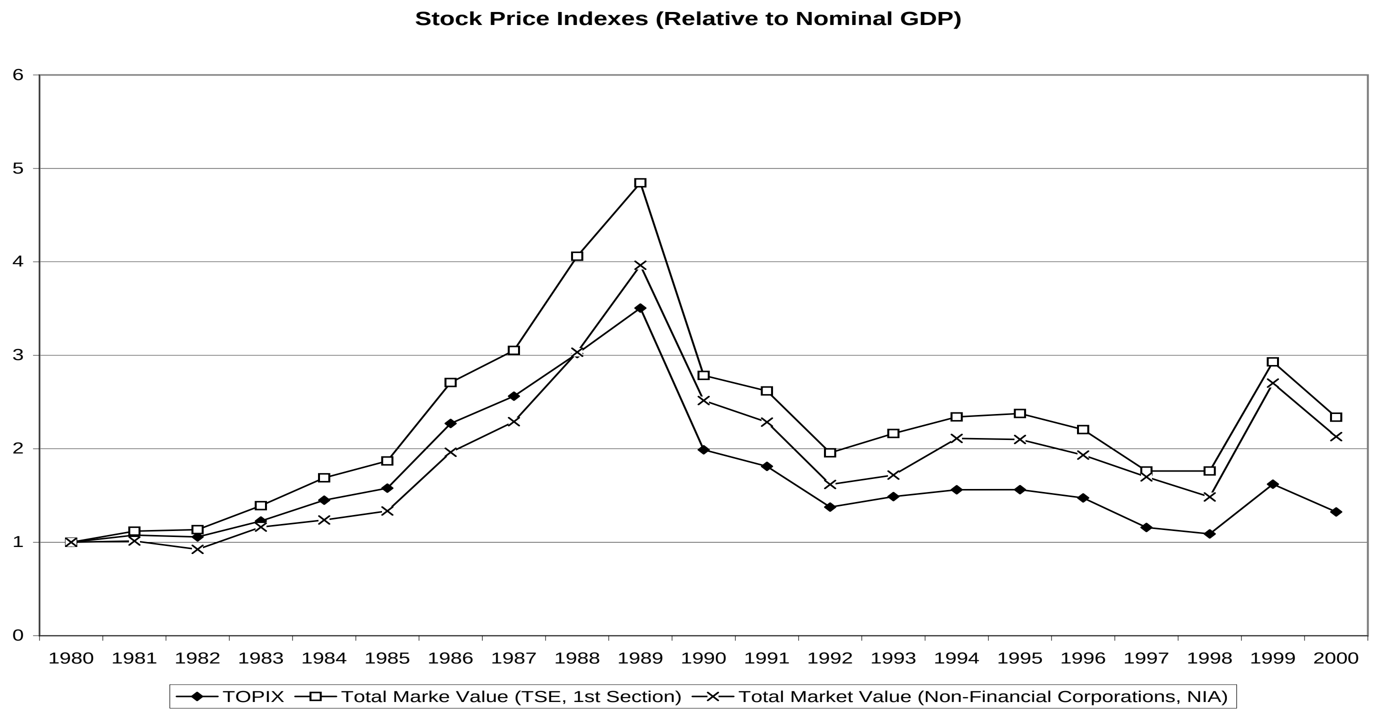

Figure 2: Stock price indexes: 1980-2000 (Ratios to nominal GDP)

