Arrow's Impossibility Theorem: Preference Diversity in a Single-Profile World

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Abstract

In this paper we provide a simple new version of Arrow's impossibility theorem, in a world with only one preference profile. This theorem relies on a new assumption of preference diversity, and we explore alternative notions of preference diversity at length.

Keywords: Arrow's theorem, single-profile, preference diversity.

JEL classifications: D60, D70, D71.

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1. Introduction.

In 1950 Kenneth Arrow (1950, 1963) provided a striking answer to a basic abstract problem of democracy: how can the preferences of many individuals be aggregated into social preferences? The answer, which has come to be known as Arrow's impossibility theorem, was that every conceivable aggregation method has some flaw. That is, a handful of reasonable-looking axioms, which one hopes an aggregation procedure would satisfy, lead to impossibility: the axioms are mutually inconsistent. The impossibility theorem created a large literature and major field called social choice theory; see for example, Suzumura's (2002) Introduction to the *Handbook of Social Choice and Welfare*, and Campbell and Kelly (2002) in the same volume. The theorem has also had a major influence on the larger fields of economics and political science, as well as on distant fields like mathematical biology. (See, e.g., Bay and McMorris (2003).)

Single-profile versions of Arrow's theorem, in which there is just one profile of individual preferences, were devised in response to an argument of Paul Samuelson (1967) against Arrow. Samuelson claimed that Arrow's model, with varying preference profiles, is irrelevant to the problem of maximizing a Bergson-Samuelson-type social welfare function (Bergson (1938)), which depends on a given set of ordinal utility functions, that is, a fixed preference profile. But single-profile Arrow theorems established that bad results (dictatorship, or illogic of social preferences, or, more generally, impossibility of aggregation) could be proved with one fixed preference profile (or set of ordinal utility functions), provided the profile is "diverse" enough. (See Parks

(1976), Hammond (1976), Kemp and Ng (1976), Pollak (1979), Roberts (1980), and Rubinstein (1984).)

This paper has two purposes. The first is to provide a short and transparent single-profile version of Arrow's theorem. In addition to being short and simple, our proof, unlike earlier proofs, does not require the existence of large numbers of alternatives. Our second and related purpose is to explore the meaning of preference profile diversity. In our theorem we will use a diversity assumption, which we call diversity under minimal decisiveness (D.M.D.), that is much weaker than the assumptions used by other authors. D.M.D. requires information about the social preference relation in order to be well defined; this makes it similar to other assumptions used in the theorem (e.g., Pareto, neutrality/monotonicity, no dictator), but different from earlier diversity assumptions. Its great advantage is that it is "almost necessary" for the impossibility result. In fact we offer a near-converse to our impossibility theorem: if there is a dictator, D.M.D. must hold.

In single-profile models, Arrow's independence of irrelevant alternatives assumption is vacuous (independence requires consistency as preference profiles are varied), and neutrality, or its stronger variant neutrality/monotonicity, takes its place (neutrality requires consistency as alternative pairs are varied within a fixed preference profile). We use the stronger neutrality/monotonicity assumption in our theorem.

Recent related literature includes Fleurbaey and Mongin (2005), who argue for returning to the Bergson-Samuelson social welfare function framework, Geanakoplos (2005), who has three very elegant proofs of Arrow's theorem in the standard multi-profile context, and Ubeda (2004), who has another elegant multi-profile proof. The proofs of Geanakoplos and Ubeda, while short, are mathematically more challenging than ours. Ubeda also emphasizes the importance of (multi-profile) neutrality, somewhat similar to the (single-profile) neutrality/monotonicity assumption we use in this paper, and much stronger than Arrow's independence, and he provides several theorems establishing neutrality's equivalence to other intuitively appealing principles. Reny (2001) has an interesting side-by-side pair of (multi-profile) proofs, of Arrow's theorem and the related theorem of Gibbard and Satterthwaite.

2. The Model.

We assume a society with $n \geq 2$ individuals, and 3 or more alternatives. A specification of the preferences of all individuals is called a preference profile. In our theorem there is only one preference profile. The preference profile is transformed into a social preference relation. Both the individual and the social preference relations allow indifference. The individual preference relations are all assumed to be complete and transitive. The following notation is used: Generic alternatives are *x, y, z, w*, etc. Particular alternatives are *a, b, c, d*, etc. A generic person is labeled *i, j, k* and so on; a particular person is 1, 2, 3, and so on. Person *i*'s preference relation is R_i. xR_iy means person *i* prefers *x* to *y* or is indifferent between them; xP_iy means *i* prefers *x* to *y*; xI_iy means *i* is indifferent between them. Society's preference relation is *R*. *xRy* means society prefers *x* to *y* or is indifferent between them; *xPy* means society prefers *x* to *y*; *xIy* means society is indifferent between them. We will start with a list of relevant assumptions:

(1) **Complete and transitive social preferences**. The social preference relation *R* is complete and transitive.

