Community Structure and Market Outcomes: A Repeated Games in Networks Approach*

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Abstract

Consider a large market with asymmetric information, in which sellers choose whether to cooperate or deviate and 'cheat' their buyers, and buyers decide whether to re-purchase from different sellers. We model active trade relationships as links in a buyer-seller network and suggest a framework for studying repeated games in such networks. In our framework, buyers and sellers have rich yet incomplete knowledge of the network structure; allowing us to derive meaningful conditions that determine whether a network is consistent with trade and cooperation between every buyer and seller that are connected.

We show that three network features reduce the minimal discount factor necessary for sustaining cooperation: moderate competition, sparseness, and segregation. We find that the incentive constraints rule out networks that maximize the volume of trade and that the constrained trade maximizing networks are in between 'old world' segregated and sparse networks, and a 'global market'. (JEL: A14, C73, D82, D85, L14)

Keywords: Buyer-seller networks, repeated games, moral hazard, asymmetric information, trust, cooperation, institutions.

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1 Introduction

Economists have long noticed that it is difficult to sustain cooperation in large groups, especially when the degree of third party observability within a large group is limited.[1] Nevertheless, even as markets grow and span across geographic and cultural borders, informal agreements and cooperation continue to be an important part of markets' activity.

A number of empirical studies document interesting patterns of trade within large groups. In particular, trade and trust are often concentrated in a subset of all possible relationships.[2] This paper suggests an explanation to the observed patterns of trade and trust. We consider a market with asymmetric information. In every period, sellers with limited supply meet sequentially with buyers with limited demand and decide whether to cooperate or to defect and 'cheat' a given buyer. Only the buyer cheated observes the seller's deviation. We model active relationships as links in a buyer-seller network and ask the following question: what structures of networks are consistent with an equilibrium in which every buyer and seller that are connected trade and cooperate with each other? The answer to this question defines a set of networks in which a link between seller s and buyer b implies that b can trust s to cooperate with him when they trade.

The absence of a satisfactory model of repeated games in networks is often attributed to the inherent intractability of the problem. The richness of network environments presents non-monotonicities and discontinuities that are hard to work with in static games, let alone adding a dynamic layer. Karlan et. al. (2009) offer a static model to approximate dynamic relationships, and argue that *"networks are complicated structures, and combining them with repeated interaction can make the analysis intractable."* Lately, several researchers take on different approaches to modeling repeated games in networks (see Lippert and Spagnolo 2010, Mihm, Toth, and Lang 2009, and work in progress by Miller and Nageeb, and by Nava and Piccione). As their approach and research questions differ significantly from ours, we defer the discussion of these papers to section 9. Notably, a repeated games model that is appropriate for the analysis of networked markets with buyers and sellers has not yet been

[1]See also Kandori (1992), Greif (1993), and Ellison (1994).

[2]See also Fafchamps (1996), McMillan and Woodruff (1999), Hardle and Kirman (1995), Kirman and Vriend (2000), Weisbuch et. al. (1996), and Karlan et. al. (2009). We review this literature in section 9.

suggested.

Our framework alleviates some of the difficulties and provides a simple expression that summarizes all the network information that seller s uses when deciding whether to cooperate with buyer b or cheat him. Consider a seller s and a buyer b that are connected. The immediate benefit for s from cheating buyer b is defined by the stage game and does not depend on the network. On the other hand, the cost of cheating depends on the entire network structure. As a starting point, consider the simple case that s deviates only in an interaction with b and cooperates with all other buyers that are connected to her. In this simple case, for every period that b 'punishes' s by not purchasing from her, s loses her expected per-period future value from cooperation with b, which we denote by $FV_{s,b}$. If $FV_{s,b}$ is large, s does not take the risk of deviating and losing the option to trade with b even if her intertemporal discount factor is low and the immediate benefit from deviating is large. In Theorem 1, we establish conditions under which the following one-deviation-principle holds: $FV_{s,b}$ is a sufficient statistic for determining whether a *fully cooperative equilibrium* (an equilibrium in which every buyer and seller that are connected always cooperate with each other) exists.

Despite the significant simplification, $FV_{s,b}$ still depends on the entire network structure and can be difficult to calculate, especially in large networks. To evaluate $FV_{s,b}$, s asks the following question: "What is the probability that I will be able to sell a good to b and not be able to sell it to any other buyer?" The answer reflects the probability that s *needs* b in a given period, and depends on the network structure in two ways. First, the network structure determines the frequency of interactions between s and b; when their frequency of interaction rises, s *needs* b more, and values more their connection. Second, the network structure determines the outside options of s if she were not connected to b. When other buyers with whom s is connected are more likely to buy from s, seller s *needs* b less. Figure 1 provides an example using two simple networks.

Figure 1a **Figure 1b**

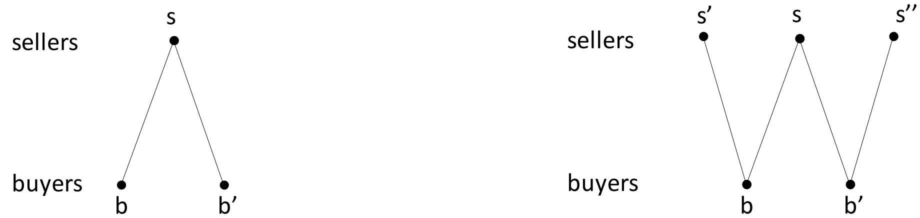

Figure 1: In every period, let meetings between buyers and sellers occur in a random order, and let each seller produce one unit of a good and each buyer have demand for one unit of a good. For both networks, assume that there exists an equilibrium in which every seller and buyer that are connected cooperate. Then, successful interactions between seller s and buyer b in the network in figure 1a are more frequent than in figure 1b, because in the latter there is a positive probability $\left(p = \frac{1}{4}\right)$ that s does not manage to sell to any buyer in a given period. However, in figure 1a s has a guaranteed outside option because buyer b' cannot transact with any other seller, whereas in figure 1b, there is a positive probability that b is the only buyer ready to buy from s, which raises the value of this connection for seller s. Focusing on figure 1b, if we eliminate the link between s' and b, the connection between s and b becomes more valuable due to higher frequency of interactions and the connection between s and buyer b' becomes less valuable due to an improved outside option for s.

The expectations of seller s with respect to $FV_{s,b}$ depend also on what s knows about the network. A novel feature of this paper is that we introduce an approach for studying repeated games with incomplete knowledge of the network structure. Our approach is similar to the one used by Jackson and Yariv (2007), and Galeotti et. al. (2010) but less restrictive.[3] In our model, sellers and buyers know who they are connected to, and the number of connections (*degree*) of each of the buyers or sellers that they are connected to. Additionally, they know the number of buyers and sellers in the network (n_b and n_s respectively), as well as some aggregate information regarding the network structure, such as the degree distribution of buyers and sellers in the network and the probability of sharing more than one neighbor with the same individual.

Theorem 1 and Equation (3) show that in any network that admits a fully cooperative equilibrium and for every seller s and buyer b that are connected, $FV_{s,b}$ can be summarized by a simple expression that captures $FV_{s,b}$ in a corresponding random tree. Using this insight, we show that three network features increase the values of links: [1] *moderate and balanced competition*: the degrees of every buyer and seller that are connected are similar (Theorem 2

[3]Galeotti et. al. (2010) and Jackson and Yariv (2007) introduce the analysis of static network games with incomplete information and are able to reduce significantly the complexity of the analysis and overcome problems of multiplicity of equilibria.

and Proposition 3); [2] *sparseness*: the degrees of sellers and buyers in the network are small (Theorem 3); and [3] *segregation*: sellers who have one buyer in common, have connections to similar sets of buyers overall (Theorem 4).[4] For fixed intertemporal discount factors, our results describe systematic constraints on the structure of networks that can sustain cooperation.[5]

If we ignore the incentive constraints and assume that sellers always cooperate, networks that maximize the expected volume of trade are dense - the exact opposite of [2] above. This difference is especially robust in stochastic environments, in which sellers' supply is subject to exogenous fluctuations (Theorem 5).

We suggest social and formal institutions that relax the constraints on the structure of networks that admit a fully cooperative equilibrium: *Reputation Networks*, *Litigation*, and *Third-Party Evaluation Services*. The direct effect of each of these institutions on cooperation is well studied. However, the integration of reduced form models of these institutions into our framework highlights a new insight: in the presence of either of these institutions, denser networks can sustain cooperation (Propositions 4 and 5); i.e. these institutions complement the network rather than substitute for it in enforcing cooperation.

Methodologically, we extend prior literature on games in networks in several ways. Most notably, while most of the literature focuses on static games (for extensive surveys, see Goyal 2007 and Jackson 2008), we analyze repeated games in networks. In addition, the current literature focuses either on complete information of the network structure,[6] or on incomplete information where an agent knows only her own degree and the degree distribution of others in the network. We allow for incomplete yet richer knowledge of the network structure. By doing so, we achieve tractability in large networks, while maintaining the ability to analyze complex changes in the network structure. We also provide a model of market activity in which incomplete knowledge of the network structure persists even with Bayesian agents.

[4] This notion of segregation in two sided (bipartite) networks is related to the concept of *clustering* in the graph theory literature, and to the notion of network closure in Granovetter (1974) and Burt (2001). From a more applied perspective, segregation is related to the size of each community within a society; a market is more segregated if each separate community is smaller.

[5] This is different from most of the networks literature, in which the constraints on the network structure come from an exogenous cost of creating or sustaining links.

[6] E.g. Ballester et. al. (2006), Bramoulle, Kranton, and D'Amours (2009), Chwe (2000), Galeotti (2005), and Goyal and Moraga-Gonzalez (2001).

