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Peace and War in Territorial Disputes

Herschel I. Grossman

Brown University

Abstract

Why do sovereign states sometimes fail to settle territorial disputes peacefully? Also, why do even peaceful settlements of territorial disputes rarely call for the resulting border to be unfortified? This paper explores a class of answers to these questions that is based on the following premise: States can settle a territorial dispute peacefully only if (1) their payoffs from a peaceful settlement are larger than their expected payoffs from a default to war, and (2) their promises not to attack are credible. This premise directs the analysis to such factors as the advantage of attacking over both defending and counterattacking, the divisibility of the contested territory, the possibility of recurring war, the depreciation or obsolescence of fortifications, and inequality in the effectiveness of mobilized resources.

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> Throughout history territorial disputes have been the principle source of conflict leading to war . . . [Yet,] in the post-World War II period over one-half of all territorial disputes *did not* involve the threat or use of military force . . . While most wars . . . involve conflicts over territory, many territorial disputes do not pose a high risk of war. (Paul Huth, 1996, pages 6-10).

Why do sovereign states sometimes fail to settle territorial disputes peacefully? Also, why do even peaceful settlements of territorial disputes rarely call for the resulting border to be unfortified? These questions arise because in fact most territorial disputes are settled peacefully, and because, war being costly, it is natural to conjecture that every territorial dispute should afford a peaceful settlement. Also, in the same spirit, it is natural to conjecture that peaceful settlements of territorial disputes should avoid the costs of fortifying the resulting border.

Attempts to explain why states sometimes choose to go to war commonly focus on the effects of incomplete information.[1] The present paper, without denying the historical importance of incomplete information as a cause of war, explores another class of answers to the question of why some territorial disputes result in war. This class of answers is based on the following premise:

> *States can settle a dispute peacefully only if (1) their payoffs from a peaceful settlement are larger than their expected payoffs from a default to war, and (2) their promises not to attack are credible.*

[1] Dagobert Brito and Michael Intriligator (1985) show how incomplete information, by causing a state to be overly optimistic about its prospects for winning a war cheaply, can prevent a peaceful settlement of a dispute. James Fearon (1995) and Herschel Grossman (2004) discuss historical examples in which overoptimism seems to have been an important factor in the decision to go to war. Also, because of incomplete information, going to war can be a worthwhile investment in reputation. As Fearon (1995, page 400) puts it, "States employ war itself as a costly signal of privately known and otherwise unverifiable information about willingness to fight...[and] as a credible means to reveal private information about their military capabilities."

In other words, states can settle a dispute peacefully only if a peaceful settlement is on their contract curve and also is subgame perfect. Given this premise the question of why states sometimes choose to go to war becomes why do some territorial disputes not afford peaceful settlements that satisfy these two conditions.

Recognition of the problem of credibility also suggests an answer to the question of why borders are usually fortified:

> *Although fortifications are costly, fortifications can help to make a peaceful settlement credible.*

This answer is embodied in the popular observation, "Good fences make good neighbors." More generally, the possibility that fortifications are the key to peaceful settlements is embodied in the prescription: *Qui desiderat pacem, preparet bellum.* In this context, fortifications include but are not limited to defensive works that are literally on the border.

Analytical Framework

Assume that two sovereign states, State 1 and State 2, face a newly arisen dispute over control of a valuable territory. Either these states settle this dispute peacefully or by default they fight a war to settle the dispute. A peaceful settlement would give each state control of a non-negative fraction of the contested territory, with the resulting border being either unfortified or fortified. A war would give each state a non-negative probability of winning control of the entire contested territory.

Within this framework a state's strategy set consists of the following elements:

- the amount to spend to mobilize resources in the event of a default to war;
- the amount to spend to construct fortifications in the event of a peaceful settlement;
- whether or not to agree to a peaceful settlement;
- whether to keep or to break a promise not to attack;
- the amount to spend to mobilize resources if it were to break a promise not to attack;

- the amount to spend to mobilize resources for a counterattack if the other state were to break a promise not to attack.

The core choice-theoretic assumption is that the states choose the elements of their strategy sets to maximize the expected payoff for which each element is relevant. For example, each state chooses the amount to spend to mobilize resources in the event of a default to war to maximize its expected payoff from a default to war. In addition the analysis assumes that in choosing its strategy set each state takes the other state's strategy set as given.[2]

To circumscribe the analytical framework the model maintains the following simplifying assumptions:

- The two states put the same value on controlling the contested territory. They also put the same value on controlling any part of the contested territory. These valuations take no account of any preference of the inhabitants of the contested territory to be affiliated with one state or the other.[3]
- Attacking has an advantage over defending in the sense that with equal amounts spent to mobilize resources for an attack and to construct fortifications the probability of the attacker prevailing would be larger than one half. In the same sense, attacking has an advantage over counterattacking — that is, there is an advantage to striking first.
- The states behave as if they are unitary and autonomous agents. Neither their internal politics nor their relations with other states bear on this dispute.[4]

[2]An alternative would be to assume that one state is a Stackelberg leader along the lines of the models in Dmitriy Gershenson and Grossman (2000) and in Grossman and Mendoza (2004).

[3]In contrast Alberto Alesina and Enrico Spolaore (1997, 2003, 2004) analyze models in which the preferences of inhabitants for public goods determine the incorporation of regions into sovereign states.

[4]Recent studies of the relation between politics and interstate war include Michelle Garfinkel (1994), Gregory Hess and Athanasios Orphanides (1995, 2001), and Paul Huth (1996). In a study of civil conflicts Barbara Walter (2002) argues that in the past the intervention of third parties has been critical in making peaceful settlements credible.

- In evaluating the costs of a war the states consider only the amounts spent to mobilize resources. The states ignore the possible havoc of war.[5]
- In choosing its strategy set neither state faces a binding constraint on its ability to mobilize resources. This assumption implies that no elements of the strategy set call for a total mobilization of resources.
- All relevant parameters are common knowledge.

This last assumption implies that the states can correctly assess the credibility of each other's promises not to attack. Hence, this model abstracts from the possibility of a surprise attack that contravenes a peaceful settlement. In this model war occurs only if the states fail to reach a peaceful settlement.

To focus on the essential elements of the choice between peace and war, the analysis also initially makes the following working assumptions:

- The whole of the territory is neither more not less valuable than the sum of its parts.
- The winner of a war would get permanent control of the contested territory.
- Fortifications neither depreciate nor become obsolete.
- The states have the same effectiveness of amounts spent to mobilize resources.

Eventually the analysis relaxes each of these working assumptions.

The Consequences of a Default to War

Let $N_i, \quad i \in \{1, 2\}$, denote the expected payoff for State i from a default to war. Given that the winner of a war would get permanent control of the contested territory, we have

$$N_i = p_i V - R_i, \tag{1}$$

where p_i, $p_i \in [0, 1]$, denotes the probability that State i would win a war that these

[5] If a war could cause significant havoc, then taking account of the expected havoc would decrease the expected payoff from a default to war and would enhance the possibility of a peaceful settlement.

states fight by default, where V, $V \in (0, \infty)$, denotes the present value for either state of having permanent control of the contested territory, and where R_i, $R_i \in (0, V)$, denotes the amount that State i would spend to mobilize resources in the event of a default to war. Equation (1) says that the expected payoff for State i from a default to war equals the product of the probability that it would win the war and the value of having permanent control of the contested territory, minus the amount that it would spend to mobilize resources.

To determine the probability that State i would win a war that these states fight by default, assume that p_i depends on R_i and R_j, $j \in \{1, 2\}$, $j \neq i$, according to the contest-success function,

$$p_i = \frac{R_i}{R_i + R_j}. \tag{2}$$

In equation (2) p_i is an increasing concave function of the ratio, R_i/R_j. Equation (2) incorporates both the assumption that the dispute is newly arisen and, hence, that the contested territory is initially unfortified and the working assumption that the states have the same effectiveness of amounts spent to mobilize resources.[6]

In the event of a default to war, to maximize its expected payoff State i would choose R_i to satisfy the first-order condition,[7]

$$\frac{dN_i}{dR_i} = V \frac{\partial p_i}{\partial R_i} - 1 = 0.$$

Using equation (2) to calculate $\partial p_i / \partial R_i$, this first-order condition implies the reaction function,

[6]Related analyses of the causes of war in Fearon (1995) and in Grossman (2004) take the probabilities of winning a war and the costs of a war as given. In contrast, the present analysis, which follows the lead of Garfinkel (1990) and Robert Powell (1993), starts by specifying contest-success functions, given by equation (2) and by equation (9) below, and then shows how the probabilities of winning a war and the credibility of promises not to attack derive from the choices that states would make to mobilize resources.

[7]This first-order condition, as well as other first-order conditions specified below, accords with the assumption that neither state faces a binding constraint on its ability to mobilize resources.

$$R_i = \sqrt{VR_j} - R_j. \tag{3}$$

Solving equations (3) for R_1 and R_2 and substituting into equations (1) and (2) we obtain

$$R_1 = R_2 = V/4 \quad \text{and} \quad N_1 = N_2 = V/4. \tag{4}$$

Equations (4) imply that in a default to war spending to mobilize resources would dissipate half of the value of having permanent control of the contested territory.