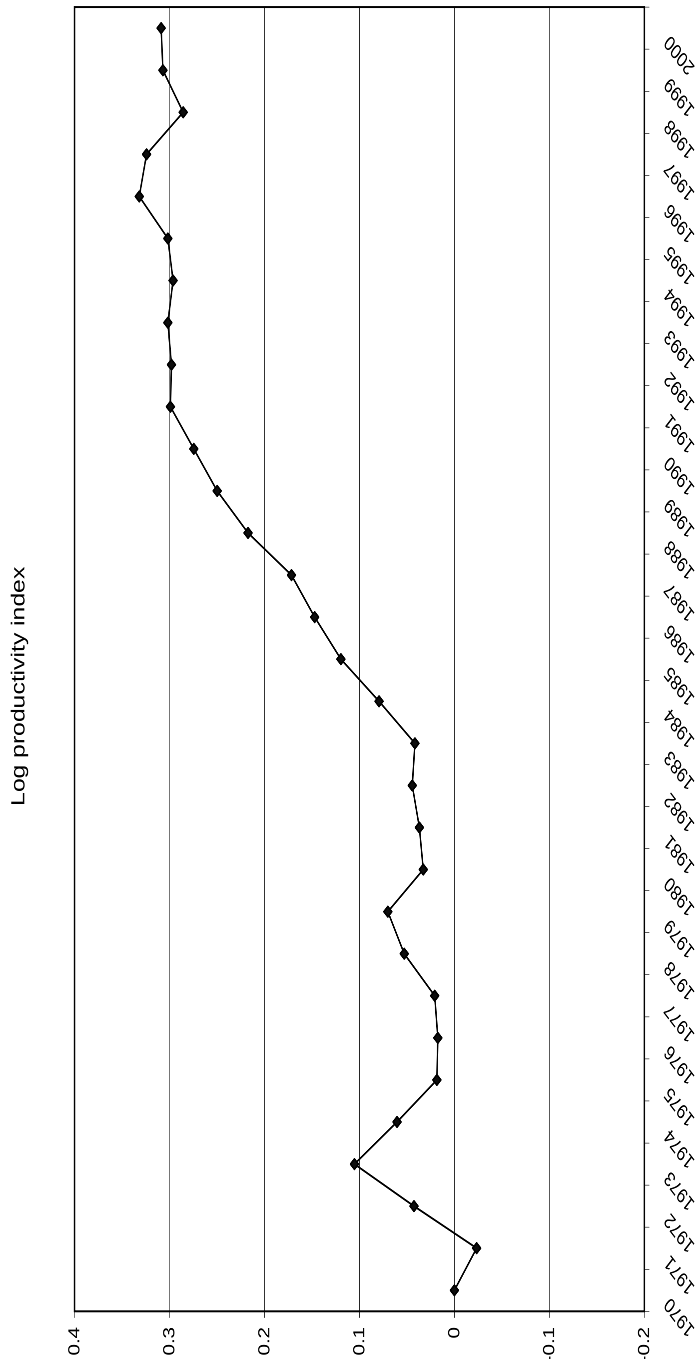

Figure 3: Productivity index: 1970-2000

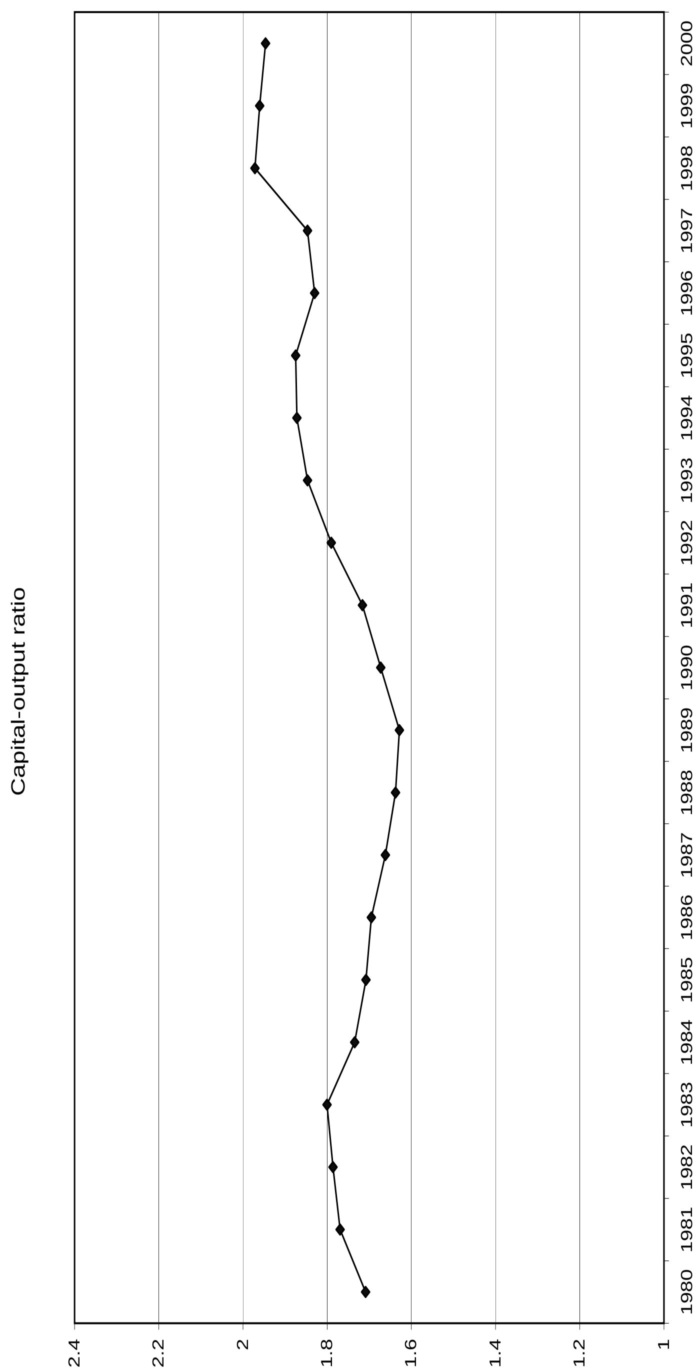

Figure 4: Capital-output ratios: 1980-2000

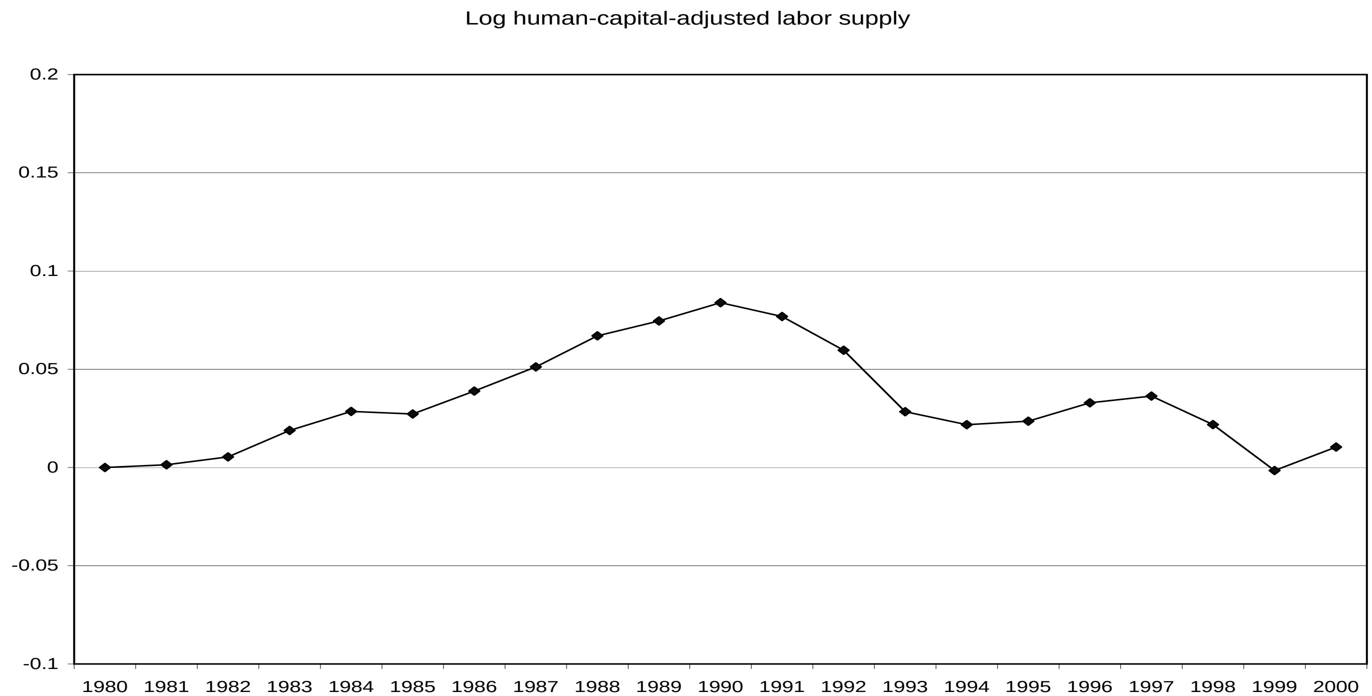