Finally, most related to our paper is Fainmesser and Goldberg (2010) - hereby FG - who analyze how the structure of an informational network between buyers affects the ability to sustain cooperation between buyers and sellers. FG show that the impact of the entire structure of the buyer-seller network on the incentives of a seller to cooperate can be approximated by focusing on the seller's local neighborhood - a small network that includes only buyers and sellers that are *close* to the seller. Furthermore, when sellers have a sufficient level of uncertainty with respect to the network structure, FG find that a seller expects her local neighborhood to look approximately like a random tree - a network that has no cycles and in which the degrees of buyers and sellers in the network are drawn independently at random from some degree distribution. We make use of these graph theoretic results in our characterization of large network for which a fully cooperative equilibrium exists.

The paper is organized as follows. The following section offers two motivating examples. In section 3, we present the model, and in section 4 we characterize the future value of links in large networks and derive conditions to determine whether a network admits a fully cooperative equilibrium. Sections 5 and 6 characterize differences in the future values of different links within and across networks and relate these differences to the constraints on the structure of networks that admit fully cooperative equilibria. Section 7 investigates the trade-off between sustaining cooperation and maximizing trade volumes. In section 8 we study institutions that affect the ability to sustain cooperation. Section 9 offers a discussion of related literature and empirical evidence, and section 10 offers concluding remarks.

2 Examples

To motivate our analysis, we briefly describe two examples of relevant applications.

2.1 Example 1: job recommendations

The importance of social networks for getting jobs has been long recognized. Granovetter (1974) documents that more than half of (white-collar) workers use personal connections to obtain jobs. Bewley (1999) summarizes 24 other U.S. studies that point to similar results. Fainmesser (2010) shows that transmission of information over social networks can affect the timing of hiring in entry-level labor markets.

Consider a group of recommenders (teachers / past employers / head hunters) that have workers to recommend and a group of firms that are seeking to hire. A recommender receives a positive payoff from getting a job for her worker. A worker's ability can be either high or low, and is observed by the firm only after the worker is hired (the recommender knows the ability of the worker). Assuming that firms want to hire only high quality workers, a recommender who has a low ability worker can benefit from recommending the worker to the firm as having high ability.

Suppose that we are able to calculate the per-period future value that recommender s has for the possibility that firm b hires her worker (given the structure of the market), and suppose that we are able to establish a one-deviation-principal in this setup. Then, the maximal cost for s from deviating in an interaction with b is a simple discounted sum of this future value and we would be able to predict whether it is sustainable to have truthtelling by recommender s in interactions with firm b.

2.2 Example 2: catering and food deliveries

The catering and food deliveries industry is a multi-billion dollar industry.[7] Consider a group of caterers and a group of repeated clients that order food frequently.[8] Providing good service costs caterers more than providing low quality service. In the absence of sufficient future payoffs that are contingent on providing high quality service, a caterer may shirk and provide low quality service. When sufficient future incentives are in place, clients are able to trust their caterer to provide high quality service.

As in example 1, it will be useful to be able to calculate the future value for every caterer s from interactions with any client b. Moreover, as eating is a social experience, one might expect that clients share among themselves information about past experiences with different caterers. In that case, being able to calculate the future value provides a useful benchmark

[7] According to online publications, the catering industry alone includes 10,000 companies with combined annual revenue of $5 billion (http://www.firstresearch.com/Industry-Research/Catering-Services.html).

[8] For example, a repeated client can be a professional event coordinator, or a person in a firm or a social or professional group, who organizes the food orders.

to evaluate the impact of information sharing between buyers.[9]

3 Model

Consider a market with a set $S = \{1, 2, ..., n_s\}$ of sellers (recommenders / caterers) and a set $B = \{1, 2, ..., n_b\}$ of buyers (firms / clients). Time is discrete. Sellers live forever and seller s has a discount factor δ_s. Periods are ex-ante identical. In every period, a seller has one high quality unit capacity with probability μ and no high quality capacity with probability $(1 - \mu)$, i.i.d. across sellers and periods. μ is common knowledge.[10] In a given period, a seller s that has one high quality unit capacity can produce either one low quality good at no cost, or one high quality good with a cost of $c_s \geq 0$. A seller with no high quality capacity can produce only one low quality good at no cost. Goods are non-durable and cannot be transferred across periods. Buyers live forever and have unit demand (for a high quality good only) in every period. Each seller s has an *active relationship* with only a subset of buyers, denoted by B_s. We first define a buyer-seller network that captures the active relationships of all sellers and later provide the activity rules that define the notion of an active relationship.

Let $m = \langle S, B, E \rangle$ be a network, where E is a set of seller-buyer pairs such that $(s, b) \in E$ if and only if seller s and buyer b are connected (linked). Let $B_s(m) = \{b \in B | (s, b) \in E\}$ be the set of buyers that are connected to seller s, and let $S_b(m) = \{s \in S | (s, b) \in E\}$ be the set of sellers that are connected to buyer b. The *degree* of seller s, $d_s = d_s(m) = |B_s(m)|$ is the number of buyers that are connected to s, and the degree of buyer b, $d_b = |S_b(m)|$, is the number of sellers that are connected to b. A *path* between buyer b and buyer b' in network m is a sequence of buyers and sellers $(b = b_0, s_1, b_1, s_2, ..., s_n, b_n = b')$ such that for every $i \in \{1, 2, .., n\}$, $\{(s_i, b_{i-1}), (s_i, b_i)\} \subset E$. The *length* of a path is the number of edges along the path. A path $(b = b_0, s_1, b_1, s_2, ..., s_n, b_n = b')$ is also a *cycle* if $b = b'$. A *tree* is a network that has no cycles.

For the statement of several of our results it will also be useful to define a notion of a

[9] In section 8.1 we allow for information sharing between buyers and study its effect on market structure and cooperation.

[10] The assumption that μ is constant across sellers is without loss of generality.

degree distribution in a network. Let $g = \langle g^S, g^B \rangle$ be a pair of probability distributions such that if we choose a link $(s, b) \in E$ uniformly at random, $g^B(d)$ is the probability that buyer b has degree d, and $g^S(d)$ is the (unconditional) probability that seller s has degree d. Let $G = \langle G^S, G^B \rangle$ be such that G^S is the CDF of g^S and G^B is the CDF of g^B. We refer to G as the degree distribution in the network.[11]

The dynamics of meetings between sellers and buyers in the market is fully determined by the network structure, and captured by the following process.

Assumption 1 *In every period, meetings between buyers and sellers that are connected in the network occur in a random order, i.i.d. across periods. Formally, in every period, links from E are drawn uniformly at random and without replacement (all links are chosen in every period). When a link (s, b) is chosen, s and b meet and get an opportunity to engage in trade unless either s or b has already traded (with anyone else) in the same period.*[12]

3.1 Trade

When seller s meets with buyer b, seller s decides whether to invest in producing high quality (if possible) and whether to tell b that the good is of high quality or of low quality. Buyer b decides whether to purchase the good from s or not. If b purchases the good, seller s receives a payoff of π (minus any production costs). Buyer b receives a positive net payoff if the good is of high quality, and a negative net payoff otherwise. Payoffs are realized at the end of the period, and buyers and sellers who do not manage to trade in a given period have utility 0.[13,14]

[11] Defining the degree distribution in terms of a probability distribution over the degree of a seller (buyer) at the end of a link is due to Galeotti et. al. (2010) and Jackson and Yariv (2007).

[12] The idea that interactions in markets have a random component is not new and is formalized in many models of market activity. As our focus is on the network structure and not on transient and irregular frictions in markets, we follow much of the networks literature and take the random component as exogenous (see also Bala and Goyal 2000, Manea 2010, and Pongou and Serrano 2009).

[13] In our model, depending on the realization of μ, a seller's deviation is either by not investing in high quality when she has the option to do so, or by telling a buyer that she has a high quality good in a period that she can produce only a low quality good. Our setup converges to a standard asymmetric prisoner's dilemma when $\mu = 1$ and $c > 0$, or when $\mu < 1$ and $c = 0$.

[14] The model and all of the results extend immediately to games in which both parties have incentives to deviate, such as the standard prisoner's dilemma. Similarly, our analysis remains the same for stochastic games in which payoffs vary across periods.

Definition 1 *We say that buyer b and seller s* ***cooperate*** *if when they meet:*

1. *If s does not have high quality capacity, she truthfully conveys that to b, and if s has high quality capacity she invests in producing high quality if b purchases the good.*

2. *Buyer b chooses to purchase the good if and only if s claims to have high quality capacity.*

Note that the profit for seller s from not cooperating may depend on the application through μ and c_s. Let $\Pi^{s,D}$ be the maximal additional payoff that s can ever gain from deviation. In the adverse selection problem in example 1, seller (recommender) s deviates by saying that a worker is of high quality when she is not. As a result, s gets benefits of trade that would not have occurred had she told the truth, and $\Pi^{s,D} = \pi$. In the moral hazard problem in example 2, a deviation by a seller (caterer) is saving on effort costs, and $\Pi^{s,D} = c_s$.

Remark 1 *We assume that the payoff for a seller from a single transaction (π) does not depend on the network structure. Introducing bargaining over the network increases the complexity significantly. Moreover, any reasonable bargaining model will preserve the main insights of this paper.*[15] *Incidentally, in a bargaining procedure in which sellers make take-it-or-leave-it offers, it is straightforward to construct equilibria for which our analysis goes through without changes.*[16]

3.2 Large networks and the knowledge of the network

Our goal is to provide a framework that is suitable for the analysis of large markets. This has proven to be a difficult task even in the study of static games, and especially when agents have complete knowledge of the network structure. With repeated interactions, the analysis soon becomes intractable. Several authors suggest studying environments in which

[15] The literature on bargaining in networks is in its early stages. Excellent examples of models of bargaining in networks are provides by Corominas-Bosch (2004), Manea (2010), and Abreu and Manea (2010).