A Peaceful Settlement

Suppose that these states, seeking an alternative that is preferable to a default to war, were to attempt to settle their territorial dispute peacefully. Specifically, suppose that these states were to consider a peaceful settlement with three provisions:

- The states divide control of the contested territory with State i getting permanent control of the non-negative fraction k_i, where $k_i + k_j = 1$.
- Each state can construct a permanent fortification on the resulting border. State i spends R_i^*, $R_i^* \in [0, \infty)$, to construct its fortification. If R_i^* equals zero, then the border is unfortified.
- Each state promises not to attack in an attempt to gain control of the entire contested territory.

State i would prefer this peaceful settlement to a default to war only if its payoff from this peaceful settlement would be larger than its expected payoff from a default to war.[8] Given the working assumption that the whole of the territory is neither more not less valuable than

[8] Powell (1993) considers the possibility of a peaceful settlement with a fortified border, but in this analysis he takes the division of the contested territory as given, rather than as subject to negotiation as in the present analysis. Powell (1999) considers the possibility of a negotiated division of the contested territory, but in this analysis he takes each state's prospects for success in war as given.

the sum of its parts, the value of permanently controlling the fraction k_i of the contested territory would be $k_i V$, and the payoff to State i from this peaceful settlement would be $k_i V - R_i^*$. Accordingly, State i would prefer this peaceful settlement to a default to war only if k_i is large enough and R_i^* is small enough that this peaceful settlement satisfies

$$k_i V - R_i^* \ > \ N_i, \tag{5}$$

where, from equation (4), N_i equals $V/4$.

Given that $k_1 + k_2 = 1$, condition (5) is satisfied for both State 1 and State 2 if and only if R_1^*, R_2^*, and k_1 satisfy

$$1 - \frac{N_2 + R_2^*}{V} \ > \ k_1 \ > \ \frac{N_1 + R_1^*}{V}. \tag{6}$$

With both N_1 and N_2 and both R_1 and R_2 equal to $V/4$, there exists at least one value of k_1 that can satisfy condition (6) if and only if the sum, $R_1^* + R_2^*$, satisfies

$$R_1^* + R_2^* \ < \ V/2 \ = \ R_1 + R_2. \tag{7}$$

Condition (7) says that there exists a division of the contested territory such that both states would prefer a peaceful settlement to a default to war only if the total amount that the two states would spend to construct fortifications under this peaceful settlement would be smaller than total amount that the two states would spend to mobilize resources in the event of a default to war.

The Expected Payoff from Attacking

In a peaceful settlement each state promises not to attack in an attempt to gain control of the entire contested territory. With $R_1^* + R_2^*$ smaller than $R_1 + R_2$, would such promises be credibility? Given that the states choose their strategy sets to maximize their expected payoffs, a promise by State i not to attack would be credible only if the expected payoff for State i from keeping its promise would at least as large as its expected payoff from breaking its promise and attacking.

Let $\tilde{N}_i$ denote the expected payoff to State i from breaking its promise not to attack. Given that the winner of a war would get permanent control of the contested territory, and that State i already has spent R_i^* to fortify the border, we have

$$\tilde{N}_i = \tilde{p}_i V - (R_i^* + \tilde{R}_i), \tag{8}$$

where $\tilde{p}_i$, $\tilde{p}_i \in [0,1]$, denotes the probability that, if State i attacks, it would win the resulting war, and where $\tilde{R}_i$, $\tilde{R}_i \in (0,V)$, denotes the amount that State i would spend to mobilize resources for an attack.

Let R_j^{**}, $R_j^{**} \in (0,V)$, denote the amount that, if State i were to attack, State j would spend to mobilize resources for a counterattack. In this formulation resources mobilized for a counterattack differ from fortifications in that states decide to mobilize resources for a counterattack only in response to an attack. In other words, State j would choose R_j^{**} as a reaction to $\tilde{R}_i$, whereas State j spends R_j^* to fortify the border before State i chooses whether or not to attack.

To determine the probability that, if State i attacks, it would win the resulting war, assume that $\tilde{p}_i$ depends on $\tilde{R}_i$, R_j^*, and R_j^{**} according to the contest-success function,

$$\tilde{p}_i = \frac{\theta \tilde{R}_i}{\theta \tilde{R}_i + \phi R_j^* + \gamma R_j^{**}}, \quad \theta \in [0,\infty), \quad \phi \in [0,\theta), \quad \gamma \in [0,\theta). \tag{9}$$

In equation (9) the parameter θ measures the effectiveness of amounts spent to mobilize resources for an attack, the parameter ϕ measures the effectiveness of amounts spent to construct fortifications, and the parameter γ measures the effectiveness of amounts spent to mobilize resources for a counterattack. The specification that ϕ is smaller than θ formalizes the assumption that attacking has an advantage over defending. The specification that γ is smaller than θ formalizes the assumption that attacking has an advantage over counterattacking. In accord with our working assumption that the states have the same effectiveness of amounts spent to mobilize resources, equation (9) assumes that θ, ϕ, and γ are the same for both states.

Given that both γ and ϕ are smaller than θ, each state would agree to a peaceful settlement only if the other state's promise not to attack is credible. Specifically, State j would have a larger expected payoff from a default to war than it would have if it were to agree to a peaceful settlement and State i were to break its promise not to attack.

If State i were to attack, then to maximize its expected payoff from attacking, taking as given the strength of the fortifications and counterattacks with which it would have to contend, State i would choose $\tilde{R}_i$ to satisfy the following first-order condition:

Either $\dfrac{d\tilde{N}_i}{d\tilde{R}_i} = 0$ and $\tilde{R}_i > 0$, or $\dfrac{d\tilde{N}_i}{d\tilde{R}_i} \leq 0$ and $\tilde{R}_i = 0$, where $\dfrac{d\tilde{N}_i}{d\tilde{R}_i} = V\,\dfrac{\partial \tilde{p}_i}{\partial \tilde{R}_i} - 1$.

Using equation (9) to calculate $\partial \tilde{p}_i / \partial \tilde{R}_i$, this first-order condition implies the reaction function,

$$\theta \tilde{R}_i \;=\; \max\left\{\sqrt{\theta V(\phi R_j^* + \gamma R_j^{**})} \;-\; (\phi R_j^* + \gamma R_j^{**}),\;\; 0\right\}. \tag{10}$$

If State i were to attack, how much would State j spend to mobilize resources for a counterattack? Let N_j^* denote the expected payoff to State j if State i were to attack. Given, again, that the winner of a war would get permanent control of the entire contested territory, we have

$$N_j^* = (1 - \tilde{p}_i)V - (R_j^* + R_j^{**}). \tag{11}$$

If State i were to attack, then State j to maximize its expected payoff from a counterattack, having already spent R_j^* to fortify the border, and taking $\tilde{R}_i$ as given, would choose R_j^{**} to satisfy the following first-order condition:

Either $\dfrac{dN_j^*}{dR_j^{**}} = 0$ and $R_j^{**} > 0$, or $\dfrac{dN_j^*}{dR_j^{**}} \leq 0$ and $R_j^{**} = 0$, where $\dfrac{dN_j^*}{dR_j^{**}} = -V\,\dfrac{\partial \tilde{p}_i}{\partial R_j^{**}} - 1$.

Using equation (9) to calculate $\partial \tilde{p}_i / \partial R_j^{**}$, this first-order condition implies the reaction function,

$$\gamma R_j^{**} \;=\; \max\left\{\sqrt{\gamma V \theta \tilde{R}_i} \;-\; (\theta \tilde{R}_i + \phi R_j^*),\;\; 0\right\}. \tag{12}$$

Combining equations (10) and (12) yields the following solutions for $\tilde{R}_i$ and R_j^{**}:

$$\theta\tilde{R}_i = \begin{cases} \left(\dfrac{\theta}{\theta+\gamma}\right)^2 \gamma V & \text{for} \quad \phi R_j^* < \left(\dfrac{\gamma}{\theta+\gamma}\right)^2 \theta V \\ \sqrt{\theta V \phi R_j^*} - \phi R_j^* & \text{for} \quad \left(\dfrac{\gamma}{\theta+\gamma}\right)^2 \theta V \leq \phi R_j^* \leq \theta V \\ 0 & \text{for} \quad \phi R_j^* > \theta V. \end{cases} \tag{13}$$

$$\gamma R_j^{**} = \max\left\{\left(\frac{\gamma}{\theta+\gamma}\right)^2 \theta V - \phi R_j^*, \ 0\right\}. \tag{14}$$

In equation (13) the first line accounts for the possibility that R_j^* is so small that both $\tilde{R}_i$ and R_j^{**} would be positive, the second line accounts for the possibility that R_j^* is large enough that, although $\tilde{R}_i$ would be positive, R_j^{**} would be zero, and the third line accounts for the possibility that R_j^* is so large that both $\tilde{R}_i$ and R_j^{**} would be zero.