Figure 5: Human-capital adjusted labor supply: 1980-2000

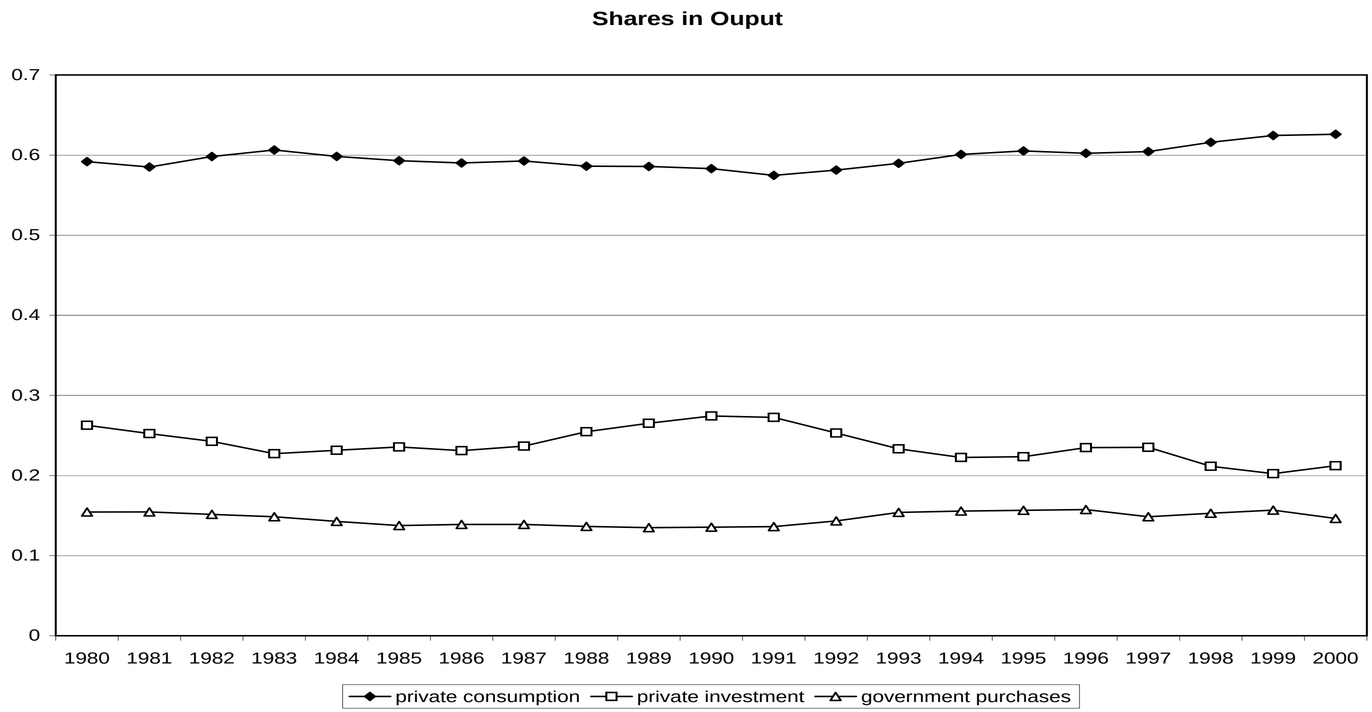

Figure 6: Shares of consumption and investment in GDP: 1980-2000

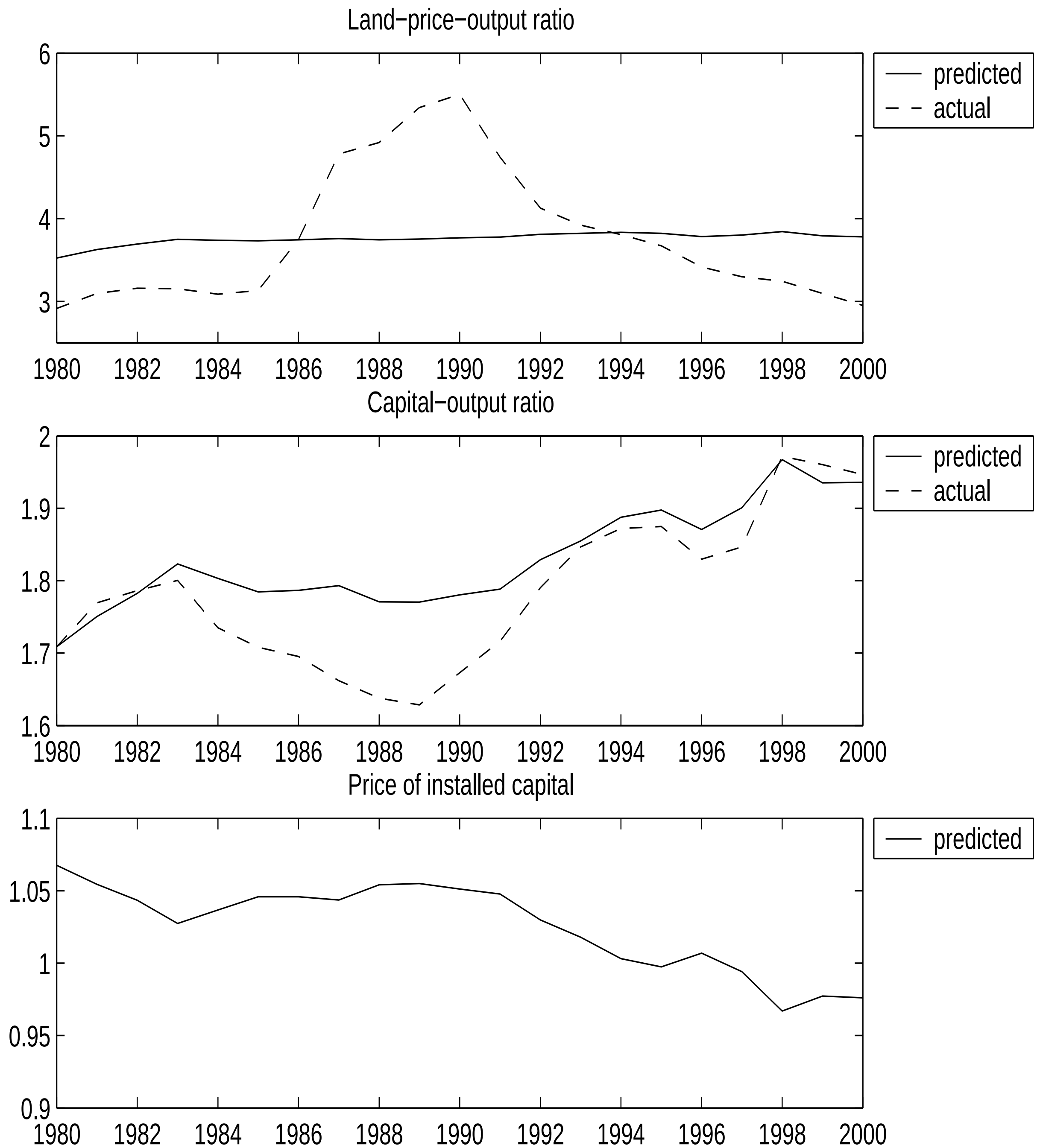