(2) **Pareto principle**. For all *x* and *y*, if xP_iy for all *i*, then *xPy*.

(3) **Neutrality/monotonicity**. Suppose the support for *w* over *z* is as strong or stronger than the support for *x* over *y*, and suppose the opposite support, for *z* over *w*, is as weak or weaker than the support for *y* over *x*. Then, if the social preference is for *x* over *y*, the social preference must also be for *w* over *z*. More formally: For all *x*, *y*, *z*, and *w*, assume that for all *i*, xP_iy implies wP_iz, and that for all *i*, zP_iw implies yP_ix. Then *xPy* implies *wPz*.

(4) **No dictator**. Individual *i* is a *dictator* if, for all *x* and *y*, xP_iy implies *xPy*. There is no dictator.

(5.a) **Simple diversity (S.D.).** There exists a triple of alternatives *x*, *y*, *z*, such that xP_iy for all *i*, but opinions are split on *x* vs. *z*, and on *y* vs. *z*. That is, some people prefer *x* to *z* and some people prefer *z* to *x*, and, similarly, some people prefer *y* to *z* and some people prefer *z* to *y*.

Note that our Pareto assumption is what is often called the "weak Pareto" principle. Note also that we are using the neutrality/monotonicity assumption for our theorem, rather than a weaker assumption of neutrality. (Blau & Deb (1977), call the multi-profile analog of assumption 3 "full neutrality and monotonicity"; Sen (1977) calls it NIM; and Pollak (1979) calls it "nonnegative responsiveness.") Simple diversity, assumption 5.a, is so numbered because we will introduce an alternative later. Observe that S.D. is similar to previous diversity assumptions in the literature, in the sense that its definition is independent of the social preference relation.

Note that the no dictator property is slightly different in the single-profile world from what it is in a multi-profile world. For example, in the single-profile world, if all individuals have the same preferences, and if weak Pareto holds, then by definition everyone is a dictator. Or, if individual *i* is indifferent among all the alternatives, he is by definition a dictator. So in the single-profile world a dictator may be innocuous. But, if preferences are diverse enough, and indifference is limited enough, dictatorship remains objectionable, even in this world.

3. Some Examples.

We will illustrate with a few simple examples. For these there are 2 or more people and 3 or more alternatives, and we assume no individual indifference between any pair of alternatives. Preferences of the people are shown by listing the alternatives from top (most preferred) to bottom (least preferred). In our examples, the last column of the table shows what is being assumed about society's preferences. The comment below each example indicates which desired property is breaking down. The point of examples 1 through 5, in which $n = 2$, is that if we are willing to discard any one of the five basic assumptions 1 through 5.a, the remaining 4 may be mutually consistent. But if we insist on all 5, we get an Arrow impossibility result.[1] The point of example 6 is that, when $n = 3$, assumptions 1 through 5.a may actually be consistent, with no Arrow

[1] In Feldman and Serrano (2006b) we provide a very simple version of Arrow's theorem, for the $n = 2$ case, based on assumptions 1 through 5.a.

impossibility. Example 6 therefore shows that the S.D. assumption must be modified if we are to get an impossibility result when $n > 2$.

Example 1

	Person 1	Person 2	Society (Majority Rule)
	a	*c*	
	b	*a*	*aPb, aIc & bIc*
	c	*b*	

Breakdown: Transitivity for social preferences fails. Transitivity for *R* implies transitivity for *I*. This means *aIc & cIb* should imply *aIb*. But we have *aPb.*

Example 2

	Person 1	Person 2	Society
	a	*c*	
	b	*a*	*aIbIc*
	c	*b*	

Breakdown: Pareto fails, because aP_1b & aP_2b should imply *aPb*. But we have *aIb*.

Example 3

	Person 1	Person 2	Society
	a	*c*	*a*
	b	*a*	*c*
	c	*b*	*b*

Breakdown: Neutrality/monotonicity fails. Compare the social treatment of *a* vs. *c*, where the two people are split and person 1 gets his way, to the social treatment of *b* vs. *c*, where the two people are split and person 2 gets his way.