Substituting equations (9), (13), and (14) into equation (8) to determine the payoff that State i would expect if it were to break its promise not to attack, we obtain

$$\tilde{N}_i = \begin{cases} \left(\dfrac{\theta}{\theta+\gamma}\right)^2 V - R_i^* & \text{for} \quad \phi R_j^* < \left(\dfrac{\gamma}{\theta+\gamma}\right)^2 \theta V \\ \left(\sqrt{V} - \sqrt{\phi R_j^*/\theta}\,\right)^2 - R_i^* & \text{for} \quad \left(\dfrac{\gamma}{\theta+\gamma}\right)^2 \theta V \leq \phi R_j^* \leq \theta V \\ -R_i^* & \text{for} \quad \phi R_j^* > \theta V. \end{cases} \tag{15}$$

Credibility Conditions

Recalling that the payoff to State i from a peaceful settlement would be $k_i V - R_i^*$, the promise of State i not to attack would be credible only if k_i and R_j^* taken together are large enough to satisfy the credibility condition, $k_i V - R_i^* \geq \tilde{N}_i$. Equivalently we can express this credibility condition as

$$k_i \geq Z_i, \quad \text{where} \quad Z_i \equiv \frac{\tilde{N}_i + R_i^*}{V}. \tag{16}$$

If condition (16) is satisfied, then for State i the payoff from a peaceful settlement is at least as large as the expected payoff from attacking. In other words, if condition (16) is satisfied, then for State i the possibility of gaining $k_j V$ if it attacks and then wins the resulting war does not outweigh the cost of mobilizing resources for an attack together with the possibility of losing $k_i V$ if it attacks and does not win the resulting war.

Given that $k_1 + k_2 = 1$, condition (16) is satisfied both for State 1 and for State 2 if and only if k_1 satisfies

$$1 - Z_2 \geq k_1 \geq Z_1. \tag{17}$$

There exists at least one value of k_1 that satisfies condition (17) if and only if the sum, $Z_1 + Z_2$, is not larger than one.

An Unfortified Border?

Suppose that the states are considering a peaceful settlement with an unfortified border. With both R_1^* and R_2^* equal to zero, equation (15) and the definition of Z_i imply that both Z_1 and Z_2 equal $\left[\theta/(\theta+\gamma)\right]^2$. Hence, both R_1^* and R_2^* equal to zero is consistent with $Z_1 + Z_2$ being not larger than one if and only if the ratio of the parameters γ and θ satisfies

$$\frac{\gamma}{\theta} \geq \sqrt{2} - 1. \tag{18}$$

Condition (18) implies the following proposition:

(I) *If amounts spent to mobilize resources for a counterattack would be sufficiently effective relative to amounts spent to mobilize resources for an attack, then the states can reach a credible peaceful settlement with an unfortified border.*[9,10]

[9]Even if condition (18) is satisfied, a peaceful settlement with an unfortified border is not a unique (Nash) equilibrium. But, if condition (18) is satisfied, then we can presume that the promises of both states not to attack serve to allow the states to coordinate on this equilibrium.

[10]The model developed in Grossman (2004) and applied to the American Civil War abstracts from the

A Peaceful Settlement with a Fortified Border?

Suppose that the ratio γ/θ does not satisfy condition (18). In that case we see from equation (15) and the definition of Z_i that, in addition to not being consistent with both R_1^* and R_2^* equal to zero, the sum, $Z_1 + Z_2$, being not larger than one would not be consistent with both ϕR_1^* and ϕR_2^* being smaller than $\theta V\Big[\gamma/(\theta+\gamma)\Big]^2$. Accordingly, the states would be limited to considering peaceful settlements with both ϕR_1^* and ϕR_2^* not smaller than $\theta V\Big[\gamma/(\theta+\gamma)\Big]^2$.

With ϕR_j^* not smaller than $\theta V\Big[\gamma/(1+\gamma)\Big]^2$, we have, from equation (15) and the definition of Z_i,

$$(19) \qquad Z_i \;=\; \begin{cases} \dfrac{\phi R_j^*}{\theta V} \;-\; 2\sqrt{\dfrac{\phi R_j^*}{\theta V}} \;+\; 1 & \text{for} \quad \left(\dfrac{\gamma}{\theta+\gamma}\right)^2 \theta V \;\le\; \phi R_j^* \;\le\; \theta V \\ 0 & \text{for} \quad \phi R_j^* \;>\; \theta V. \end{cases}$$

Solving the quadratic equation obtained by setting $Z_1 + Z_2$ equal to one, we find that the minimum values of the R_1^* and R_2^* that would satisfy the condition, $Z_1 + Z_2 \le 1$, are

$$(20) \qquad R_1^* \;= R_2^* \;=\; \frac{\theta V}{\phi}\left(1 - \frac{1}{\sqrt{2}}\right)^2.$$

Recall that for both states the payoff from a peaceful settlement would be larger than the expected payoff from a default to war only if the sum, $R_1^* + R_2^*$, is smaller than $R_1 + R_2$, which equals $V/2$. For $R_1^* + R_2^*$, as implied by equation (20), to be smaller than $V/2$, the ratio of the parameters ϕ and θ must satisfy

$$(21) \qquad \frac{\phi}{\theta} \;>\; 4\left(1 - \frac{1}{\sqrt{2}}\right)^2 \;\equiv\; \left(2 - \sqrt{2}\right)^2.$$

Condition (21) implies the following proposition:

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distinction between attacking and counterattacking. Hence, that model implicitly assumes that γ equals θ, and, accordingly, that the analog of condition (18) is satisfied. That model focuses on overoptimism about the prospects of winning a war cheaply as a necessary part of an explanation for the choice to go to war.

(II) *If a peaceful settlement with an unfortified border would not be credible, but if amounts spent to construct fortifications would be sufficiently effective relative to amounts spent to mobilize resources for an attack, then the states can reach a peaceful settlement with a fortified border that is credible and preferable to a default to war.*

In addition, equation (20) implies that in such a peaceful settlement the amount that each state would have to spend to construct fortifications would be a decreasing function of ϕ/θ. Figure 1 depicts the derivation of this result.

According to Proposition (I) the possibility of a credible peaceful settlement with an unfortified border depends on the relative effectiveness of amounts spent to mobilize resources for a counterattack, whereas according to Proposition (II) the possibility of a credible peaceful settlement with a fortified border depends on the relative effectiveness of amounts spent to construct fortifications. The converse of Propositions (I) and (II) is also worth emphasizing.

(III) *If both amounts spent to mobilize resources for a counterattack and amounts spent to construct fortifications would be ineffective relative to amounts spent to mobilize resources for an attack, then the states cannot reach a credible peaceful settlement, with the resulting border either unfortified or fortified, that would be preferable to a default to war.*

Figure 2 depicts conditions (18) and (21) and Propositions (I), (II), and (III).

Divisibility

We now turn to relaxing our working assumptions, beginning with the assumption that the whole of the territory is neither more nor less valuable than the sum of its parts. To be more general, assume that the value of permanently controlling the fraction k_i of the contested territory would be $k_i^{1/\sigma}V$, where $\sigma \in (0,\ 1]$. The parameter, σ, calibrates the divisibility of the territory.

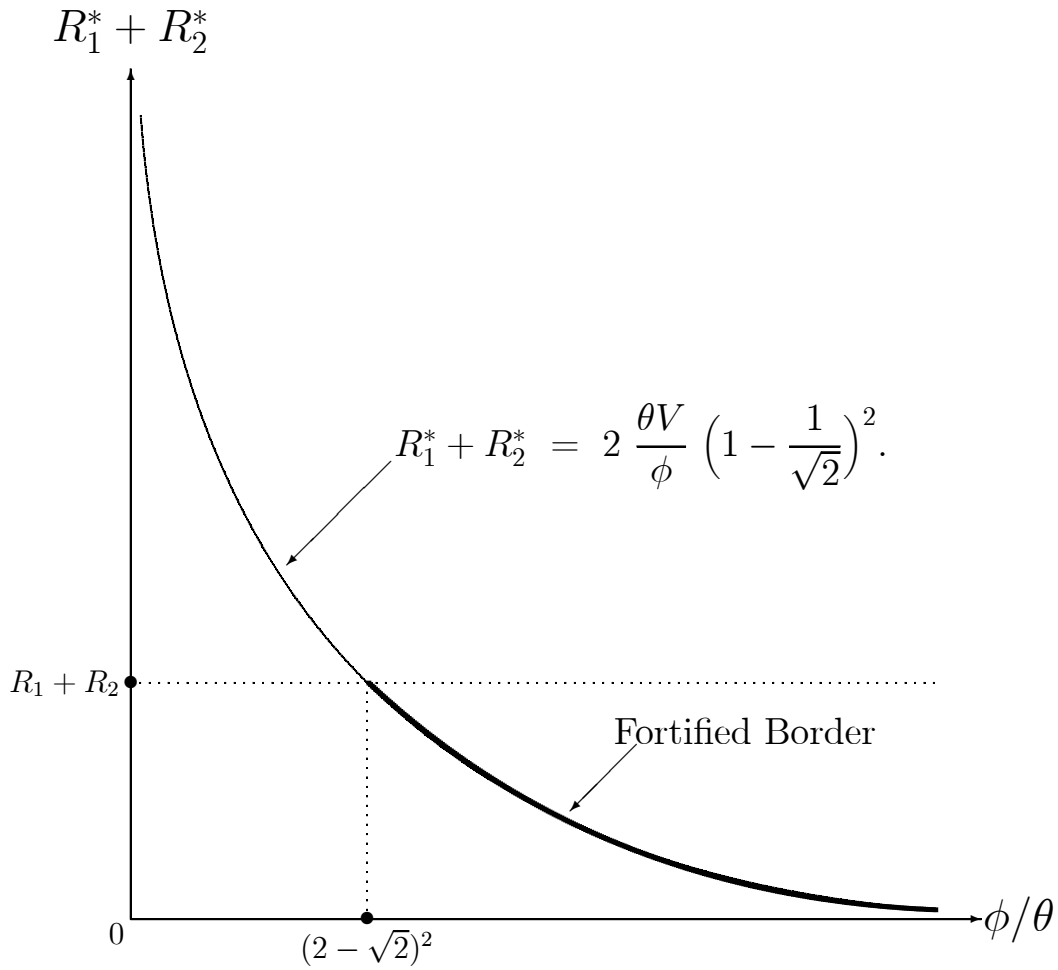

Figure 1: A Peaceful but Fortified Border?