[16] In section 9, we review evidence from markets in which price seems to be only of second order relative to the ability to sustain cooperation. In other markets, as in the job recommendations example, there are no money transfers and the payoffs represent intrinsic utilities from trade.

agents have incomplete information of the network structure. In particular, Jackson and Yariv (2007), and Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2010) focus on static network games in which agents know only their own degree and the degree distribution in the network.

A novel feature of this paper is that we introduce an approach that is similar to Jackson and Yariv (2007), and Galeotti et. al. (2010) but less restrictive, and apply it to the study of repeated games. In particular, sellers (and buyers) know their own degree and the degrees of every buyer or seller that is connected directly to them, as well as the number of buyers and sellers in the network (n_b and n_s respectively) and the degree distribution G.[17]

Assumption 2 *Let K_s (K_b) be the knowledge that seller s (buyer b) has with respect to the network structure. We assume that $K_s = \langle d_s, \{d_{b'}\}_{b' \in B_s}, n_s, n_b, G \rangle$ and $K_b = \langle d_b, \{d_{s'}\}_{s' \in S_b}, n_s, n_b, G \rangle$.*

What seller s knows about the network is illustrated in figure 2.

Figure 2

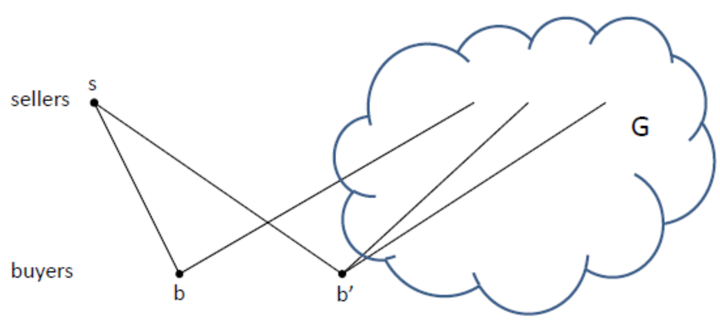

Figure 2: The network from the point of view of seller s who is connected to buyers b and b'. Seller s knows the identities and degrees of buyers that are connected to her directly and the degree distribution G of buyers and sellers that are not connected to her directly.

While clearly stylized, assumption 2 captures the idea that participants in the market have some information on the alternative trading opportunities of their potential trading partners. Restricting further the knowledge of the sellers and buyers does not change our analysis. However, we find that outside opportunities of trading partners have a first order effect on the incentives to cooperate. Extending the knowledge of sellers and buyers beyond K_s and K_b may only refine their evaluation of their trading partners' outside opportunities.

[17] Alternative assumptions that allow us to study the effects of segregation on the ability to sustain cooperation are analyzed in section 6.2.

As a result, allowing sellers and buyer to have additional information complicates our analysis significantly without changing the nature of our results.

To capture the idea that K_s and K_b contain all of the information that sellers and buyers have with respect to the network structure, we suggest the following assumption.

Assumption 3 *A seller s (buyer b) attaches identical probability to the network being any of the possible networks conditional on K_s (K_b).*[18]

Remark 2 *The knowledge that individuals are expected to have in a repeated games setup deserves further discussion. Clearly, repeated interactions provide sellers and buyers with opportunities to learn about their environment. However, even excluding purely behavioral considerations, there are several reasons for market participants not to be able to learn beyond their close local network and some aggregate characteristics of the global environment. In appendix A, we provide further discussion of our informational assumptions and provide an example of an environment in which incomplete knowledge of the network structure persists even if individuals are Bayesian and interact repeatedly in the network.*

Large networks. To capture the idea of large markets, it is useful to consider the following notion of an increasing sequence of networks.[19]

Definition 2 *Consider a finite support degree distribution G, and let $m(n_b, G)$ be a network with n_b buyers and a degree distribution G. We say that $\{m(n_b^i, G)\}_{i=1}^{\infty}$ is* ***an increasing sequence of networks*** *if for every $j > i$, $n_b^j > n_b^i$.*

For some (n_b, G) a network $m(n_b, G)$ may not exist. In particular, for $m(n_b, G)$ to exist two conditions must be satisfied: [1] n_b must to be such that G^B can be induced by some vector $(d_b^1, d_b^2, ..., d_b^{n_b})$; and [2] there must exist some n_s and a vector $(d_s^1, d_s^2, ..., d_s^{n_s})$ such that $(d_s^1, d_s^2, ..., d_s^{n_s})$ is consistent with G and $\sum_{i=1}^{n_s} d_s^i = \sum_{i=1}^{n_b} d_b^i$. However, for every G, and starting from some n_b, there exists an increasing sequence as required. Moreover, given n_b and G, n_s is uniquely determined.

[18]Assumptions 2 and 3 are consistent with a seller (buyer) having a uniform prior over the set of all networks given n_s and n_b. A seller (buyer) then updates her prior using K_s (K_b).

[19]See Ozsoylev and Walden (2009), and Golub and Jackson (2010) for a similar formulation of large networks in the context of information diffusion in networks.

4 The value of a relationship

In this section we define a notion of a per-period future value (FV) of a connection that follows a greedy calculation: assume that in all networks all buyers and sellers always cooperate and consider a seller's single deviation followed by cooperation with all other buyers connected to her. Then, the future value of the connection (s,b) in network m is the difference between the expected payoff of seller s in network m and her expected payoff in network $m\backslash(s,b)$. Theorem 1 establishes conditions under which: [1] the naively calculated future values of links are sufficient statistics for determining whether a fully cooperative equilibrium exists, and [2] the future values of links in a network m can be calculated as if m is a random tree. The following example demonstrates the simple conditions for cooperation in our model in a market with a single seller and a single buyer.

Example (a market with one seller and one buyer). *Consider a single seller s who has unit capacity with probability μ and a single buyer b. Conditional on cooperation, with probability μ, s needs b in order to trade with a payoff $\pi - c_s$. Note that $\frac{\delta_s}{1-\delta_s} \cdot \mu \cdot (\pi - c_s)$ equals the maximal punishment that b can punish s (by not purchasing goods from s in subsequent periods). Therefore, an equilibrium in which seller s and buyer b cooperate exists if and only if $\frac{\delta_s}{1-\delta_s} \cdot \mu \cdot (\pi - c_s) \geq \Pi^{s,D}$.*

In networked markets with multiple sellers and buyers, the analysis is no longer straightforward. The maximal effective punishment that could be imposed on a seller s by a given buyer b depends on: [1] the outside option of the seller, and [2] the frequency of interaction between s and b. Moreover, both [1] and [2] depend on the strategies of all of the buyers and seller in the market. To this end, we restrict attention to equilibria in which buyers and sellers use 'trigger strategies' that take the following form.

Definition 3 *We say that buyer b and seller s that are connected in the network use **trigger strategies** if when s and b meet they cooperate as long as neither of them deviated unilaterally in the last T periods and deviate otherwise.*

Although milder restrictions are sufficient to establish our results, focusing on trigger strategies has several advantages. Beyond being intuitive and well studied in the economic literature, trigger strategies are found to be commonly used by subjects in the lab,[20] and are often used by people to describe their own intentions and behavior. In relation to example 2 above, a quick glance in online discussion boards and blogs shows that an extreme version of a trigger strategy, in which $T \to \infty$, is often claimed to be used by many clients who had a bad experience with their caterer or food delivery provider.[21]

Definition 4 *A* ***Strict Trigger Nash Equilibrium (STNE)*** *is a strict Nash equilibrium in which all buyers and sellers employ trigger strategies.*[22]

In the remainder of the paper, we focus on STNE unless stated otherwise. Extending the analysis to a corresponding version of Perfect Bayesian Equilibrium complicates the analysis significantly, but does not change our results.

Note that by definition, in any STNE, every buyer and seller that are connected cooperate with each other. Thus, given a STNE, periods are ex-ante identical. For each seller s and buyer b, let $I_m^t(s,b)$ denote the indicator of the event that s sold a unit of a good to b in period t in a STNE in network m. We note that [1] $I_m^t(s,b)$ is fully determined by the realizations of who of the sellers are active in period t and of the order of meetings in period t; and [2] ex-ante $\Pr\left(I_m^t(s,b)\right)$ is independent of t. A greedy calculation of the per-period future value of the link (s,b) for seller s yields:

$$FV_{s,b}(m) = \mu \cdot \left[\sum_{b' \in B_s(m)} \Pr\left(I_m^t(s,b')\right) - \sum_{b' \in B_s(m) \backslash b} \Pr\left(I_{m \backslash (s,b)}^t(s,b')\right) \right] \cdot (\pi - c_s) \qquad (1)$$

Note that it is not at all obvious that we can apply the same logic as in the example above and claim that seller s cooperates with buyer b as long as $\frac{\delta_s}{1-\delta_s} \cdot FV_{s,b}(m) > \Pi^{s,D}$. In particular, we are required to consider more complex deviations of seller s and cannot assume that her best strategy after deviating in an interaction with buyer b is to always cooperate with all other buyers in $B_s \backslash b$. Moreover, even if $FV_{s,b}(m)$ is a sufficient statistic for the existence of a

[20] Engle-Warnick and Slonim (2006) find that trigger strategies are often used by subjects in repeated trust games.

[21] For a not unusual example see http://www.grubhub.com/chicago/feast/. Note especially the "I will not do delivery from them again." and the extreme version: "There's no way in hell I'd ever order from here again". In other web pages shorter punishment phases are suggested.

[22] In a strict Nash equilibrium all players play a strict best response.

STNE, $\Pr\left(I_m^t(s,b)\right)$ is a complex object, making is costly to compute and analyze $FV_{s,b}(m)$. In fact, given the sellers' information sets $\{K_s\}_{s\in S}$, a direct calculation will require each seller s to compute $\{\Pr\left(I_m^t(s,b)\right)\}_{b\in B_s(m)}$ for each network m that is possible given her information, and then average over all such networks. Theorem 1 resolves both of these issues.