Example 4

	Person 1	Person 2	Society (1 is Dictator)
	a	*c*	*a*
	b	*a*	*b*
	c	*b*	*c*

Breakdown: Person 1 is a dictator.

Note that examples 1 through 4 all use the same profile of individual preferences, which satisfies the S.D. assumption. Example 5 modifies the individual preferences so that S.D. no longer holds. With S.D. dropped, majority rule works fine.

	Person 1	Person 2	Society (Majority Rule)
Example 5	*a*	*c*	
	c	*a*	*aIc*
	b	*b*	*aPb* & *cPb*

Breakdown: S.D. fails. Opinions are no longer split over two pairs of alternatives.

In example 6 we start with the same individual preferences as in examples 1 through 4, and we add a 3rd person and a 4th alternative. Now assumptions 1 through 5.a are all satisfied. So with $n > 2$ and S.D., there may be no Arrow impossibility.

	Person 1	Person 2	Person 3	Society (Majority Rule)
Example 6	*a*	*c*	*a*	*a*
	b	*a*	*c*	*c*
	c	*b*	*d*	*b*
	d	*d*	*b*	*d*

Breakdown: None. The complete and transitive social preferences assumption is satisfied, as are Pareto, neutrality/monotonicity, S.D., and no dictator. Majority rule works fine. There is no Arrow impossibility.

4. Diversity.

In this section we will modify the diverse preferences assumption.

Before doing so, let's consider the assumption when $n = 2$. When there are only two people, S.D. says there must exist a triple of alternatives x, y, z, such that xP_iy for i = 1, 2, but such that opinions are split on x vs. z and on y vs. z. That is, one person prefers x to z, while the other prefers z to x, and one person prefers y to z, while the other prefers z to y. Given our assumption that individual preferences are transitive, it must be the case that the two people's preferences over the triple can be represented as follows:

Simple diversity (S.D.) array, *n* = 2.

Person *i*	Person *j*
x	*z*
y	*x*
z	*y*

Note that this is exactly the preference profile pattern of examples 1, 2, 3 and 4. The preferences in this table are two thirds of the Condorcet voting paradox preferences, as shown below:

Condorcet voting paradox array.

Person *i*	Person *j*	Person *k*
x	*z*	*y*
y	*x*	*z*
z	*y*	*x*

A similar array of preferences is used by Arrow in the proof of his impossibility theorem (e.g. Arrow (1963), p. 58), and by many others since, including us (Feldman & Serrano (2006a), p. 294). For the moment, assume V is any non-empty set of people in society, that V^C is the complement of V, and that V is partitioned into two non-empty subsets V_1 and V_2. (Note that V^C may be empty.) The standard preference array used in many versions of Arrow's theorem looks like this:

Standard Arrow array.

People in V_1	People in V_2	People in V^C
x	*z*	*y*
y	*x*	*z*
z	*y*	*x*

Now, let's return to the question of how to modify the diverse preferences assumption. Example 6 shows that we cannot stick with the S.D. array and still get an impossibility result. We might start with the Condorcet voting paradox array, but if $n \geq 4$, we would have to worry about the preferences of people other than *i*, *j* and *k*. That suggests using something like the standard Arrow array. However, assuming the existence of a triple *x*, *y*, and *z*, and preferences as per that array, for *every subset of people V and every partition of V*, is an unnecessarily strong diversity assumption.

An even stronger diversity assumption was in fact used by Parks (1976), Pollak and other originators of single-profile Arrow theorems. Pollak (1979) is clearest in his definition. His condition of "unrestricted domain over triples" requires the following: Imagine "any logically possible sub-profile" of individual preferences over 3 "hypothetical" alternatives *x*, *y* and *z*. Then there exist 3 actual alternatives *a*, *b* and *c* for which the sub-profile of preferences exactly matches that "logically possible sub-profile" over *x*, *y* and *z*. We will call this *Pollak diversity*. Let us consider what this assumption requires in the simple world of strict preferences, 2 people, and 3 alternatives. Pollak diversity would require that every one of the following arrays be represented, somewhere in the actual preference profile of the two people over the actual alternatives:

Pollak diversity arrays, *n* = 2.