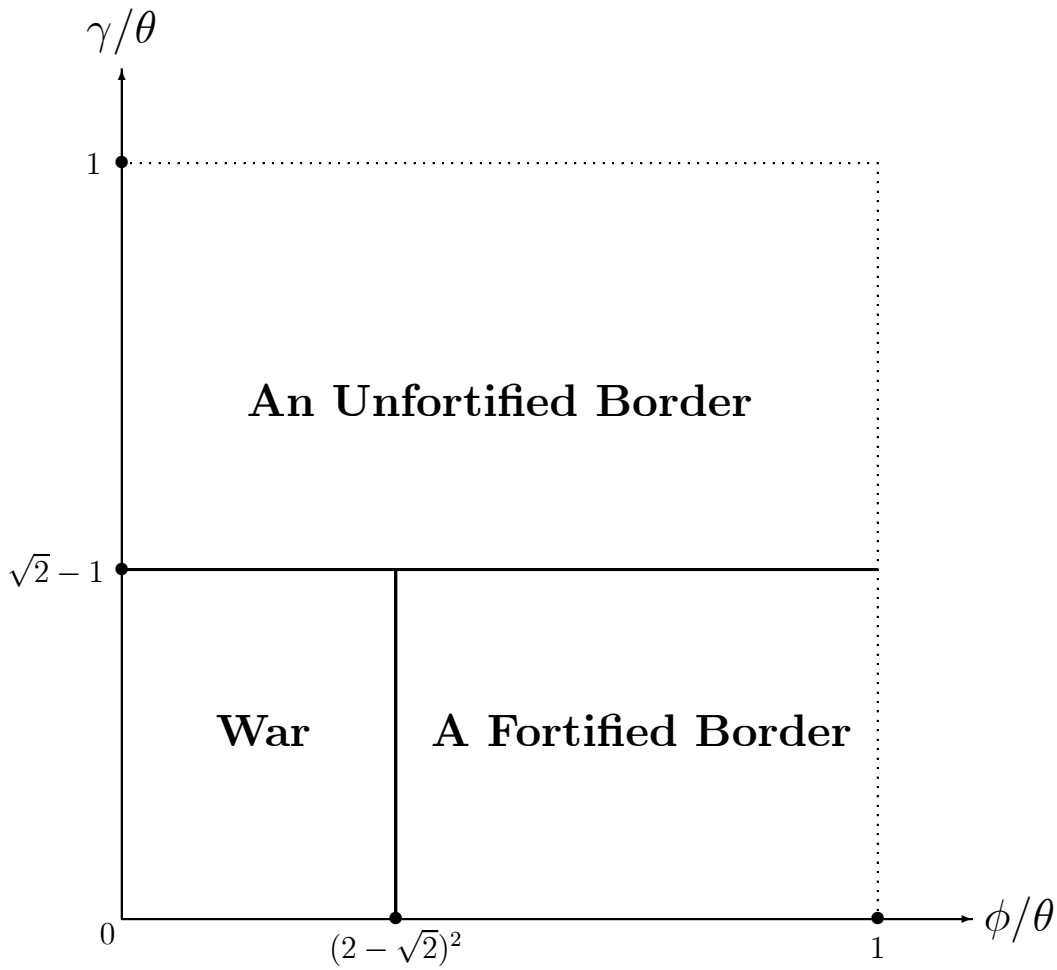

Figure 2:

An Unfortified Border, a Fortified Border, or War?

If σ equals one, as implicitly we have been assuming, the value of controlling the fraction k_i of the territory would be exactly equal to k_i times the value of controlling the entire territory. In this case dividing the territory would be costless.

In contrast, if σ is smaller than one, then the value of controlling the fraction k_i of the territory would be smaller than k_i times the value of controlling the entire territory. In this case the whole of the territory is more valuable than the sum of its parts, and dividing the territory would be costly.[11]

Given that the winner of a war would gain permanent control of the contested territory, the expected payoffs to a default to war and to breaking a promise not to attack do not depend on σ. But, as σ becomes smaller than one, the payoff to any peaceful settlement that divides control of the contested territory becomes smaller.[12]

To analyze the effect of costly divisibility, begin by replacing k_i in condition (5) with $k_i^{1/\sigma}$. With this generalization condition (6) becomes

$$(6\sigma) \qquad 1 - \left(\frac{N_2 + R_2^*}{V}\right)^{\sigma} > k_1 > \left(\frac{N_1 + R_1^*}{V}\right)^{\sigma}.$$

There exists at least one value of k_1 that can satisfy condition (6σ) if and only if the sum, $\left[(N_1 + R_1^*)/V\right]^{\sigma} + \left[(N_2 + R_2^*)/V\right]^{\sigma}$, is smaller than one.

Using condition (6σ) suppose that σ is equal to or smaller than one half. With this parameterization, controlling half of the territory would be one quarter or less as valuable as controlling the entire territory. With σ equal to or smaller than one half, and given that both N_1 and N_2 equal $V/4$, the set of nonnegative values of R_1^* and R_2^* such

[11] As σ approaches zero, the territory becomes indivisible. An example would be a territory that either one of the states perceives, for either geopolitical or symbolic reasons, to be essential for its survival.

[12] If side payments in the form of monetary or other compensation are possible, then the states could reach a peaceful settlement that avoids a costly division of a contested territory. Fearon (1995) argues that side payments "typically" are feasible. Hence, he claims that costly divisibility does not provide a "compelling" reason for failure to settle territorial disputes peacefully. Other scholars, however, claim that in fact indivisibility often prevents peaceful settlements. See, for example, Monica Duffy Toft (2003).

that $\left[(N+R_1^*)/V\right]^\sigma + \left[(N+R_2^*)/V\right]^\sigma$ is smaller than one is empty. Thus, condition (6σ) implies that, if σ were equal to or smaller than one half, then, because the winner of a war would control the entire contested territory, the payoff from a peaceful settlement, even with the resulting border unfortified, could not be larger for both states than the expected payoff from a default to war. Thus, we have the following extension of Proposition (III):

> (IIIσ) *If the whole of the contested territory is worth sufficiently more than the sum of its parts, then the states cannot reach a peaceful settlement, with the resulting border either unfortified or fortified, that would be preferable to a default to war.*

Suppose, alternatively, that σ is larger than one half. In this case, condition (6σ) implies that there exist peaceful settlements that both states would prefer to a default to war. But, would such peaceful settlements be credible?

To answer this question, replace k_i in condition (16) with $k_i^{1/\sigma}$. With this generalization condition (17) becomes

$$(17\sigma) \qquad 1 - Z_2^\sigma \ \geq k_1 \ \geq \ Z_1^\sigma .$$

There exists at least one value of k_1 that satisfies condition (17σ) if and only if the sum, $Z_1^\sigma + Z_2^\sigma$, is not larger than one.

Consider again the possibility of a peaceful settlement with an unfortified border. From equation (15) and the definition of Z_i we find that both R_1^* and R_2^* equal to zero is consistent with the sum, $Z_1^\sigma + Z_2^\sigma$, being not larger than one if and only if the parameters γ, θ, and σ satisfy the following generalization of condition (18):

$$(18\sigma) \qquad \frac{\gamma}{\theta} \ \geq \ 2^{1/2\sigma} - 1.$$

Condition (18σ) implies the following generalization of Proposition (I):

(Iσ) *If amounts spent to mobilize resources for a counterattack would be sufficiently effective relative to amounts spent to mobilize resources for an attack,* ***and if dividing the territory would not be too costly,*** *then the states can reach a credible peaceful settlement with an unfortified border.*

If γ/θ and σ do not satisfy condition (18σ), then the states again would be limited to considering peaceful settlements with both ϕR_1^* and ϕR_2^* not smaller than $\theta V \left[\gamma/(\theta+\gamma)\right]^2$. Solving the quadratic equation obtained by setting $Z_1^\sigma + Z_2^\sigma$ equal to one, we find that the minimum values of R_1^* and R_2^* that would satisfy the condition, $Z_1^\sigma + Z_2^\sigma \leq 1$, are

$$R_1^* = R_2^* = \frac{\theta V}{\phi}\left[1 - \left(\frac{1}{2}\right)^{1/2\sigma}\right]^2. \tag{20σ}$$

According to condition (6σ), both states would prefer a peaceful settlement to a default to war only if R_1^* and R_2^* are sufficiently small that the sum, $\left[(N_1+R_1^*)/V\right]^\sigma + \left[(N_2+R_2^*)/V\right]^\sigma$, is smaller than one. For the minimum values of R_1^* and R_2^* given by equation (20σ) to satisfy this condition, the parameters ϕ, θ, and σ must satisfy

$$\frac{\phi}{\theta}\left[\left(\frac{1}{2}\right)^{1/\sigma} - \frac{1}{4}\right] > \left[1 - \left(\frac{1}{2}\right)^{1/2\sigma}\right]^2 \tag{21σ}$$

Condition (21σ) implies the following generalization of Proposition (II):

(IIσ) *If a peaceful settlement with an unfortified border would not be credible, but if amounts spent to construct fortifications would be sufficiently effective relative to amounts spent to mobilize resources for an attack,* ***and if dividing the territory would not be too costly,*** *then the states can reach a peaceful settlement with a fortified border that is credible and preferable to a default to war.*

In addition, equation (20σ) implies that in such a peaceful settlement the amount that each state would have to spend to construct fortifications would be a decreasing function of σ. Figures 3 and 4 depict Propositions (Iσ), (IIσ), and (IIIσ).