Consider a network m with a degree distribution G, and a seller $s \in S$ with degree d_s. Let $\bar{b}_s \in (Z^+)^{d_s}$ be a sorted vector of the degrees of all buyers in $B_s(m)$. Now, let $T^d(m,s)$ denote the random depth$-d$ tree such that the root r has degree d_s, the sorted vector of degrees of the children of r is $\bar{b}_s$, all subsequent non-leaf nodes at an even depth have a degree drawn i.i.d. from G^S, all subsequent non-leaf nodes at an odd depth have a degree drawn i.i.d. from G^B. Note that $T^d(m,s)$ can have more or less buyers and sellers than m. Let $FV_{s,b}\left(T^\infty(m,s)\right) \triangleq \lim_{d\to\infty} FV_{s,b}\left(T^d(m,s)\right)$.

Theorem 1 establishes that $FV_{s,b}\left(T^\infty(m,s)\right)$ exists and that for a large network m, $\{FV_{s,b}\left(T^\infty(m,s)\right)\}_{s\in S, b\in B}$ are sufficient statistics to determine whether there exists a STNE with network m.

Theorem 1 *For any network m, $\{FV_{s,b}\left(T^\infty(m,s)\right)\}_{s\in S,b\in B}$ exist. Moreover, let G be any finite support degree distribution. Then, for any increasing sequence of networks $\{m(n_b^i,G)\}_{i=1}^\infty$ there exists $\bar{i}$ such that for any $i>\bar{i}$ a STNE with network $m(n_b^i,G)$ exists if and only if for every seller s and buyer b that are connected in $m(n_b^i,G)$,*

$$\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^\infty\left(m\left(n_b^i,G\right),s\right)\right) > \Pi^{s,D} \tag{2}$$

Theorem 1 implies that we can analyze the existence of an STNE in any large network *as if* the network is a random tree. The proof of Theorem 1 extends recent results by FG and is deferred to Appendix B. We now consider the implications of Theorem 1. Consider a network $m = \langle S,B,E\rangle$ with a degree distribution G, and a link $(s,b)\in E$. Let $T^d(m,s,b)$ denote the random depth$-d$ tree such that the root r has degree 1, the degree of the only child of r is d_b, all subsequent non-leaf nodes at an even depth have a degree drawn i.i.d. from G^S, all subsequent non-leaf nodes at an odd depth have a degree drawn i.i.d. from G^B. In words, $T^d(m,s,b)$ is constructed in the same way as the subtree of $T^d(m,s)$ that results from disconnecting all buyers in $B_s \backslash b$ from seller s. In the context of the bigger network

$T^d(m,s)$, $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$ captures the probability that buyer b does not purchase a good before meeting seller s.

Then, the future value of a link in a random tree $T^d(m,s)$ can be re-written as:

$$FV_{s,b}\left(T^d(m,s)\right) = \mu\cdot(\pi - c_s)\cdot \Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)\cdot \Pi_{b'\in B_s\setminus b}\left[1-\Pr\left(I^t_{T^d(m,s,b')}(s,b')\right)\right]. \quad (3)$$

With respect to $T^d(m,s)$, the expression $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)\cdot\Pi_{b'\in B_s\setminus b}\left[1-\Pr\left(I^t_{T^d(m,s,b')}(s,b')\right)\right]$ captures the probability that in a period t, buyer b has demand for a good when he meets seller s AND no other buyer $b'\in B_s\setminus b$ has demand when their link with seller s is chosen. Thus, seller s sells a good if she is connected to b, but would not have been able to sell a good has she not been connected to buyer b. The simple expression is due to the the tree structure that guarantees the independence of $\left\{I^t_{T^d(m,s,b)}(s,b)\right\}_{b\in B_s}$ of each other. Moreover, for every seller s and buyer $b'\in B_s$, the tree structure and the independence of the degrees across subtrees guarantee that comparative statics over $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$ are governed by the following simple rule.

Lemma 1 *Suppose that for all $d\geq 1$, the random tree $T^d_2 = T^d(m,s,b)$ can be constructed (on the same probability space) from the random tree $T^d_1 = T^d(m',s',b')$ by performing only the two operations: 1. appending (as children) subtrees to seller nodes in an arbitrary way, and 2. removing (as children) subtrees from buyer nodes in an arbitrary way. Then* $\Pr\left(I^t_{T^d_2}(s,b)\right)\geq \Pr\left(I^t_{T^d_1}(s,b)\right)$

5 Network structure, competition, and cooperation

A higher $FV_{s,b}(T^\infty(m,s))$ implies more cooperation in a large network in two ways: [1] holding $\Pi^{s,D}$ fixed, a higher $FV_{s,b}(T^\infty(m,s))$ means that a lower δ_s is sufficient to allow for cooperation to be sustained over the link (s,b), and [2] holding δ_s fixed, a higher $FV_{s,b}(T^\infty(m,s))$ means that cooperation can be sustained over the link (s,b) even when the temptation of s to deviate $\left(\Pi^{s,D}\right)$ is higher.

In this section, we examine the relationship between the structure of network m and $FV_{s,b}(T^\infty(m,s))$. We later relate our results to the level of competition between sellers in m. If many sellers with low degrees are connected to buyers with high degrees, we say that the network exhibits *fierce competition*. On the other hand, if sellers with high degrees are connected to many buyers with low degrees, we say that the network exhibits *weak competition*. If in the entire network sellers and buyers have similar degrees, we say that the network exhibits *moderate and balanced competition*. In particular, we find that the future values of links are higher when a network exhibits a more *moderate and balanced competition*.

We start by showing that in a given network m, $FV_{s,b}(T^{\infty}(m,s))$ is lower for links in which the buyer (seller) has a high degree than for links in which the buyer (seller) has a low degree.

Proposition 1 *:*

1. *Consider buyers $b, b' \in B_s$ such that $d_{b'} \geq d_b$. Then, $FV_{s,b'}(T^{\infty}(m,s)) \leq FV_{s,b}(T^{\infty}(m,s))$.*
2. *Consider sellers $s, s' \in S$ such that $d_{s'} \geq d_s$ and $\{d_b\}_{b \in B_s} \subseteq \{d_{b'}\}_{b \in B_{s'}}$, and consider buyers $b_1 \in B_s$ and $b'_1 \in B_{s'}$ such that $d_{b'_1} = d_{b_1}$. Then, $FV_{s',b'_1}(T^{\infty}(m,s')) \leq FV_{s,b_1}(T^{\infty}(m,s))$.*

Proof. By Lemma 1, as $d_{b'} > d_b$ we have that $\Pr\left(I^t_{T^d(m,s,b')}(s,b')\right) \leq \Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$, which when combined with Theorem 1 and Equation (3) completes the proof of Part 1. Part 2 follows directly from Theorem 1 and Equation (3). ■

If $d_{b'} > d_b$, seller s expects fewer periods with demand from b' than periods with demand from b $\left(\Pr\left(I^t_{T^{\infty}(m,s,b')}(s,b')\right) \leq \Pr\left(I^t_{T^{\infty}(m,s,b)}(s,b)\right)\right)$. In part 2, seller s' has connections to buyers with the same degrees as the buyers that s is connected to and is also connected to some additional buyers. As a result, s' has a better outside option in the case that buyer b'_1 does not purchase the good from her (compared with the outside option of seller s in case that buyer b_1 does not purchase the good from her).

On the other hand, if the degrees of buyers in $B_s \backslash b$ are large, s is more likely to *need* buyer b in order to make a sale in period t. For example, in figure 1a, if we add a connection between buyer b and some seller s' that we add to the figure, the connection (s,b) becomes less valuable, whereas the connection (s,b') becomes more valuable.

Proposition 2 *Consider two sellers s and s' such that [1] $d_{s'} = d_s$; and [2] there exist $b_1 \in B_s$ and $b_1' \in B_{s'}$ such that $d_{b_1'} \geq d_{b_1}$ and $\{d_b\}_{b \in B_s \backslash b_1} \equiv \{d_{b'}\}_{b \in B_{s'} \backslash b_1'}$. Then, for every $b \in B_s$ and $b' \in B_{s'}$ such that $d_{b'} = d_b$, $FV_{s',b'}(T^{\infty}(m, s')) \geq FV_{s,b}(T^{\infty}(m, s))$.*

Proof. By Lemma 1, if $d_{b_1'} > d_{b_1}$, $\Pr\left(I^t_{T^d\left(m,s',b_1'\right)}(s', b_1')\right) < \Pr\left(I^t_{T^d(m,s,b_1)}(s, b_1)\right)$. Plugging this inequality into Equation (3) for some $b \in B_s$ and $b' \in B_{s'}$ such that $d_{b'} = d_b$ yields that $FV_{s',b'}(T^{\infty}(m, s')) \geq FV_{s,b}(T^{\infty}(m, s))$. ■

Proposition 2 captures the positive externality of links: if $d_{b_1'} > d_{b_1}$ seller s' expects less periods with demand from b_1' than periods with demand that seller s expects from b_1. As a result, s' (more than s) is likely to need her other connections in order to sell the good.

Now let

$$\underline{FV}_s(m) \triangleq \min_{b \in B_s} \{FV_{s,b}(T^{\infty}(m, s))\}.$$

From Proposition 1, if d_b is large, $FV_{s,b}(T^{\infty}(m, s))$ is small. From Proposition 2, this is mitigated if for every $b' \in B_s \backslash b$, $d_{b'}$ is also very large. Thus, networks in which buyers that are connected to the same seller have 'similar' degrees have a larger $\{\underline{FV}_s(m)\}_{s \in S}$.