Figure 7: Random walk with constant drift: Benchmark parameter values

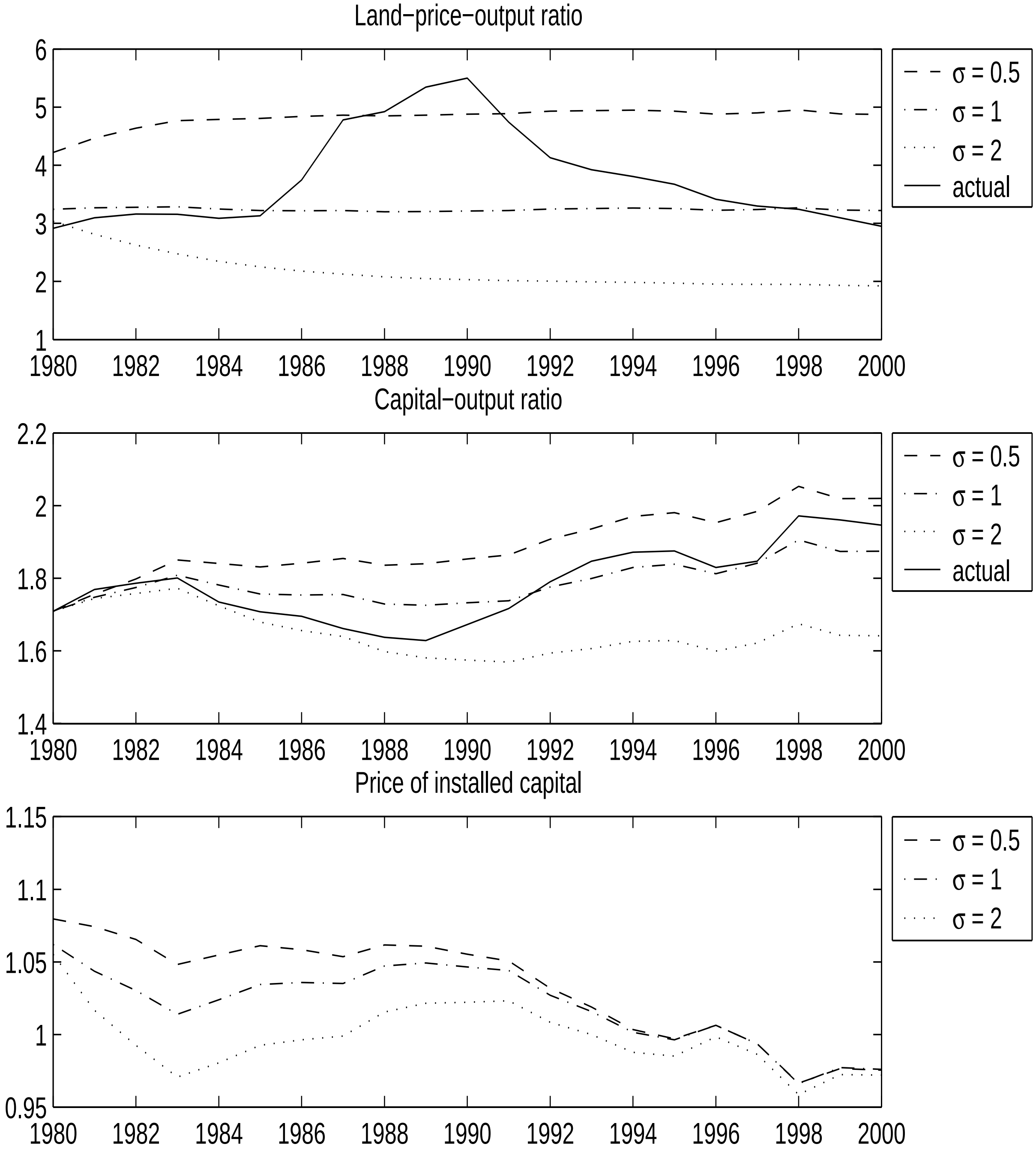

Figure 8: Random walk with constant drift: Different values of σ

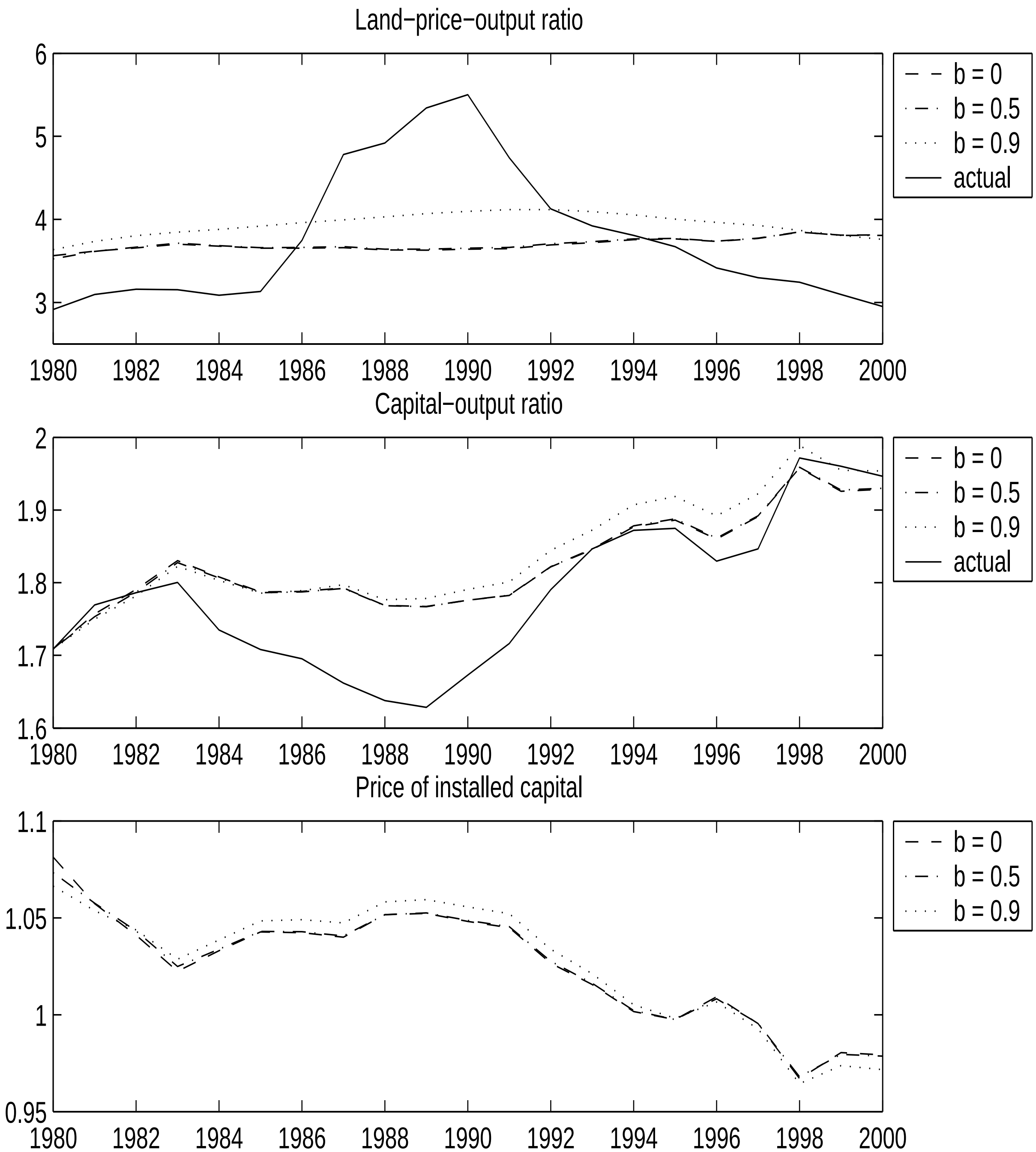