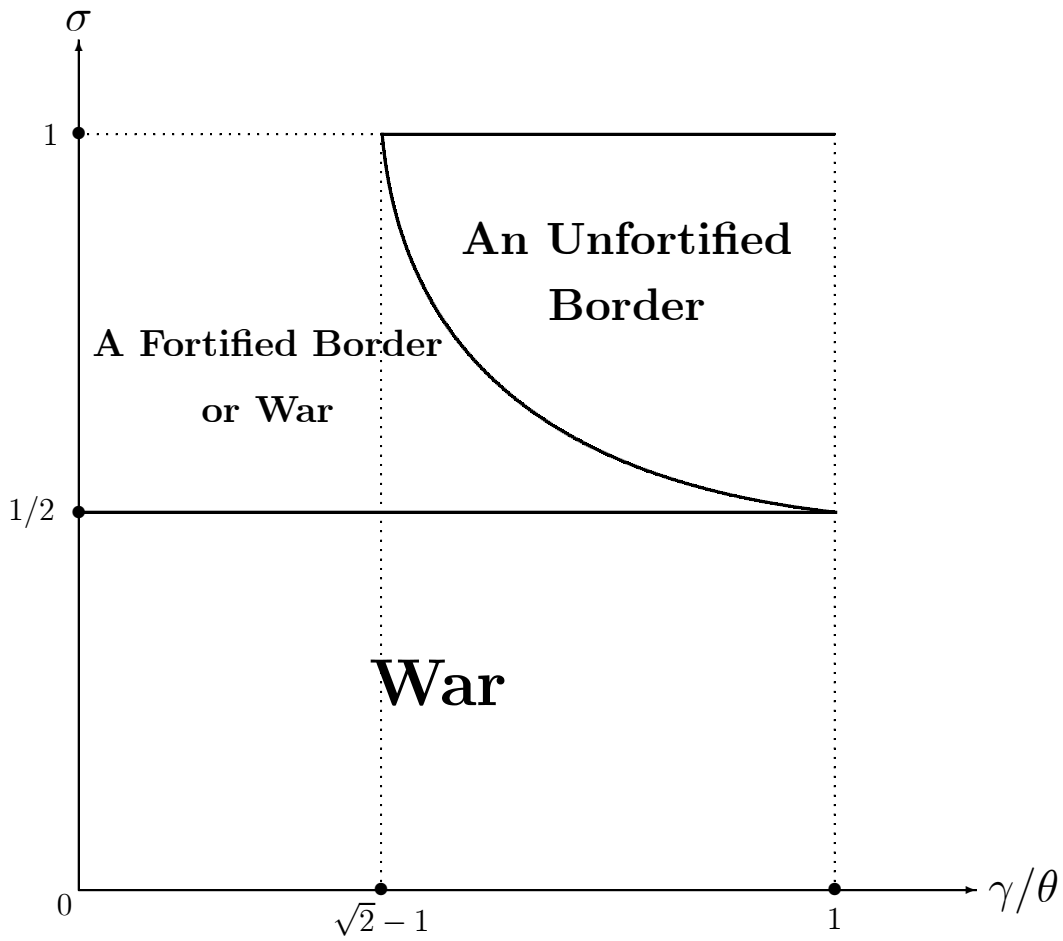

Figure 3:

An Unfortified Border with Costly Divisibility?

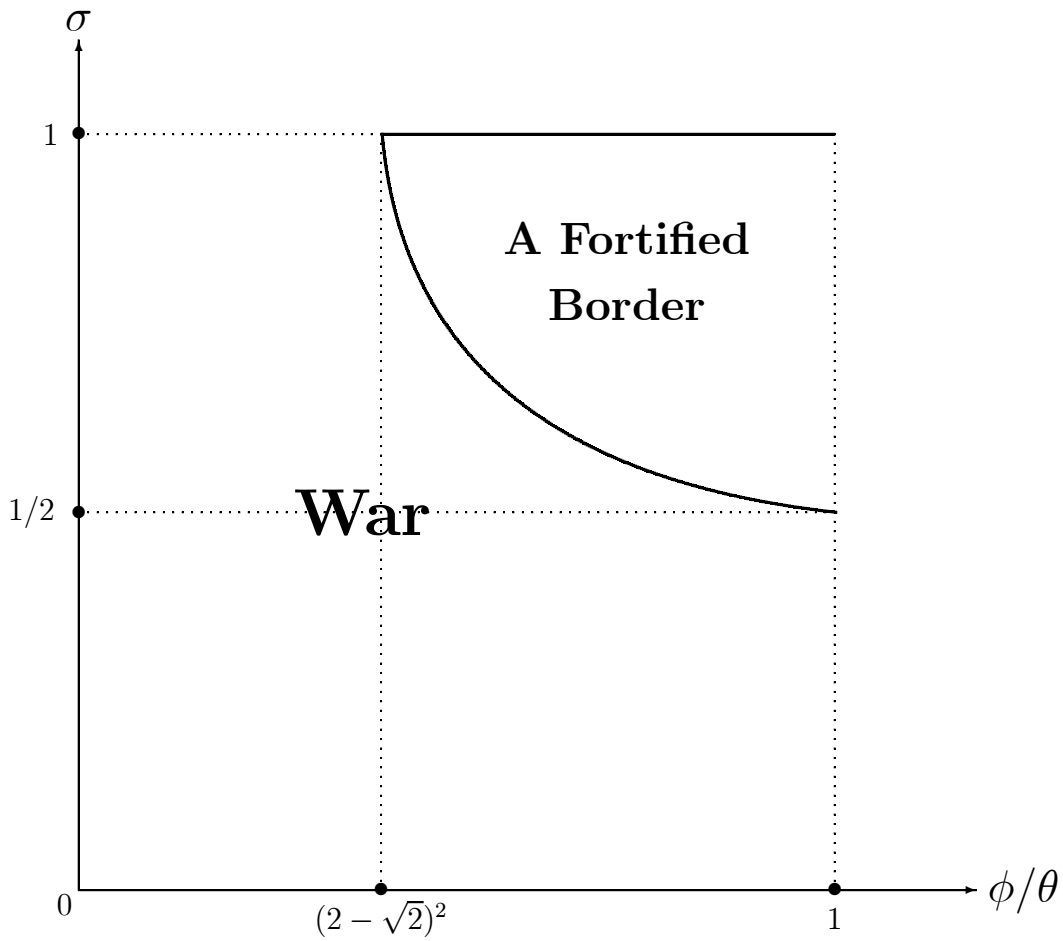

Figure 4:

A Fortified Border or War with Costly Divisibility?

A Recurring Possibility of War

So far we have assumed that a war would settle the territorial dispute permanently. To relax this assumption, assume that a state that wins a war would gain control of the contested territory only for a single finite period, where a period is the amount of time, measured in standard units of time, such as years, that it would take for the states to rearm and prepare for a new war. Under this assumption the states face a recurring possibility of war.

Let v, $v \in (0, \infty)$, denote the value of having control of the entire contested territory for a single period. Assuming that v does not vary over time, the present value, V, of having permanent control of the entire contested territory equals $v/(1-\rho)$, where ρ, $\rho \in [0, 1)$, is the discount factor that corresponds to the length of a period. Given V, the shorter is a period, the smaller is v, and the larger is ρ. Specifically, if q, $q \in [0, 1)$, is the discount factor applicable to a periodicity of one year, and if it would take m years to rearm and prepare for a new war, then ρ equals q^m. The working assumption that a war would settle the territorial dispute permanently corresponds to the limiting case of v equal to V, and ρ equal to zero.

To focus on the effects of a recurring possibility of war, abstract from the ability to counterattack by assuming that γ equals zero. Condition (18) implies that, under the assumption that the winner of a war would get permanent control of the contested territory, a credible peaceful settlement with an unfortified border would not be possible with γ equal to zero. Also, abstract from the problem of costly divisibility by again assuming that σ equals one. Condition (21) implies that, with σ equal to one, a peaceful settlement with a fortified border that is credible and preferable to a default to war requires that ϕ/θ be larger than $\left(2-\sqrt{2}\right)^2$.

Assuming that, in the event of a default to recurring war, the expected payoff for State i from each recurring war would be $p_i v - R_i$, equation (1) becomes

$$(1\rho) \qquad N_i = \frac{p_i v - R_i}{1-\rho} = p_i V - \frac{R_i}{1-\rho},$$

where N_i now denotes the expected payoff for State i from a default to recurring war. Given equation (1ρ), equations (4) become

$$(4\rho) \qquad R_1 \;=\; R_2 \;=\; v/4 \;=\; (1-\rho)V/4 \qquad \text{and} \qquad N_1 \;=\; N_2 \;=\; \frac{v/4}{1-\rho} \;=\; V/4.$$

Comparing equations (4ρ) with equations (4) we see that, given V, because the amounts that the states would spend to mobilize resources in the event of a default to war are proportionate to v, the expected payoff from a default to war does not depend on whether or not war would be recurring.