Beyond the immediate neighborhood, Theorem 1 and Lemma 1 allow us to evaluate the effect of differences in the degree distribution across networks. Let $\widehat{G^{\widehat{B}}}$ FOSD G^B and G^S FOSD $\widehat{G^{\widehat{S}}}$, and consider two sellers $s \in S$ and $\widehat{s} \in \widehat{S}$ that have an identical local neighborhood, so that the only difference between K_s and $K_{\widehat{s}}$ is difference in the degree distribution in the networks that s and $\widehat{s}$ are embedded in (m and $\widehat{m}$ respectively). Theorem 2 shows that [1] if $d_s = d_{\widehat{s}}$ are large enough then the future values of links of seller $\widehat{s}$ are higher then the future values of links of s; and [2] if $d_s = d_{\widehat{s}}$ are small enough then the future values of links of seller $\widehat{s}$ are smaller then the future values of links of s.

Theorem 2 *Let $\widehat{G^{\widehat{B}}}$ FOSD G^B, and G^S FOSD $\widehat{G^{\widehat{S}}}$, and let $m = \langle S, B, E \rangle$ and $\widehat{m} = \left\langle \widehat{S}, \widehat{B}, \widehat{E} \right\rangle$ be two networks with degree distributions G and $\widehat{G}$ respectively. Consider sellers $s \in S$ and $\widehat{s} \in \widehat{S}$ such that $d_{\widehat{s}} = d_s$ and $\left\{d_{\widehat{b}}\right\}_{\widehat{b} \in B_{\widehat{s}}} \equiv \{d_b\}_{b \in B_s}$. Then, there exist $\overline{d_s}(m, \widehat{m})$ and $\underline{d_s}(m, \widehat{m})$ such that if $d_s < \underline{d_s}$, then $\underline{FV}_{\widehat{s}}(T^{\infty}(\widehat{m}, \widehat{s})) \leq \underline{FV}_s(T^{\infty}(m, s))$, and if $d_s > \overline{d_s}$, then $\underline{FV}_{\widehat{s}}(T^{\infty}(\widehat{m}, \widehat{s})) \geq \underline{FV}_s(T^{\infty}(m, s))$.*

The differences in degree distribution that are analyzed in Theorem 2 are illustrated in figure 3.

Figure 3

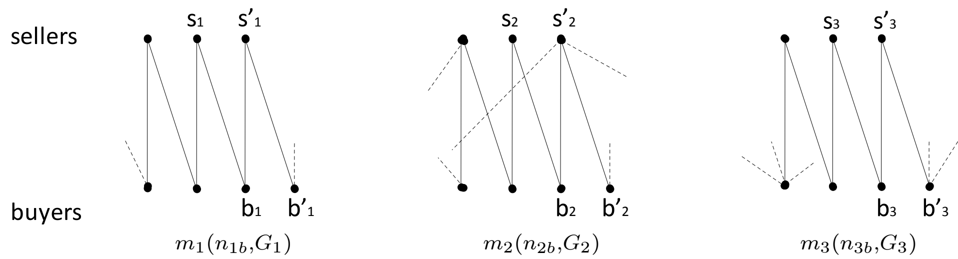

Figure 3: In networks m_1, m_2, and m_3 above, the broken lines represent links to buyers and sellers that are not in the diagram. For illustration, counting only the buyers and sellers in the figure, G_2^S FOSD G_1^S and G_3^B FOSD G_1^B.

If G^S FOSD $\widehat{G^{\widehat{S}}}$ the aggregate demand per seller in network m is larger that in $\widehat{m}$. This difference in effective demand is captured by the difference between $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$ and $\Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(\widehat{s},\widehat{b})\right)$ even if locally the small environments around (s,b) and around $\left(\widehat{s},\widehat{b}\right)$ look the same. In particular, $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right) > \Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(\widehat{s},\widehat{b})\right)$. If sellers s and $\widehat{s}$ are connected to many buyers, the difference between $\sum_{b\in B_s}\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$ and $\sum_{\widehat{b}\in B_{\widehat{s}}}\Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(\widehat{s},\widehat{b})\right)$ makes s very likely to sell even if she had less connections, whereas $\widehat{s}$ has higher values for her links. To illustrate the effect when d_s and $d_{\widehat{s}}$ are small, consider the case that $d_s = d_{\widehat{s}} = 1$, the difference between $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right)$ and $\Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(\widehat{s},\widehat{b})\right)$ implies that $\widehat{s}$ is less likely to be able to sell to $\widehat{b}$, while s is more likely to be able to sell to b and values the link more. The impact of differences in buyers' degree distributions follow a similar logic.

Theorem 2 - Proof. If G^S FOSD $\widehat{G^{\widehat{S}}}$ then by Lemma 1, for every $b \in B_s(m)$ and $\widehat{b} \in B_{\widehat{s}}(\widehat{m})$ such that $d_{\widehat{b}} = d_b$, $\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right) > \Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(\widehat{s},\widehat{b})\right)$. If $d_s = 1$ than $FV_{\widehat{s},\widehat{b}}\left(T^\infty\left(\widehat{m},\widehat{s}\right)\right) \leq FV_{s,b}\left(T^\infty\left(m,s\right)\right)$ is immediate from Theorem 1 and Equation (3). On the other hand,

$$lim_{d_s=d_{\widehat{s}}\to\infty}\left\{\Pi_{b'\in B_s(m)\backslash b}\left[1-\Pr\left(I^t_{T^d(m,s,b')}(s,b')\right)\right]\right\} / \left\{\Pi_{\widehat{b'}\in B_{\widehat{s}}(\widehat{m})\backslash\widehat{b}}\left[1-\Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b'}\right)}(s,\widehat{b'})\right)\right]\right\} = 0 \tag{4}$$

and at the same time

$$\Pr\left(I^t_{T^d(m,s,b)}(s,b)\right) / \Pr\left(I^t_{T^d\left(\widehat{m},\widehat{s},\widehat{b}\right)}(s,\widehat{b})\right) \tag{5}$$

is independent of d_s and $d_{\widehat{s}}$. Combining (4), (5) and (3) and evaluating $\frac{FV_{s,b}}{FV_{\widehat{s},\widehat{b}}}$ for any $b \in B_s$ and $\widehat{b} \in B_{\widehat{s}}$ such that $d_b = d_{\widehat{b}}$ yields the result that $\underline{FV}_{\widehat{s}}\left(T^\infty\left(\widehat{m},\widehat{s}\right)\right) \geq \underline{FV}_s\left(T^\infty\left(m,s\right)\right)$. ■

The implications of Theorem 2 extend those of propositions 1 and 2. A network allows for a STNE if: [1] buyers have degrees that are similar enough, [2] sellers have degrees that are similar enough, and [3] buyers' degrees are not too small or too large relative to those of the sellers that are connected to them. An immediate implication is that there exists a bliss point to the ratio of buyers to sellers in any small area of the network, as well as in the network as a whole.

We interpret our results in this section as suggesting that networks that exhibit *moderate and balanced competition* support a STNE for a large range of discount factors. Proposition 3 illustrates our interpretation by considering simple networks in which all sellers have the same degree and same production costs, and all buyers have the same degree. In this special case, if sellers have degrees that are very high relative to the degrees of buyers (or vice versa), the future values of links are low. Let $c_s = c$ for every $s \in S$, and let $G\left(d^B, d^S\right)$ be a degree distribution such that $g^B\left(d^B\right) = 1$ and $g^S\left(d^S\right) = 1$. Note that in a network m with a degree distribution $G\left(d^B, d^S\right)$, for every $(s,b),(s',b') \in E$, $FV_{s,b}\left(T^\infty\left(m,s\right)\right) = FV_{s',b'}\left(T^\infty\left(m,s'\right)\right)$ and denote this value by $FV^T\left(d^B, d^S\right)$. The proof of Proposition 3 is deferred to Appendix B.

Proposition 3 *Let $c_s = c$ for all $s \in S$. Hold fixed d^B, d^S, and μ. There exist $\overline{d^S}\left(d^B,\mu\right)$, $\overline{d^B}\left(d^S,\mu\right)$, $\overline{\mu}\left(d^S,d^B\right)$, and $\underline{d^S}\left(d^B,\mu\right)$, $\underline{d^B}\left(d^S,\mu\right)$, $\underline{\mu}\left(d^S,d^B\right)$ such that:*

1. *If $d^S > \overline{d^S}$ then $FV^T\left(d^B+1,d^S\right) > FV^T\left(d^B,d^S\right) > FV^T\left(d^B,d^S+1\right)$ and if $d^S < \underline{d^S}$ then $FV^T\left(d^B+1,d^S\right) < FV^T\left(d^B,d^S\right)$.*

2. *If $d^B < \underline{d^B}$ then $FV^T\left(d^B+1,d^S\right) > FV^T\left(d^B,d^S\right) > FV^T\left(d^B,d^S+1\right)$ and if $d^B > \overline{d^B}$ then $FV^T\left(d^B+1,d^S\right) < FV^T\left(d^B,d^S\right)$.*

3. *If $\mu < \underline{\mu}$ then $FV^T\left(d^B+1, d^S\right) > FV^T\left(d^B, d^S\right)$ and if $\mu > \overline{\mu}$ then $FV^T\left(d^B+1, d^S\right) < FV^T\left(d^B, d^S\right)$.*

It is interesting to see how part 2 of Proposition 3 aggregates our results from propositions 1 and 2, and from Theorem 2: a larger d^B corresponds to a combination of a larger degree of a specific buyer (Proposition 1), larger degrees of other buyers connected to the same seller (Proposition 2) and buyers' degree distribution that is larger in a FOSD sense (Theorem 2). The first implies lower links' values, the second implies higher links' values, and the third has a non-monotonic effect. As illustrated in figure 4, the result is non-monotonic in nature.[23]

Figure 4

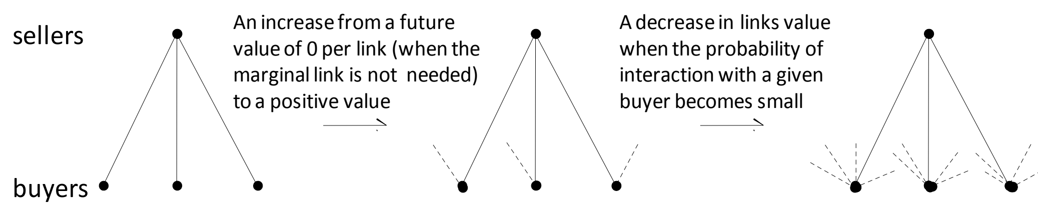

Figure 4: When d^B is low (e.g. the leftmost network in Figure 4), each of the buyers connected to a seller is likely to have demand when meeting the seller, and the seller is guaranteed to sell even if she has fewer connections, raising d^B a little decreases the probability of a sale and the seller needs more connections. However, raising d^B too much reduces the frequency with which a seller interacts with each buyer and the value of each link decreases.