Figure 9: Random walk with constant drift: Different values of b

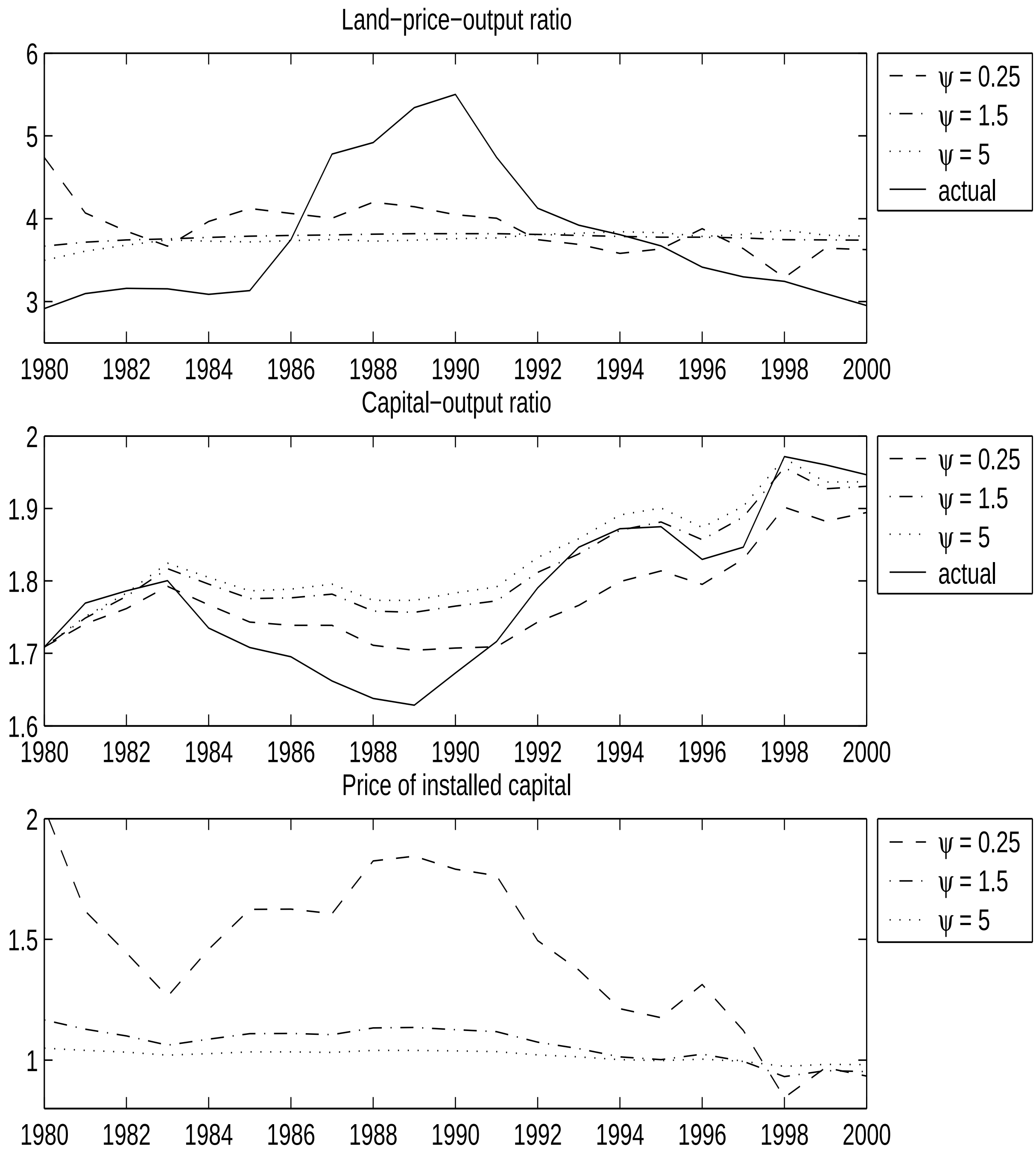

Figure 10: Random walk with constant drift: Different values of ψ