1	2	1	2	1	2	1	2	1	2	1	2
x	*x*	*x*	*x*	*x*	*y*	*x*	*y*	*x*	*z*	*x*	*z*
y	*y*	*y*	*z*	*y*	*x*	*y*	*z*	*y*	*x*	*y*	*y*
z	*z*	*z*	*y*	*z*	*z*	*z*	*x*	*z*	*y*	*z*	*x*

Note that the number of arrays in the table above is 3! = 6. If *n* were equal to 3 we would have triples of columns instead of pairs, and there would have to be $(3!)^2 = 36$ such triples. With *n* people, the number of required *n*-tuples of columns would be $(3!)^{n-1}$. In short, the number of arrays required for Pollak diversity rises exponentially with *n*. The number of alternatives (which is larger that the number of arrays) rises with the number of required arrays, although not as fast because of array overlaps. Parks (1976) uses an assumption ("diversity in society") that is very similar to Pollak's, although not so clear, and he indicates that it "requires at least 3^n alternatives..."

We believe Pollak diversity is much stronger than necessary, and we will proceed as follows. We will not assume the existence of a triple *x*, *y* and *z* to give every conceivable array of preferences on that triple. We will not even assume a triple *x*, *y* and *z* to give every possible array for given V, V_1, V_2, and V^C, as per the description of the standard Arrow array. We will only assume the existence of the required Arrow-type triple, and we will only assume that much when the Arrow array matters. For the purposes of our proof, the Arrow array assumption only matters if *V* is a decisive set of minimal size, and if it has 2 or more members.

We say that a set of people *V* is *decisive* if it is non-empty and if, for all alternatives *x* and *y*, if xP_iy for all *i* in *V*, then *xPy*. *V* is a *minimally sized* decisive set if there is no decisive set of smaller cardinality.

It is appropriate to make a few comments about the notion of decisiveness. First, note that if person *i* is a dictator, then *i* by himself is a minimally sized decisive set, although without 2 or more members, and any set strictly containing *i* is also decisive, but not minimally sized. Also, note that the Pareto principle implies the set of all people is decisive. Second, in a multi-preference profile world, decisiveness for *V* would be a far stronger assumption that it is in the single-profile world, since it would require that (the same) *V* prevail no matter how preferences might change. We only require that *V* prevail under the given fixed preference profile.

Our diversity assumption is now modified as follows:

(5.b) **Diversity under minimal decisiveness (D.M.D.)**. For any minimally-sized decisive set *V* with 2 or more members, there exists a triple of alternatives *x*, *y*, *z*, such that xP_iy for all *i* in *V*; such that yP_iz and zP_ix for everyone outside of *V*; and such that *V* can be partitioned into non-empty subsets V_1 and V_2, where the members of V_1 all put *z* last in their rankings over the triple, and the members of V_2 all put *z* first in their rankings over the triple.

The assumption of D.M.D. means that for any minimally-sized decisive set *V* with 2 or more members, there is a triple *x*, *y*, and *z*, and a partition of *V*, which produces exactly the standard Arrow array shown above.

It's appropriate to make a few comments about this definition of preference diversity. First, unlike other definitions of diversity, it requires information about the induced social preference relation. This is a disadvantage, because it makes the determination of diversity more complex; it no longer suffices to look at individual preferences to determine whether or not diversity is satisfied. But it is not a logical problem, and in fact assumptions 1 through 4 (including Pareto, neutrality/monotonicity, and no dictator) also require information about the induced social preference relation. Second, it is obviously implied by Pollak diversity but not vice versa. Third, it requires the existence of far fewer alternatives than Pollak diversity; for instance, with $n = 3$, we can easily construct a D.M.D. example with just 3 alternatives (see the comments following example 7 below). Fourth, when $n = 2$, S.D. and D.M.D. are equivalent, provided the Pareto and no dictator assumptions hold. Fifth, in some contexts D.M.D. may be easier to determine than one might think; for instance, if one is analyzing majority rule, one can start the search for minimally-sized decisive sets by examining majority coalitions.[2] Sixth and finally, a very important advantage of the diversity under minimal decisiveness definition is that is just strong enough to get the desired impossibility result.

Referring back to example 6 of the previous section, consider persons 2 and 3. Under simple majority rule, which was assumed in the example, they constitute a decisive coalition. They are a coalition of 2 or more members. They are a minimally-sized decisive coalition, because there is no dictator. However the D.M.D. assumption

[2] In a recent paper, Bossert and Suzumura (2007) characterize "consistent" preference aggregation procedures in a multi-profile world. They provide a theorem showing an aggregation procedure has a list of attractive properties if and only if it counts numbers of people who prefer *x* to *y*, and who prefer *y* to *x*, and those counts satisfy certain simple inequalities. Such a counting rule may help identify minimally-sized decisive sets.

fails in that example, because there is no way to define the triple *x, y, z* so as to get the standard Arrow array, when $V_1 = \{2\}$, $V_2 = \{3\}$, and $V^C = \{1\}$. Therefore D.M.D. excludes example 6.