This section concludes that networks that facilitate moderate and balanced competition are better in sustaining cooperation.[24] Consequently, one might expect to find moderate competition in networked markets that manage to rely on bilateral cooperation. In the following section, we show that the need to enforce cooperation may also constrain (or be constrained by) the overall connectivity in a network.

[23] Proposition 3 also sheds light on the role of μ. We illustrate that using our job recommendations example. Recall that a low μ implies that only a small fraction of the teachers have high ability students in every period. Hence, more teachers are required to be connected to every firm to prevent the competition from being 'too weak'. On the other hand, high μ implies that a large fraction of the teachers have high ability students in every period and in order to restrain the fierce competition a low degree for firms or a high degree for teachers is required.

[24] In a related work on competition and seller's reputation in an environment with price competition and no network, Bar-Isaac (2005) finds that competition can both aid and hinder reputation for quality.

6 Cooperation as a constraint on connectivity

Throughout the paper, we do not consider exogenous forces that influence on the structures of networks, but rather focus on understanding how the incentive constraints determine the structure of networks that facilitate STNE. Nevertheless, trade networks are affected by exogenous changes in the economy. For example, the structure of networks of trade opportunities is expected to change following processes of modernization that reduce the costs of creating and sustaining links.[25] Using our framework, we can study whether such changes are consistent with sustaining cooperation.[26]

We focus on two trends that are suggested in the literature. First, with mobile phones, E-mail, and modern transportation, maintaining connections is 'cheaper' and people and businesses can afford to have more links. Second, the collapse of geographic barriers may lead to the rise of large communities and to a decrease in geographical segregation: in the past people from the same village knew (and traded with) the same set of people, whereas in the developed world technology enables a 'global village,' with only little overlap in the sets of connections of even close neighbors.[27]

6.1 Connectivity and congestion

Even with moderate competition, if the degrees of buyers and sellers are 'too' large, cooperation becomes impossible to sustain. Intuitively, the pivotal probability that seller s manages to sell to a specific buyer b, but would not have managed to sell to any other buyer, is negligible when sellers and buyers have many connections.

Theorem 3 *Let $m(\alpha, D)$ be some network in which $min_b\{d_b\} = D$ and $min_s\{d_s\} = \alpha \cdot D$. For every α, μ, and $\overline{FV} > 0$ there exist $\overline{D}(\alpha, \mu)$ such that if $D > \overline{D}$ then*

$$\min_{s\in S, b\in B}\{FV_{s,b}(T^{\infty}(m(\alpha, D), s))\} < \overline{FV}.$$

[25] See Watts (2003) for a non technical survey.

[26] Kranton (1996) studies a different aspect of the impact of modernization on cooperation. In her model, producers and consumers that rely on bilateral cooperation are exposed to markets in which the same goods are traded with no need for cooperation. Kranton shows that cooperation is limited when access to markets is allowed.

[27] See also Mobius and Rosenblat (2004).

Proof. Let $d_b^{\max}$ be the maximal degree of any buyer in network m (so $d_b^{\max} > D$), and let

$$I_{T^d(m,s,b)}^{\min} \triangleq \min_{s \in S, b \in B} \Pr\left(I_{T^d(m,s,b)}^t (s,b)\right) = \left(\Pr\left(I_{T^d(m,s,b)}^t (s,b)\right) | d_b = d_b^{\max}\right). \quad (6)$$

Consider a seller s' that is connected to a buyer b' such that $d_{b'} = d_b^{\max}$. Then, substituting (6) and $d_s = \alpha \cdot D + k$ in (3) yields

$$FV_{s,b}\left(T^{\infty}\left(m\left(\alpha, D\right), s\right)\right) \leq \mu \cdot (\pi - c) \cdot I_{T^d(m,s,b)}^{\min} \cdot \left[1 - I_{T^d(m,s,b)}^{\min}\right]^{\alpha \cdot D + k - 1}.$$

Because $0 \leq I_{T^d(m,s,b)}^{\min} \leq 1$; we have that $lim_{D \to \infty} FV_{s,b}\left(T^{\infty}\left(m\left(\alpha, D\right), s\right)\right) \leq 0$ for any $k \in \mathbb{Z}^+$. ■

Theorem 3 is conceptually related to a broader discussion in both the Market Design literature and the Networks literature in Economics that recognize circumstances in which lack of coordination in markets creates congestion that reduces the volume of transactions in markets and harms assortative and Pareto efficiency.[28] In the Market Design literature, congestion is often due to lack of time to complete search and transactions in the market. In both literatures, congestion is driven by lack of ability to coordinate on who transacts with whom.

According to Theorem 3, congestion can occur also at the fundamental level of deciding who to cooperate with and who to trust. When anyone can potentially cooperate with everyone else, the value of a cooperating partner goes down as each partner has only a small influence on outcomes. This leads to a congested market in which being potentially able to cooperate with everyone means that there is no ability to really cooperate with anyone. A major value creating role of the network is to provide coordination and specify who cooperates with who. This necessary coordination is lost when sellers and buyers have many links.

[28] See also Roth nand Xing (1997), Calvo-Armengol and Zenou (2005), and Fainmesser (2010).

6.2 Community size and segregation

In this section, we show that, holding all else equal, the ability to define small communities according to real or artificial boundaries might increase the future values of links and thus improve the ability to sustain cooperation.

For simplicity, let $c_s = c$ and let $d_s = d^S$, and $d_b = d^B$ for every seller s and buyer b throughout this section. We extend our analysis to consider networks that are divided into islands, such that there are no links between a buyer and a seller that are from different islands. In each island there are $\Psi \cdot d^B$ sellers and $\Psi \cdot d^S$ buyers. $\Psi \in \mathbb{N}$ represents the size of each 'island community'.[29] When comparing across networks with different Ψ, we keep d^B and d^S identical across the compared networks. This allows us to discriminate between the effect of differences in community sizes, and the effects of differences in the degrees of sellers and buyers. Figure 5 provides an illustration of a sample of networks with $d^S = d^B = 2$ and different values of Ψ.

Figure 5

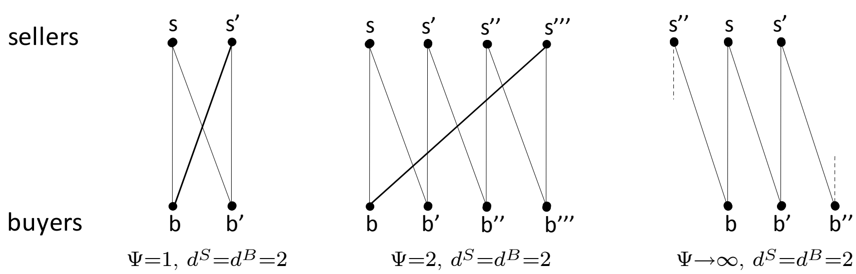

Figure 5: If seller s is informed that $\Psi = 1$, she knows that some seller s' is connected both to buyer b and to b'. Thus, seller s knows that there is perfect overlap between B_s and $B_{s'}$ and between S_b and $S_{b'}$. If seller s is informed that $\Psi \to \infty$, seller s knows that apart from herself, there is no other seller to whom both buyer b and b' are connected. It is also interesting to note that Theorem 1 implies that in a large network that is chosen uniformly at random conditional on $G\left(d^B, d^S\right)$, n_s, and n_b, and without restrictions on Ψ, seller s behaves *as if* apart from herself, there is no seller to whom both buyer b and b' are connected.

Varying Ψ continuously raises technical difficulties and is beyond the scope of this paper. Instead, we focus on two interesting limit cases.

Definition 5 *We say that a network is* ***segregated*** *if* $\Psi = 1$. *A segregated network is divided into small islands in which each of a group of* d^S *buyers is connected to each of a group of*

[29]The economic interpretation of 'a community' depends on the application and can be defined according to geographical region, interests, race, social status, culture, etc.

*d^B sellers. We say that a network is **global** if it is chosen uniformly at random conditional on $G\left(d^B, d^S\right)$, n_s, and n_b, and without restrictions on Ψ.*

We are interested in the following question: When does the existence of a STNE in a segregated network imply that a STNE exists in the corresponding global network and vice versa? Given that sellers' actions are driven by their expectations of future trade, allowing for different network architectures makes a difference for the existence of a STNE only if sellers are aware of the differences. Thus we make the following assumption,

Assumption 4 *Sellers know whether the network is segregated or global.*[30]

In a segregated network $m_{\mathcal{S}}$, it is still true that a STNE exists if and only if for every seller s and buyer b that are connected, $\frac{\delta_s}{1-\delta_s} \cdot FV_{s,b}\left(m_{\mathcal{S}}\right) > \Pi^{s,D}$. Consequently, Theorem 4-1-a suggests that in *sparse* networks that *exhibit moderate competition*, the following claim is true: if there exists a STNE in a global network, a STNE also exists in a segregated network with the same degree distribution. The proof is deferred to the appendix.