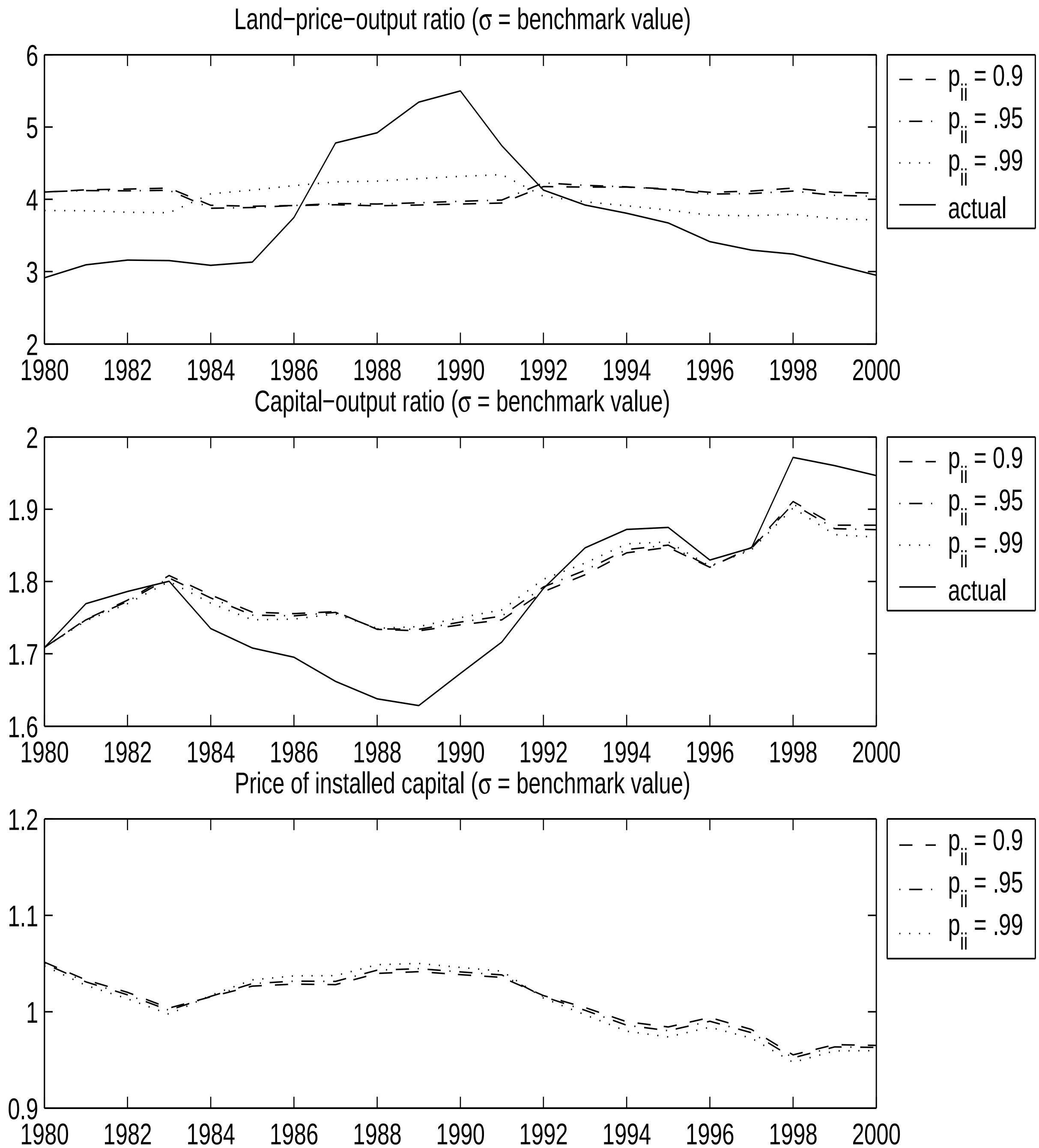

Figure 11: Random walk with Markov-switching drift: $\sigma = 0.781$

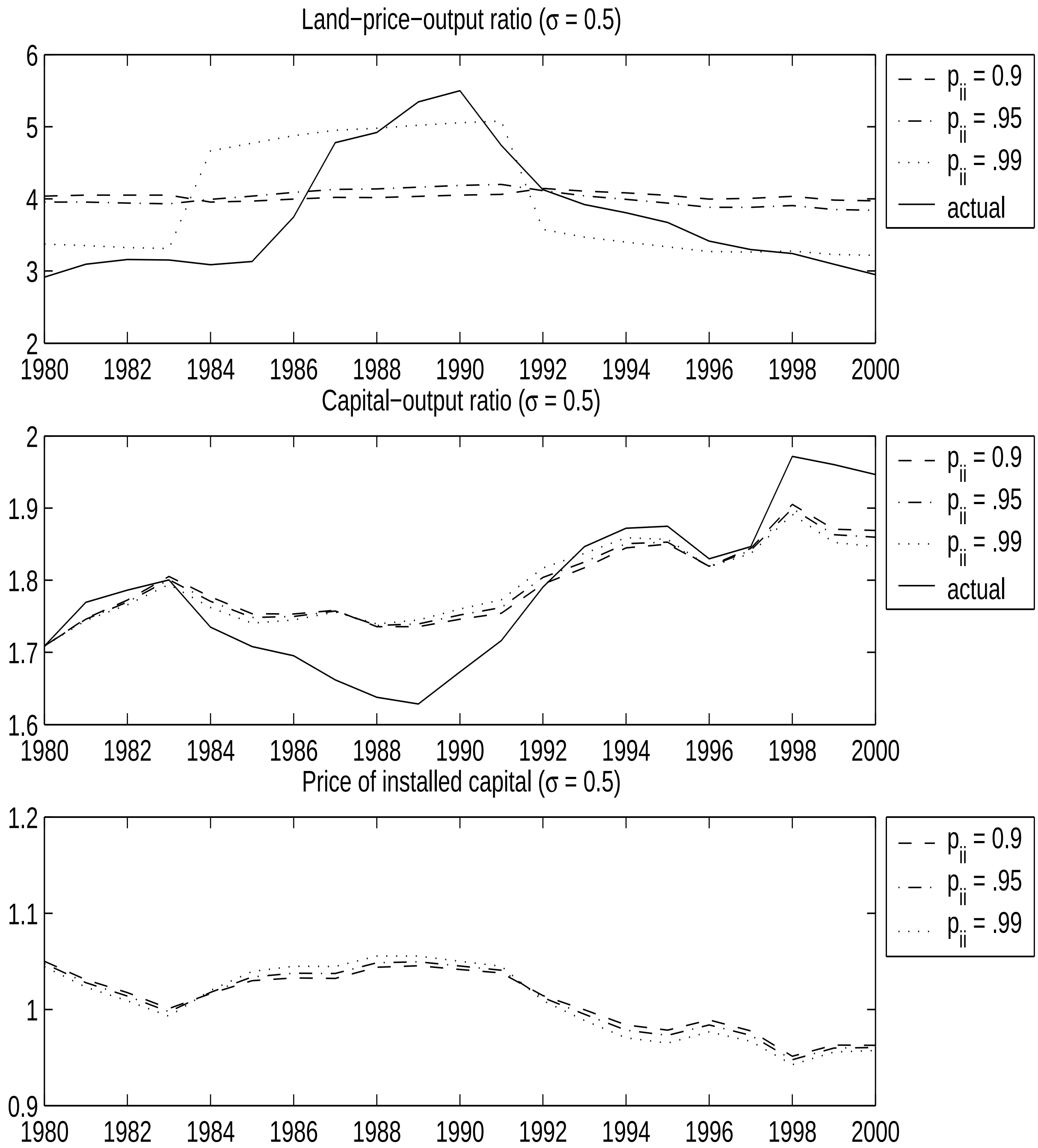

Figure 12: Random walk with Markov-switching drift: $\sigma = 0.5$