As in the standard theory of repeated games, assume that, with states facing a recurring possibility of war, as long as both states keep their promises not to attack, they can continue to make credible promises not to attack. But, if either state were to break its promise not to attack, then future peaceful settlements would be precluded. In that event, starting in the next period, the states would have to bear the costs of a recurring default to war.[13]

Under these assumptions the expected payoff to State i from breaking its promise not to attack becomes

$$(8\rho) \qquad \tilde{N}_i \;=\; \tilde{p}_i v \;-\; (R_i^* + \tilde{R}_i) \;+\; \rho V/4 \;=\; \tilde{p}_i(1-\rho)V \;-\; (R_i^* + \tilde{R}_i) \;+\; \rho V/4,$$

Equation (8ρ) amends equation (8) in accord with the assumption that a state that attacks and wins a war would gain control of the entire contested territory for the current period, but would expect to obtain in future periods only the expected value of a default to recurring war. Given equation (8ρ), and assuming that γ equals zero, equation (15) becomes

$$(15\rho) \quad \tilde{N}_i \;=\; \begin{cases} \left(\sqrt{(1-\rho)V} - \sqrt{\phi R_j^*/\theta}\,\right)^2 \;-\; R_i^* \;+\; \rho V/4 & \text{for} \quad \phi R_j^* \;\leq\; (1-\rho)\theta V \\ -R_i^* \;+\; \rho V/4 & \text{for} \quad \phi R_j^* \;>\; (1-\rho)\theta V, \end{cases}$$

[13] These assumptions accord with the Garfinkel's (1990) analysis of the possibility of armed peace. In Garfinkel's model states contest control over capital stocks that are endogenously determined. The present analysis is simplified by taking the value of controlling the contested territory to be exogenous.

Again the credibility condition is $k_i \geq Z_i$, where $Z_i \equiv (\tilde{N}_i + R_i^*)/V$, and again there exists at least one value of k_1 that satisfies these credibility conditions if and only if the sum, $Z_1 + Z_2$, is not larger than one.

Consider again the possibility of a peaceful settlement with an unfortified border. Given equation (15ρ) and the definition of Z_i, both R_1^* and R_2^* equal to zero is consistent with $Z_1 + Z_2$ being not larger than one if and only if the discount factor, ρ, satisfies

$$(18\rho) \qquad \rho \;\geq\; 2/3.$$

To interpret condition (18ρ), observe that if, for example, the annual discount factor equals about 9/10, then, because 2/3 equals approximately $(9/10)^4$, a value of ρ equal to or larger than 2/3 corresponds to a potential recurrence of war more frequently than approximately every four years.

Condition (18ρ) implies the following extension of Proposition (I).

(Iρ) *If war could recur with sufficient frequency, then the states can reach a credible peaceful settlement with an unfortified border, regardless of the relative effectiveness of amounts spent to mobilize resources for a counterattack.*

Suppose that ρ does not satisfy condition (18ρ). In that case, with γ equal to zero, the states would be limited to considering the possibility of a peaceful settlement with a fortified border. For positive values of R_1^* and R_2^* equation (19) becomes

$$(19\rho) \qquad Z_i \;=\; \begin{cases} \dfrac{\phi R_j^*}{\theta V} \;-\; 2\sqrt{(1-\rho)\dfrac{\phi R_j^*}{\theta V}} \;+\; 1 \;-\; \dfrac{3\rho}{4} & \text{for} \quad \phi R_j^* \;\leq\; (1-\rho)\theta V \\[2ex] \dfrac{\rho}{4} & \text{for} \quad \phi R_j^* \;>\; (1-\rho)\theta V. \end{cases}$$

Solving the quadratic equation obtained by setting $Z_1 + Z_2$ equal to one, we find that equation (20), which gives the minimum values of R_1^* and R_2^* that would satisfy the condition, $Z_1 + Z_2 \leq 1$, becomes

$$(20\rho) \qquad R_1^* \;= R_2^* \;=\; \frac{\theta V}{\phi}\left(\sqrt{1-\rho} \;-\; \sqrt{\frac{1}{2} - \frac{\rho}{4}}\right)^2.$$

Recall that, with σ equal to one, both states would prefer a peaceful settlement to a default to war only if $R_1^* + R_2^*$ is smaller than $V/2$, which now equals $(R_1 + R_2)/(1-\rho)$. For $R_1^* + R_2^*$, as implied by equation (20ρ), to be smaller than $V/2$, the parameters ϕ, θ, and ρ must satisfy

$$(21\rho) \qquad \frac{\phi}{\theta} > 4\left(\sqrt{1-\rho} - \sqrt{\frac{1}{2} - \frac{\rho}{4}}\right)^2.$$

Condition (21ρ) implies that a peaceful settlement with a fortified border that is credible and preferable to a default to war is possible even if ϕ/θ is smaller than $\left(2-\sqrt{2}\right)^2$. Thus, condition (21$\rho$) implies the following extension of Proposition (II):

> (IIρ) *If a peaceful settlement with an unfortified border would not be credible, but war could recur with sufficient frequency, then the states can reach a peaceful settlement with a fortified border that is credible and preferable to a default to war, even if amounts spent to construct fortifications would not be highly effective relative to amounts spent to mobilize resources for an attack.*

In addition, equation (20ρ) implies that in such a peaceful settlement the amount that each state would have to spend to construct fortifications would be a decreasing function of ρ.

Figure 5 depicts conditions (18ρ) and (21ρ) and Propositions (Iρ) and (IIρ). Both of these propositions are applications of standard results from the theory of repeated games.

Depreciation and Obsolescence

We now turn to the working assumption that fortifications neither depreciate nor become obsolete. To relax this assumption, assume that fortifications must be rebuilt periodically, where a period now is the useful lifetime of fortifications, measured in standard units of time, such as years. Accordingly, the present value of the cost of maintaining a fortification that entailed an initial expenditure of R_i^* would be $R_i^*/(1-\delta)$, where δ, $\delta \in [0,1)$, is the discount factor that corresponds to the periodicity with which fortifications must be

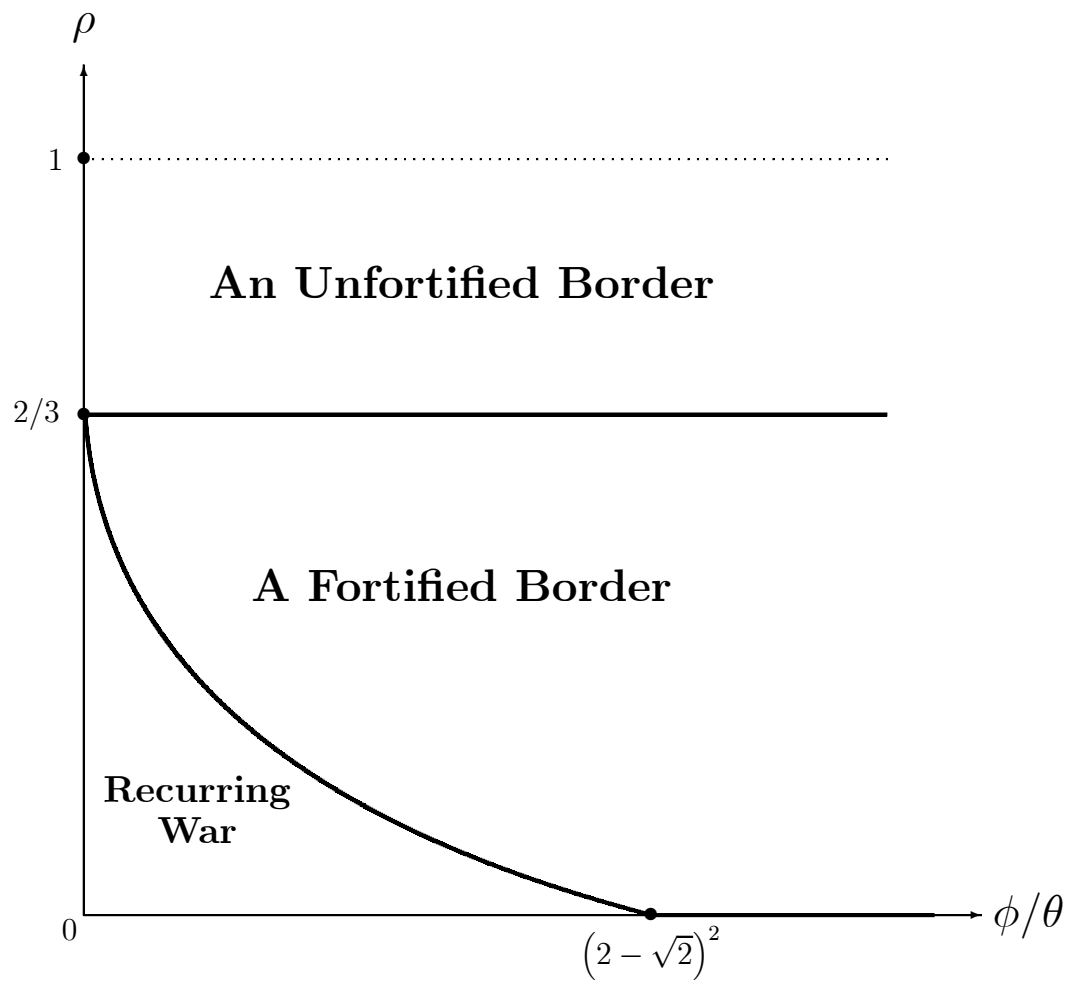

Figure 5:

An Unfortified Border, a Fortified Border, or Recurring War?

rebuilt. The shorter is a period the larger is δ. Specifically, if q is the discount factor applicable to a periodicity of one year, and if fortifications must be rebuilt every n years, then δ equals q^n. The working assumption that fortifications neither depreciate nor become obsolete corresponds to the limiting case of δ equal to zero.