Theorem 4 *Let $m_{\mathcal{S}}\left(d^B, d^S\right)$ be a segregated network with a degree distribution $G\left(d^B, d^S\right)$, and let $FV^{\mathcal{S}}\left(d^B, d^S\right) \triangleq FV_{s,b}\left(m_{\mathcal{S}}\left(d^B, d^S\right), s\right)$.*[31] *Then,*

1. *There exists $\overline{d^S} > 1$ such that for every $d^S \leq \overline{d^S}$:*

   (a) *(Moderate competition) There exist $\underline{d^B} > 1$ such that $d^S \leq d^B \leq \underline{d^B}$ implies that $FV^{\mathcal{S}}\left(d^B, d^S\right) \geq FV^{\mathcal{T}}\left(d^B, d^S\right)$.*

   (b) *(Fierce competition) For every $0 << \mu < \frac{1}{2}$ there exists $\overline{d^B}(\mu)$ such that $d^B \geq \overline{d^B}$ implies that $FV^{\mathcal{S}}\left(d^B, d^S\right) \leq FV^{\mathcal{T}}\left(d^B, d^S\right)$.*

2. *(Weak competition) If $d^S > d^B$ then $FV^{\mathcal{S}}\left(d^B, d^S\right) \leq FV^{\mathcal{T}}\left(d^B, d^S\right)$.*

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[30] When $\Psi = 1$ the network topology is unique (up to permutations on the names of buyers and sellers). Consequently, when the network is segregated sellers put probability 1 on the correct network structure and our incomplete information environment is equivalent to one of complete information.

[31] Due to the symmetry across sellers, buyers, and islands, the future value of a link in a segregated network does not depend on the size of the network (n_s and n_b) and is identical across links in the network.

Sketch of the proof. The main idea of the proof can be can be demonstrated using figure 6.

Figure 6a

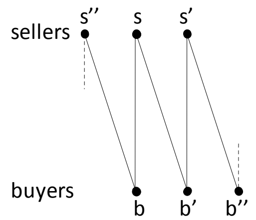
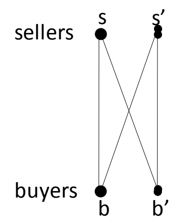


Figure 6b



Theorem 4 is driven by two countervailing forces. On the one hand, *without* the link (s, b) in both networks, the segregated network in figure 6b provides s with a higher probability of trading than the global network in figure 6a. This is because in figure 6b seller s' does not face any competition for selling to b, whereas in figure 6a s' faces competition for selling to buyer b''. Therefore, s' is more likely not to sell to b' in figure 6b. This causes the value of (s, b) in the segregated network (figure 6b) to be lower than in the global network (figure 6a).

Now consider 'adding back' (s, b) to both networks. In the global network in figure 6a the opportunity for seller s to sell the good to buyer b is *independent* of her opportunity to sell the good to buyer b'. On the other hand, in figure 6b the opportunity of seller s to sell the good to b is *negatively correlated* with her opportunity to sell the good to b'. In fact, in the segregated network in figure 6b, s is guaranteed to be able to trade if she has a link to b. This causes the value of (s, b) in the segregated network (figure 6b) to be higher than the global network (figure 6a). In the example in figure 6, the second force dominates and the value of the link between s and b is higher in the segregated network.

If d^B is large, the negative correlation is weak; not being able to trade with b indicates only that, at most, s has one less competitor for trading with buyer $b' \in B_s \backslash b$. However, it is still the case that a seller with a missing link has higher probability of trading in the segregated network.

The second part of Theorem 4 is more straightforward. In a segregated network with more buyers than sellers, a seller is guaranteed to trade with or without her marginal link. Consequently, the value of each link is zero. This is not true for a global network.

7 Welfare

In this section, we pose the following questions: [1] under what circumstances can cooperation that is supported only by repeated interaction in the network achieve the social optimum? [2] Which networks maximize aggregate welfare? Which networks maximize constrained aggregate welfare when maximal welfare is not attainable? To this end, we now pose the following two design problems: let Δm be any probability distribution over network structures. In the *unconstrained network design problem*, a planner chooses Δm and *compels* all sellers (and buyers) to always cooperate. In the *cooperation constrained network design problem*, the planner chooses Δm and *recommends* that all sellers (and buyers) always cooperate. Sellers and buyers are then informed of Δm (as well as of their own degrees and the degrees of buyers and sellers that are connected to them) and follow the planner's recommendation if and only if Δm admits a STNE. The planner cannot affect any of the parameters of the model apart from the set of links E, but is allowed to condition her choice of Δm on all of the parameters of the model.

For a given network m, let $E[V(m)] = E\left[\sum_{s\in S}\sum_{b\in B_s(m)} \Pr\left(I_m^t(s,b)\right)\right]$ denote the expected volume of trade (number of transactions) in high quality goods that is achieved in a given period if all sellers (and buyers) always cooperate. Denote by $E[V(\Delta m)]$ the corresponding value given a probability distribution Δm over networks. Let $N^{uc}(\cdot)$ $(N^{c}(\cdot))$ be the solution to the unconstrained (constrained) network design problem. Then,

$$N^{uc}(\cdot) = \underset{\Delta m}{argmax} E[V(\Delta m)]$$

and

$$N^{c}(\cdot) = \underset{\Delta m}{argmax} \underset{\text{s.t. } \Delta m \text{ admits a STNE}}{E[V(\Delta m)]}$$

Recall that transactions in high quality goods are mutually beneficial. Thus, the proportion of welfare loss due to the constraints on the structure of networks that support STNE is

$$L(\cdot) = 1 - \frac{E[V(N^{c}(\cdot))]}{E[V(N^{uc}(\cdot))]}.$$

If $L(\cdot) = 0$, then repeated interactions support the social optimum.

The following definition is useful for interpreting our results.

Definition 6 *An environment is* ***constantly over- (under-) demanded*** *if there are weakly more (less) buyers than sellers with unit capacity in every period. We call an environment* ***stationary*** *if it is constantly over- (under-) demanded and* ***stochastic*** *otherwise.*

Note also that an environment is stationary if either $n_s \leq n_b$, or $n_s > n_b$ and $\mu = 1$. An environment is stochastic if $n_s > n_b$ and $\mu < 1$. Theorem 5 shows that there exist a trade-off between sustaining cooperation and maximizing the volume of trade in stochastic environments, but not in static ones.[32]

Theorem 5 *:*

1. *Assume that for every $s \in S$, $\frac{\delta_s}{1-\delta_s} \cdot \mu \cdot (\pi - c_s) > \Pi^{s,D}$.[33] Then*
   (a) *$L(\cdot|\mu = 1) = 0$, and*
   (b) *$L(\cdot|n_s \leq n_b) = 0$.*
2. *Let $\mu \in (0,1)$ and assume that for every seller $s \in S$, $\delta_s \in (0,1)$. Then,*
   (a) *there exists $\overline{n}_b \in \mathbb{Z}^+$ such that for all $n_s > n_b > \overline{n}_b$, $L(\cdot|n_b, n_s, \mu) > 0$; and*
   (b) *for any $n_b \in \mathbb{Z}^+$ there exists $\overline{n}_s \in \mathbb{Z}^+$ such that for all $n_s > \overline{n}_s$, $L(\cdot|n_b, n_s, \mu) > 0$.*

In a stochastic environment, only the complete network (in which each seller is connected to all buyers) provides the maximal expected volume of trade. However, if there are sufficiently many buyers and sellers, a dense network cannot support a STNE. Similarly, Part 2-(b) of Theorem 5 relies on the observation that an unbalanced network in which many sellers are connected to a small number of buyers cannot support a STNE.

[32] In related work, Lee and Schwarz (2009) analyze interviewing decisions in labor markets. In their setup, the decision to interview is made after knowing that workers are of at least some minimal quality. Thus, there are no demand and supply fluctuation. Lee and Schwarz find that complete overlap in the interviewing decisions among groups of firms maximizes the number of position filled.

[33] The requirement that for every $s \in S$, $\frac{\delta_s}{1-\delta_s} \cdot \mu \cdot (\pi - c_s) > \Pi^{s,D}$ insures that we are in the more interesting case where no seller is a *pathological* defector and where for any seller s there exists some pattern of repeated interactions that is sufficient to incentivize seller s to cooperate with at least one buyer.

In a static environment it is no longer the case that the complete network is the only network that provides the maximal expected volume of trade. For example, if $\mu = 1$ or if $n_s \leq n_b$ a network in which no buyer or seller has degree higher than 1 can support cooperation and facilitate the maximal volume of trade. Even though some sellers or buyers will be excluded from the market, the expected volume of trade will not be affected.

8 Institutions and networks

Technological progress and the reduction in communication and transportation costs are often considered the drivers of the growth and 'globalization' of networks. However, our results indicate that there are fundamental constraints on the structure of networks for which a STNE exists.

In this section, we review three institutions that help to sustain cooperation and examine their effect on the set of networks for which a STNE exists. The direct effect of each of these institutions on cooperation is well studied. However, the integration of reduced form models of institutions into our framework highlights a new insight: institutions that support cooperation allow for the existence of STNEs with denser buyer-seller networks. Consequently, the difference between markets with and without institutions is underestimated if it is measured only by the level of cooperation between two parties - institutions affect indirectly also the volume of trade. This new theoretical prediction was independently suggested in empirical work on institutions and markets that is reviewed in section 9.