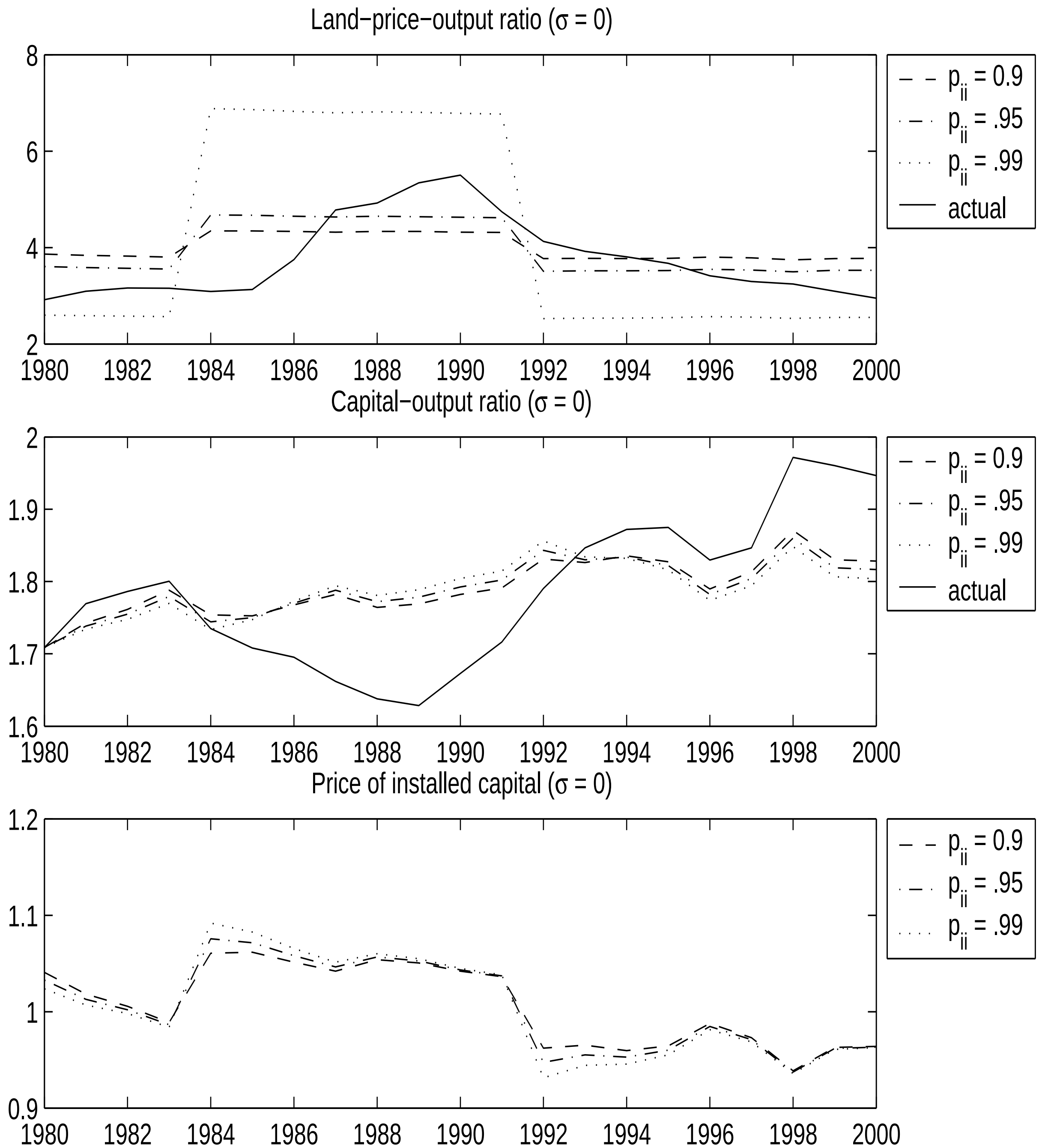

Figure 13: Random walk with Markov-switching drift: $\sigma = 0$

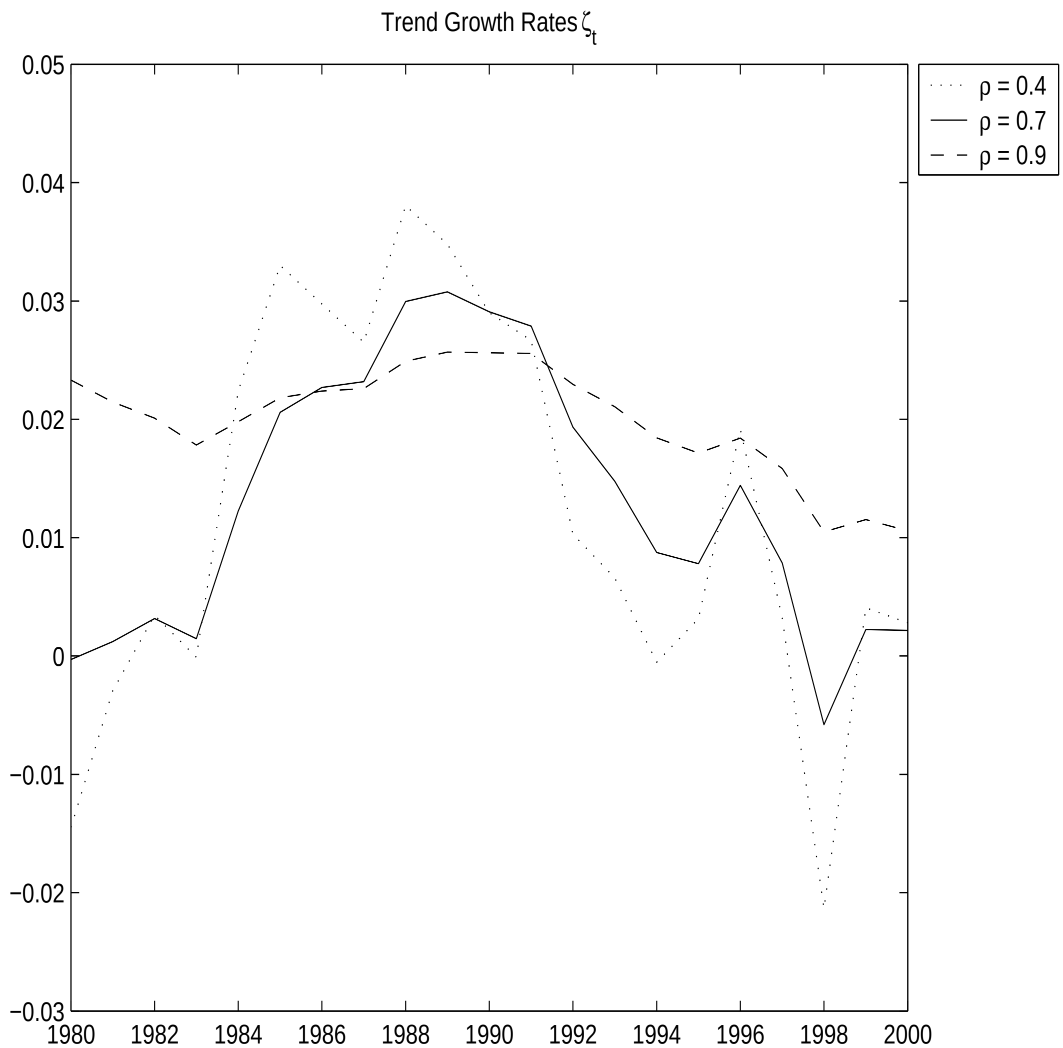

Figure 14: Expected trend growth rates for different values of ρ