To focus on the effects of depreciation and obsolescence, abstract from the problem of costly divisibility by again assuming that σ equals one. Also, assume again that the winner of a war would get permanent control of the contested territory.

With fortifications having to be rebuilt periodically, the payoff for State i from a peaceful settlement that required an initial expenditure of R_i^* on fortifications would be $k_iV - R_i^*/(1-\delta)$. Hence, condition (5) becomes

$$(5\delta) \qquad k_iV - R_i^*/(1-\delta) \;>\; N_i,$$

condition (6) becomes

$$(6\delta) \qquad 1 - \frac{N_2 + R_2^*/(1-\delta)}{V} \;>\; k_1 \;>\; \frac{N_1 + R_1^*/(1-\delta)}{V},$$

and condition (7) becomes

$$(7\delta) \qquad \frac{R_1^* + R_2^*}{1-\delta} \;<\; V/2 \;=\; R_1 + R_2.$$

Condition (7δ) says that both states would prefer a peaceful settlement to a default to war only if the present value of the cost for the two states of constructing and maintaining fortifications under a peaceful settlement would be smaller than total amount that the two states would spend to mobilize resources in the event of a default to war.

Would a peaceful settlement with $(R_1^* + R_2^*)/(1-\delta)$ smaller than $R_1 + R_2$ be credible? To answer this question observe that with the payoff from a peaceful settlement equal to $k_iV - R_i^*/(1-\delta)$, and with $\tilde{N}_i$ again given by equation (15), equation (16) becomes

$$(16\delta) \qquad k_i \;\geq\; Z_i, \quad \text{where} \quad Z_i \;\equiv\; \frac{\tilde{N}_i + R_i^*/(1-\delta)}{V}.$$

Again, there exists at least one value of k_1 that satisfies condition (17), $1-Z_2 \geq k_1 \geq Z_1$, if and only if the sum, $Z_1 + Z_2$, is not larger than one.

Depreciation and obsolescence of fortifications obviously do not affect the possibility of a credible peaceful settlement with an unfortified border. Hence, both R_1^* and R_2^* equal to zero is consistent with $Z_1 + Z_2$ being not larger than one if and only if again γ/θ satisfies condition (18), $\gamma/\theta \geq \sqrt{2} - 1$.

Suppose that γ/θ does not satisfy condition (18). In that case, the states again would be limited to considering the possibility of a peaceful settlement with a fortified border and with both ϕR_1^* and ϕR_2^* not smaller than $\theta V \left[\gamma/(\theta+\gamma)\right]^2$. With ϕR_j^* not smaller than $V \left[\gamma/(\theta+\gamma)\right]^2$, equation (19) becomes

$$(19\delta) \qquad Z_i = \begin{cases} \dfrac{\phi}{\theta}\dfrac{R_j^*}{V} - 2\sqrt{\dfrac{\phi R_j^*}{\theta V}} + 1 + \dfrac{\delta}{1-\delta}\dfrac{R_i^*}{V} & \text{for} \quad \phi R_j^* \leq \theta V \\ \dfrac{\delta}{1-\delta}\dfrac{R_i^*}{V} & \text{for} \quad \phi R_j^* > \theta V. \end{cases}$$

The quadratic equation obtained by setting $Z_1 + Z_2$, as implied by equation (19δ), equal to one implies that the minimum values of R_1^* and R_2^* that would satisfy the condition, $Z_1 + Z_2 \leq 1$, are

$$(20\delta) \qquad R_1^* = R_2^* = \frac{\theta V}{\phi}\left[\frac{1 - \sqrt{\frac{1}{2}\left(1 - \frac{\theta}{\phi}\frac{\delta}{1-\delta}\right)}}{1 + \frac{\theta}{\phi}\frac{\delta}{1-\delta}}\right]^2.$$

Equation (20δ) implies that there exist values of R_1^* and R_2^* that would satisfy the condition, $Z_1 + Z_2 \leq 1$, only if $\delta/(1-\delta)$ is not larger than ϕ/θ. Thus, equation (20δ) implies the following extension of Proposition (III):

(IIIδ) *If fortifications depreciate or become obsolete sufficiently rapidly, then the states cannot reach a peaceful settlement with a fortified border that would be credible and preferable to a default to war.*[14]

[14] The logic of this proposition is similar to the logic of the example in Garfinkel and Stergios Skaperdas

If $\delta/(1-\delta)$ is not larger than ϕ/θ, then for the sum of R_1^* and R_2^* implied by equation (20δ) to satisfy condition (7δ), the parameters ϕ, θ, and δ must satisfy

$$(21\delta) \qquad (1-\delta)\,\frac{\phi}{\theta} \;>\; 4\;\left[\;\frac{1\;-\;\sqrt{\frac{1}{2}\,(1-\frac{\theta}{\phi}\frac{\delta}{1-\delta})}}{1\;+\;\frac{\theta}{\phi}\;\frac{\delta}{1-\delta}}\;\right]^2.$$

The LHS of condition (21δ) is decreasing in δ and increasing in ϕ/θ, whereas the RHS of condition (21δ) is increasing in δ and decreasing in ϕ/θ. Thus, condition (21δ) implies that, even if $\delta/(1-\delta)$ is not larger than ϕ/θ, the larger is δ the larger must be ϕ/θ in order for a peaceful settlement with a fortified border to be possible. Thus, condition (21δ) implies the following extension of Proposition (II):

> (IIδ) *If fortifications do not either depreciate or become obsolete too rapidly, and if amounts spent to construct fortifications would be sufficiently effective relative to amounts spent to mobilize resources for an attack, then the states can reach a peaceful settlement with a fortified border that is credible and preferable to a default to war.*

Equation (20δ) also implies that in such a peaceful settlement the amount that each state would have to spend on fortifications would be an increasing function of δ.

Figure 6 depicts condition (21δ) and Proposition (IIδ). In interpreting Figure 6, observe that, if the annual discount factor equals about 9/10, then, because 7/16 equals approximately $(9/10)^8$, a value of δ smaller than 7/16 corresponds to a need to rebuild fortifications less frequently than approximately every eight years.

Unequal Effectiveness of Amounts Spent to Mobilize Resources

Finally we come to the working assumption that the two states have the same effectiveness of amounts spent to mobilize resources. To relax this assumption, let θ_i measure the

(2000) in which a war that would settle a dispute permanently can be preferable to an armed peace in which fortifications must be rebuilt periodically.

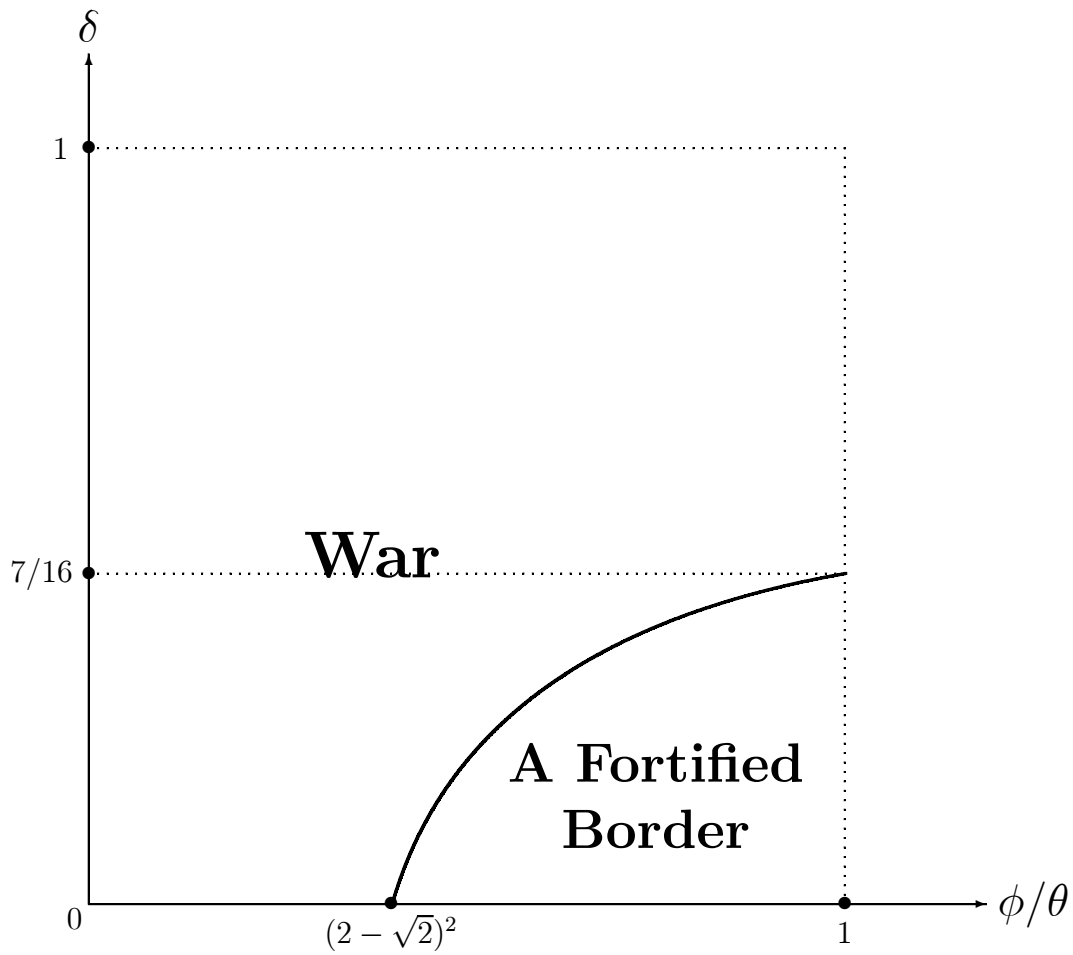

Figure 6: A Fortified Border or War with Depreciation or Obsolescence?

effectiveness of amounts spent by State i to mobilize resources either in the event of a default to war or in the event that State i were to break its promise not to attack, where θ_i and θ_j can be unequal, and let γ_i measure the effectiveness of amounts spent by State i to mobilize resources for a counterattack, where γ_i and γ_j can be unequal.