Example 7 below modifies example 6 to make it consistent with D.M.D.. (This example is created from example 6 by switching alternatives *a* and *b* in person 3's ranking.) Now that preferences have been modified to satisfy our new diversity assumption, an Arrow-type impossibility pops up.

	Person 1	Person 2	Person 3	Society (Majority Rule)
Example 7	*a*	*c*	*b*	
	b	*a*	*c*	*aPb, bPc, cPa*
	c	*b*	*d*	*aPd, bPd, cPd*
	d	*d*	*a*	

Breakdown: Transitivity for social preferences fails, with a *P* cycle among *a,b,c* .

Example 7 could be further modified by dropping alternative *d*, in which case it would become the Condorcet voting paradox array. It would then have 3 people and 3 alternatives, and would satisfy D.M.D.. Recall that Pollack diversity in the 3 person case would require at least 36 n-tuples of alternatives, and that Parks diversity would require at least $3^n = 27$ alternatives. The point is that that D.M.D. requires many fewer alternatives than Pollack diversity.

5. Arrow/Pollak Impossibility Theorem.

We now proceed to a proof of our single-profile Arrow's theorem.[3] Although Pollak made a much stronger diversity assumption than we use, and although Parks (1976), Hammond (1976), and Kemp and Ng (1976), preceded Pollak with single-profile Arrow theorems, we will call this the Arrow/*Pollak* impossibility theorem, because of the similarity of our proof to his. But first we need the following:

Proposition: Assume neutrality/monotonicity. Assume there is a non-empty group of people V and a pair of alternatives x and y, such that xP_iy for all i in V and yP_ix for all i not in V. Suppose that xPy. Then V is decisive.

Proof: Let w and z be any pair of alternatives. Assume wP_iz for all i in V. We need to show that wPz must hold. This follows immediately from neutrality/monotonicity. QED.

Arrow/Pollak Impossibility Theorem: The assumptions of complete and transitive social preferences, Pareto, neutrality/monotonicity, diversity under minimal decisiveness, and no dictator are mutually inconsistent.

Proof: By the Pareto principle, the set of all individuals is decisive. Therefore decisive sets exist. Let V be a decisive set of minimal size, that is, a decisive set with no proper subsets that are also decisive. We will show that there is only one person in V, which will make that person a dictator. This will establish Arrow's theorem.

Suppose to the contrary that V has 2 or more members. By the diverse-2

[3] We have a similar proof for a multi-profile Arrow's theorem in Feldman & Serrano (2006a).

preferences assumption there is a triple of alternatives *x*, *y*, and *z*, and a partition of *V* into non-empty subsets V_1 and V_2, giving the standard Arrow array as shown above. Since *V* is decisive, it must be true that *xPy*. Next we consider the social preference for *x* vs. *z*.

Case 1. Suppose *zRx*. Then *zPy* by transitivity. Then V_2 becomes decisive by the proposition above. But this is a contradiction, since we assumed that *V* was a decisive set of minimal size.

Case 2. Suppose *not zRx*. Then the social preference must be *xPz*, by completeness. But in this case V_1 is getting its way in the face of opposition by everyone else, and by the proposition above V_1 is decisive, another contradiction. QED.

Examples 1 through 4, and 7, show that the assumptions used in the theorem are all essential. Also note that this theorem can be put in the following way: Assume complete and transitive social preferences, Pareto, neutrality/monotonicity, and D.M.D.. Then there is a dictator.

Given our definition of D.M.D., we have an easy "near converse":

Dictatorship/Diversity Near Converse: Assume there is a dictator. Then the diversity under minimal decisiveness assumption is satisfied.

Proof: If there is a dictator, then there are no minimally sized decisive sets which have 2 or more members. Therefore D.M.D. is vacuously satisfied. QED.

In conclusion, we have presented a new, simple and transparent single-profile Arrow impossibility theorem. The theorem relies on an assumption about diversity of preferences within the given profile that is much weaker than the assumptions used by other authors, that is close to necessary for the result, and that produces impossibility even when the number of alternatives is small.

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