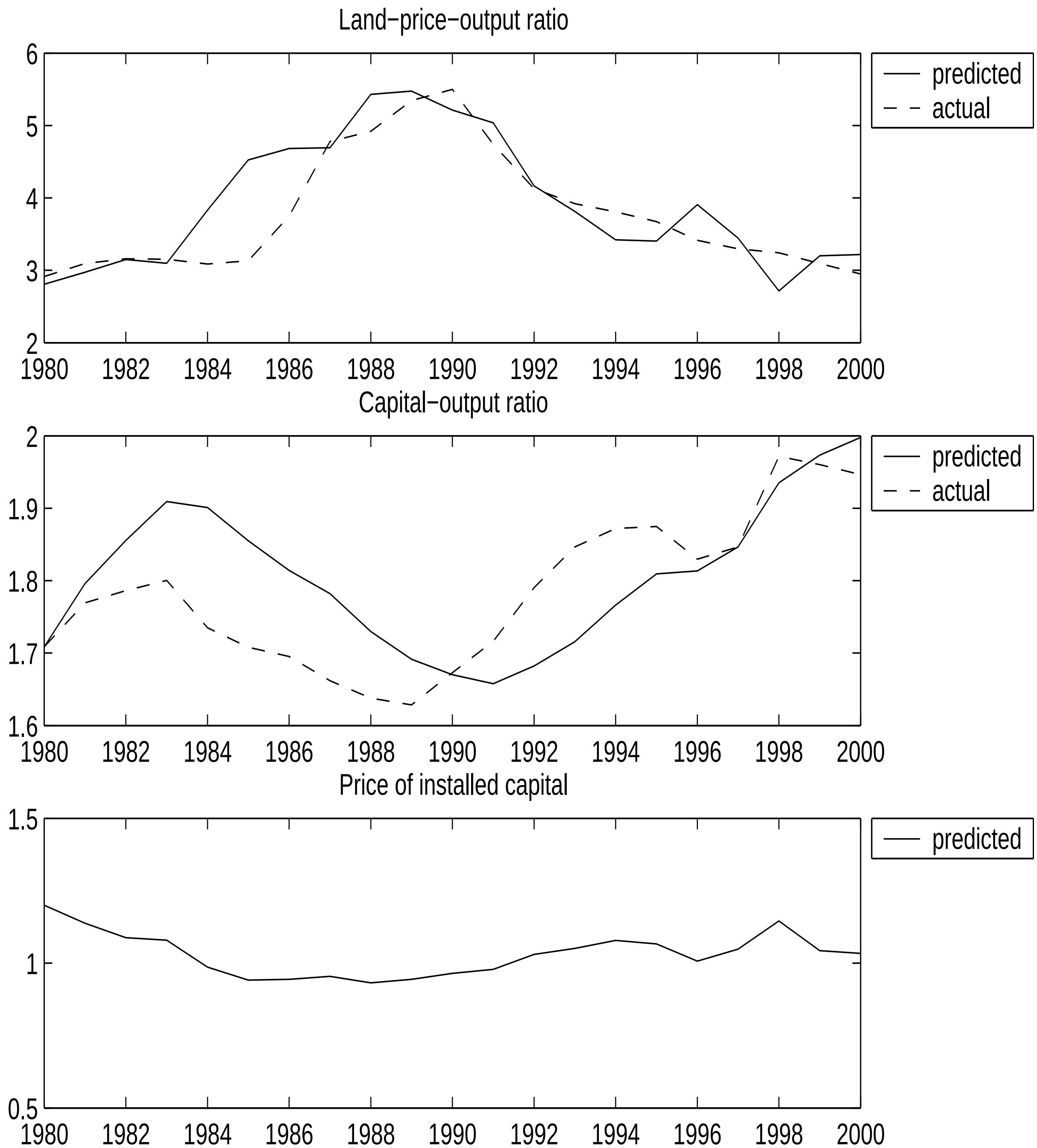

Figure 15: Random walk with adaptive expectations on future growth rates

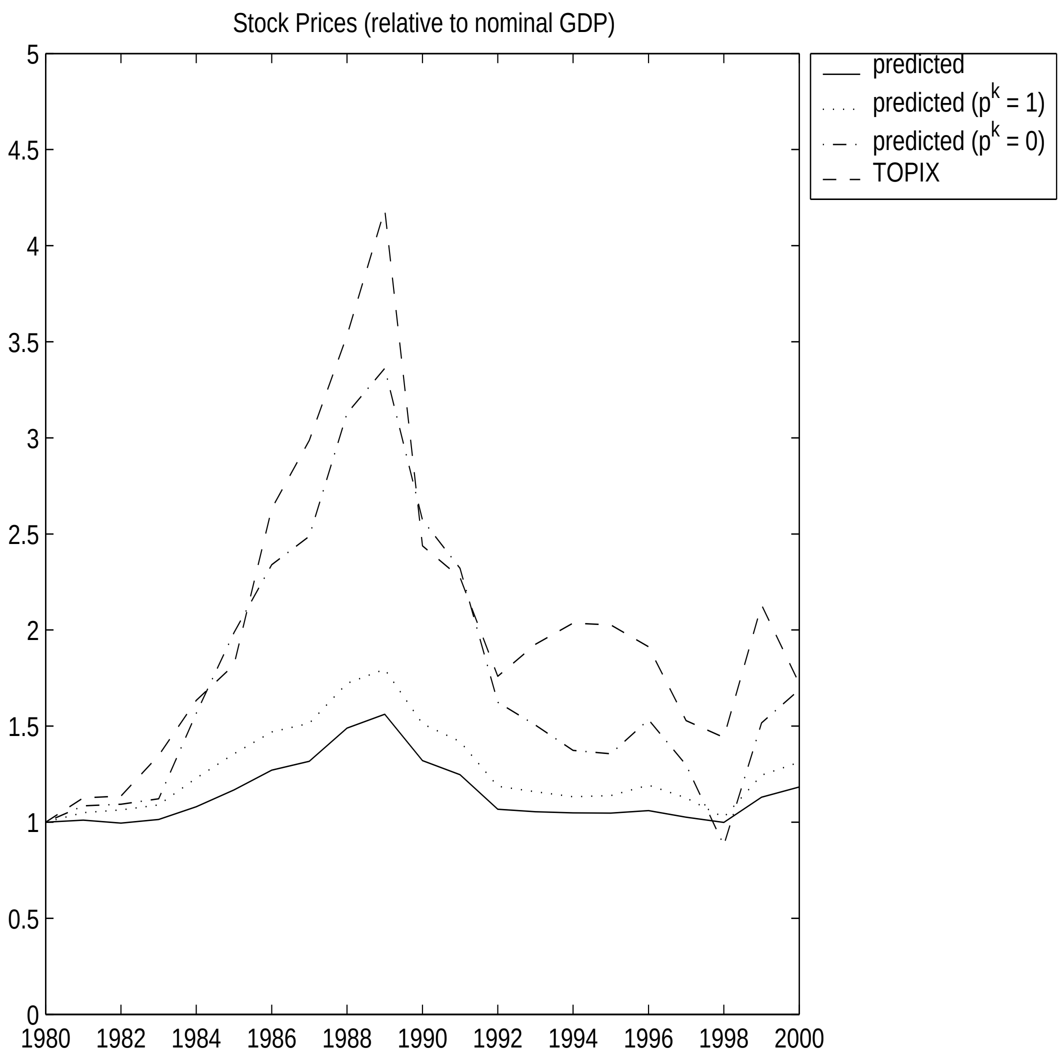

Figure 16: Predicted stock price index