Consider again the possibility of a peaceful settlement with an unfortified border. If θ_i and θ_j can be unequal, equation (2), the contest-success function that applies in the event of a default to war, becomes

$$(2u) \qquad p_i = \frac{\theta_i R_i}{\theta_i R_i + \theta_j R_j}.$$

If, in addition, γ_i and γ_j can be unequal, then equation (9), the contest-success function that applies in the event that State i were to break its promise not to attack, becomes, with R_j^* equal to zero,

$$(9u) \qquad \tilde{p}_i = \frac{\theta_i \tilde{R}_i}{\theta_i \tilde{R}_i + \gamma_j R_j^{**}}.$$

Using equations $(2u)$ and $(9u)$ in place of equations (2) and (9), and assuming again that σ equals one and that δ and ρ equal zero, we find, as is shown in the mathematical appendix, that condition (18) generalizes to

$$(18u) \qquad \left(\frac{\theta_1}{\theta_1 + \gamma_2}\right)^2 + \left(\frac{\theta_2}{\theta_2 + \gamma_1}\right)^2 \leq 1.$$

Under the working assumption of equal effectiveness of amounts spent to mobilize resources, condition $(18u)$ would be identical to condition (18), $\gamma/\theta \geq \sqrt{2} - 1$. Thus, condition $(18u)$ implies that, if both γ_2/θ_1 and γ_1/θ_2 were equal to (or larger than) $\sqrt{2} - 1$, then a credible peaceful settlement with an unfortified border would be possible.

But, condition $(18u)$ also implies that, if the average of γ_2/θ_1 and γ_1/θ_2 were equal to $\sqrt{2} - 1$, but if γ_2/θ_1 was not equal to γ_1/θ_2, then a credible peaceful settlement with an unfortified border would not be possible. Thus, condition $(18u)$ implies the following extension of Proposition (I):

(Iu) *Unequal effectiveness of amounts spent to mobilize resources can preclude a peaceful settlement that would be credible if the two states had equal effectiveness of amounts spent to mobilize resources.*

Figure (7) illustrates Proposition (Iu). This proposition follows from the nonlinearity of the contest-success functions.

Summary

This paper has analyzed a choice-theoretic model in which a territorial dispute between sovereign states can afford either a peaceful settlement with an unfortified border, or, if not, perhaps a peaceful settlement with a fortified border, or possibly only a default to war. The essential premise of the model is that states can settle a dispute peacefully only if their payoffs from a peaceful settlement are larger than their expected payoffs from a default to war, and their promises not to attack are credible. The model assumes that all parameters are common knowledge, thereby abstracting from incomplete information as a cause of war.

The following are the main results of our analysis:

- A large advantage of attacking over counterattacking precludes a peaceful settlement with an unfortified border.
- A large advantage of attacking over defending precludes a peaceful settlement with a fortified border.
- A high cost of dividing the contested territory precludes a peaceful settlement with the border either unfortified or fortified.
- The possibility of recurring war enhances the possibility of a peaceful settlement with the border either unfortified or fortified.
- Rapid depreciation or obsolescence of fortifications precludes a peaceful settlement with a fortified border.
- Unequal effectiveness of amounts spent to mobilize resources can preclude a peaceful settlement with the border either unfortified or fortified.

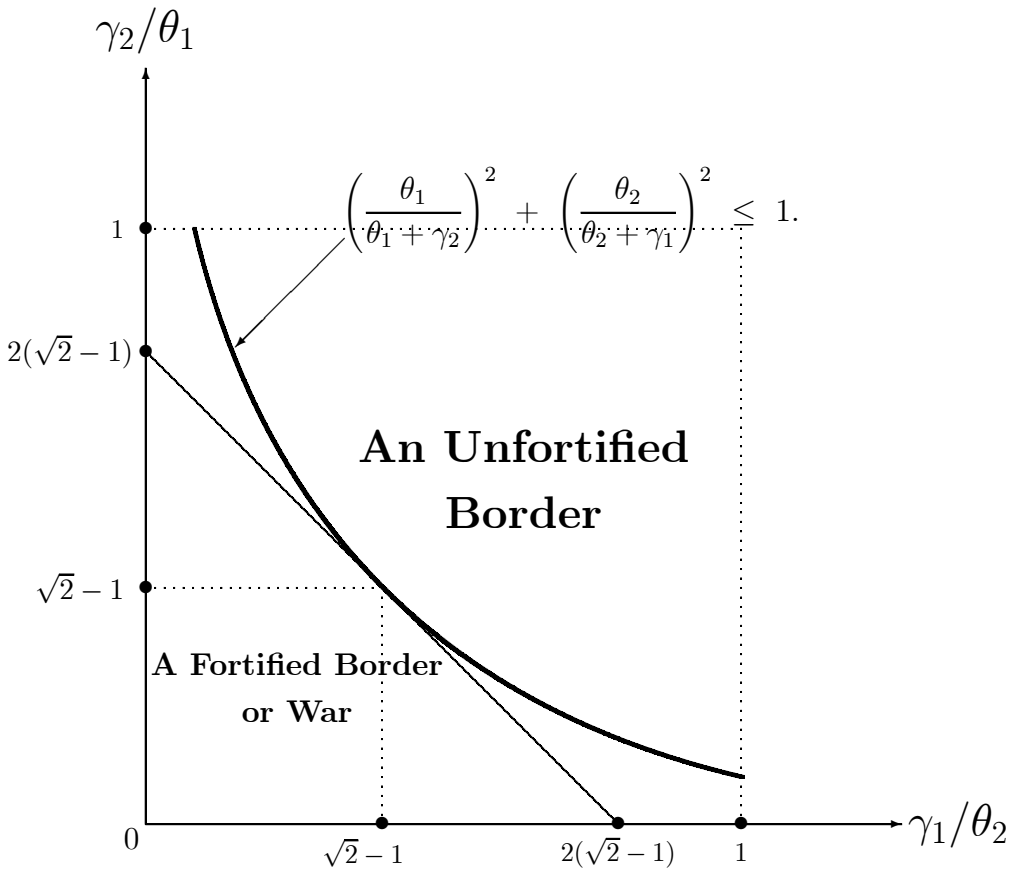

Figure 7: An Unfortified Border with Inequality in the Effectiveness of Mobilized Resources?

Mathematical Appendix

Derivation of Condition $(18u)$**:**

Using equations $(1u)$ and (2), the reaction function, equation (3), becomes

$$(3u) \qquad R_i \;=\; \sqrt{\frac{\theta_j}{\theta_i} V R_j} \;-\; \frac{\theta_j}{\theta_i} R_j,$$

and equations (4), which give the expected payoff to a default to war, become

$$(4u) \qquad R_i \;=\; \frac{\theta_i \theta_j}{(\theta_i + \theta_j)^2}\, V \quad \text{and} \quad N_i \;=\; \left(\frac{\theta_i}{\theta_i + \theta_j}\right)^2 V.$$

Using equations (8) and $(9u)$, equation (10) becomes, with R_j^* equal to zero,

$$(10u) \qquad \theta_i \tilde{R}_i \;=\; \max\left\{\sqrt{\theta_i V \gamma_j R_j^{**}} \;-\; \gamma_j R_j^{**},\;\; 0\right\}.$$

Using equations $(9u)$ and (11), equation (12), becomes, with R_j^* equal to zero,

$$(12u) \qquad \gamma_j R_j^{**} \;=\; \max\left\{\sqrt{\theta_i V \gamma_j \tilde{R}_i} \;-\; \theta_i \tilde{R}_i,\;\; 0\right\}.$$

Combining equations $(10u)$ and $(12u)$, equations (13) and (14) become

$$(13u) \qquad \tilde{R}_i \;=\; R_j^{**} \;=\; \frac{\theta_i \gamma_j}{(\theta_i + \gamma_j)^2}\, V,$$

and equation (15) becomes

$$(15u). \qquad \tilde{N}_i \;=\; \left(\frac{\theta_i}{\theta_i + \gamma_j}\right)^2 V,$$

With R_i^* equal to zero, the credibility condition (16) becomes

$$(16u) \qquad k_i \;\geq\; \tilde{N}_i / V.$$

Accordingly, condition (17) becomes

$$(17u) \qquad 1 - \tilde{N}_2/V \;\geq k_1 \;\geq\; \tilde{N}_1/V,$$

There exists at least one value of k_1 that satisfies condition $(17u)$ if and only if $(\tilde{N}_1 + \tilde{N}_2)/V$ is not larger than one. Hence, condition $(18u)$.

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