8.1 Community based institutions - reputation networks

Consider a network $\mathcal{R}$ that connects different buyers in B. Let buyers that are connected in $\mathcal{R}$ share with each other information about their past interactions with sellers. For simplicity, assume that a seller s knows whether any two buyers $b, b' \in B_s$ are connected in $\mathcal{R}$. When we add links to $\mathcal{R}$, seller s can lose more than the future value of one link after deviating. While we cannot show that the set of buyer-seller networks for which a STNE exists expands monotonically with the addition of links to $\mathcal{R}$, it is true that adding a sufficient number of links to $\mathcal{R}$ expands the set of buyer-seller networks for which a STNE exists.

Proposition 4 *(FG) Consider a market with S, B, μ, $\{c_s\}_{s\in S}$, $\{\delta_s\}_{s\in S}$, and $\left\{\Pi^{s,D}\right\}_{s\in S}$. For a given $\mathcal{R}$ let $M^{\mathcal{R}}$ be the set of buyer-seller networks for which a STNE exists. Let $\mathcal{R}^1$ be the reputation network in which every two buyers are connected. Then, for any $\mathcal{R}$, $M^{\mathcal{R}} \subseteq M^{\mathcal{R}^1}$.*

Recall that Theorems 3 and 4 suggest that in the absence of institutions, networks that are denser and less segregated have lower values of links. Following Proposition 4, high quality reputation network have the potential to enable cooperation in such networks. However, FG show that some buyer-seller networks do not admit a STNE even when the reputation network is captured by $\mathcal{R}^1$.

8.2 Transaction oriented institutions - litigation and third-party evaluation services

Litigation allows buyers that were 'cheated' to prosecute the deviating seller in order to get compensation and punish the seller directly. Third-party evaluation services inspect the goods *before* trade occurs in order to expose deviations before trade has taken place, decreasing the potential gains from deviation.

Let β^L be the probability that a buyer who was harmed by a deviation succeeds in prosecuting the deviating seller and receives a compensation λ (without loss of generality, λ is also the penalty for the seller). Let β^E be the probability that a third-party evaluation service detects that a low quality good is of low quality, in which case, the deviation of the seller is exposed even though trade does not occur, and buyers can punish the deviating seller.[34] The following Proposition shows that an increase in the quality of either institution (an increase in β^L, λ, or β^E), increases the set of networks for which a STNE exists. The proof is immediate and therefore omitted.

Proposition 5 *Consider a market with S, B, μ, $\{c_s\}_{s\in S}$, $\{\delta_s\}_{s\in S}$, and $\left\{\Pi^{s,D}\right\}_{s\in S}$. Let $M^{\beta^L,\lambda,\beta^E}$ be the set of buyer-seller networks for which a STNE exists given β^L, λ, and β^E. If $\widehat{\beta^L} \geq \beta^L$, $\widehat{\lambda} \geq \lambda$, and $\widehat{\beta^E} \geq \beta^E$ then $M^{\beta^L,\lambda,\beta^E} \subseteq M^{\widehat{\beta^L},\widehat{\lambda},\widehat{\beta^E}}$.*

[34] For simplicity, assume that an evaluation service never mistakes a good product to be of low quality.

9 Discussion

In this section, we discuss the predictions of the model, review evidence, and discuss the relation to existing literature.

9.1 Community structure and cooperation

Our model predicts that networks that are especially good in sustaining cooperation are: [1] *moderately competitive*: the degrees of a buyer and a seller that are connected are similar (Theorem 2 and Proposition 3); [2] *sparse*: the degrees of sellers and buyers in the network are small (Theorem 3); and [3] *segregated*: sellers who have one buyer in common, have connections to similar sets of buyers overall (Theorem 4).

The result that networks facilitate cooperation better when they are sparse is in contrast with some existing theoretical literature, in which adding links helps cooperation. In particular, the literature on repeated games in networks provides a couple of interesting network based extensions of the literature on community enforcement that started with Kandori (1992), Greif (1993), and Ellison (1994). In Lippert and Spagnolo (2010) and in Mihm, Toth, and Lang (2009) increasing the number of links increases the number of bilateral games that a player plays in every period. Consequently, adding links generally improves the ability to sustain cooperation.[35] Our focus is different. First, while increasing the number of links in our model can increase trade opportunities, there is a separation between the network structure and the capacity of each seller and buyer.[36] Second, we focus on buyer-seller networks in which contagion equilibria are not realistic nor feasible. As a result, our framework allows links to be substitutes or complements and additional links can improve or harm cooperation. This allows us to explain why cooperation is limited to sparse networks without assuming an exogenous cost of creating and maintaining a link.

The empirical literature provides ample evidence on the role of networks in markets and finds that additional links can either improve or harm cooperation. The literature on

[35] Kinateder (2008) offers a different model of repeated games, in which all of the players play a common, multi-player game, and a network is used for transferring information about deviations. Clearly, adding more links in this setup helps to sustain cooperation.

[36] In this aspect, our model is more similar to Jackson et. al. (2010) who study a gift exchanges game.

the microfinance industry in developing countries finds exceptionally low default rates even without a centralized credit bureau. Similar to our predictions, microfinance institutions are very local. In every developing country there are significant parts of the population that have no access to loans, independent of their economic status, while others can take multiple loans simultaneously, suggesting that some individuals are part of the market's network while others are excluded. Moreover, our model predicts that strategic default occurs when many lending institutions offer loans to the same borrowers, and do not condition the loan on repayment of debt to some of the other lenders. While the evidence is far from being conclusive, Chaudhury and Matin (2002) and McIntosh and Wydick (2005) document evidence that is consistent with this observation.

Research in other markets in developing and transition economies provide further evidence consistent with the prevalence of networks of cooperation and the requirement that they should be sparse. Fafchamps (1996) surveys manufacturing and trading firms in Ghana and finds that firms rely on repeated bilateral interactions to enforce contracts. McMillan and Woodruff (1999) who study trading networks in Vietnam find that a firm trusts its customer enough to offer credit when the customer finds it hard to locate an alternative supplier.

In developed countries, market activity is also found to be influenced by connections and repeated interactions in a network. Hardle and Kirman (1995), Kirman and Vriend (2000), and Weisbuch et. al. (1996) document a network of consistent loyalty and preferential treatment between buyers and sellers within the fish market in Marseille. Kirman and Vriend assert that the standard asymmetric information model "*seems a too loose application of the textbook argument*". They explain that this is because there is a fixed population of buyers and sellers in this market and "*every buyer (loyal or not) is a potential repeat buyer*" so "*a seller would have an incentive to deliver good quality to every single buyer*". Incidentally, the selective supply of high quality by sellers to only a subset of the population of buyers is consistent with our model - sellers do not have the incentives to maintain reputation with all of the buyers, even if all are potential repeated customers.

In labor markets, as illustrated by example 1 in section 2.1, many hiring decisions are affected by patterns of connections in the market. Fainmesser (2010) allows for truthful communication of workers' qualities along connections in a network and finds that the patterns of connections can affect not only the number and identities of workers hired, but also the timing of hiring in entry-level labor markets. This provides yet an additional motivation to study which networks can facilitate truthful revelation of private information.

9.2 Trade, institutions, and growth

Our results formalize the idea that in the absence of trust facilitating institutions, markets suffer from a significant disadvantage as they are forced to compromise on the volume of trade in order to improve the enforcement of informal contracts. Consequently, even when there is evidence of high level of trust and cooperation in individual transactions, markets with missing institutions are paying a cost via a reduction in overall trade.

While our analysis is stylized, the prediction that trust enhancing institutions allow for denser networks to sustain STNEs is consistent with empirical evidence. In developing countries, Fafchamps (1996) finds that the absence of reputation mechanisms limits the economic reach of manufacturing and trading firms in Ghana, and Johnson et. al. (2002) show that the main effect of belief in the court system is to encourage the formation of new relationships. The complementarity of networks and institutions in the context of the transition to market economies in Eastern Europe is also documented in Woodruff (2002). More generally, there is much evidence that countries with better institutions tend to trade more and grow faster (see Dollar and Kraay 2003 and reference therein).

10 Conclusion

This paper presents a framework that greatly simplifies the analysis of repeated games in networks and provides intuition relevant in many markets.

In contrast with previous literature on networks and markets (see Kranton 1996 and references therein), we do not analyze markets and networks as two mutual exclusive and competing ways to conduct the same activity. We rather focus on markets that are networked. We find that even when every agent in the market can potentially approach any other agent, the need to trust ones partners constrains the trade in the market and allows only certain networks to sustain long term cooperation. We are motivated by evidence that networks are present in many market interactions and suggest that understanding their role improves our understanding of markets.

Our results show that the network structure matters. On one hand, dense and global networks have the potential to maximize trade. On the other hand, these same networks cannot sustain cooperation in environments with asymmetric information and moral hazard. Without cooperation in these environments, there is a risk that no trade will take place at all (see Akerlof 1970).

Consistent with existing evidence, we show that welfare is maximized when proper institutions are in place, and that improving transportation and communication technologies is not enough to promote markets in the lack of trust enhancing institutions.

11 Appendix A: repeated games and incomplete knowledge of the network

In this section, we suggest that studying environments in which individuals have only incomplete knowledge of the network is insightful beyond the tractability it provides. Clearly, repeated interactions provide sellers and buyers with opportunities to learn about their environment. However, even excluding purely behavioral considerations, there are several reasons for market participants not to be able to learn beyond their close local network and some aggregate characteristics of the global environment.

First, much of the economic literature suggests that learning is costly. Consider market participants that learn optimally given the information that they acquire and process, but have costs of information acquisition and processing.[37] Assume that market participants learn directly about the network structure (e.g. viewing a person's links in social networking websites, going through old call or shipment records, or gathering other information on past interactions of a seller or buyer). It is easy to write a model in which assumption 2 is a result. For example, a model in which there are increasing costs of learning information on participants that are at a large distance. On the other hand, if participants focus on frequencies of their own trade to infer the network structure, it is not clear what sellers' beliefs are likely to converge to. In the latter case, assumption 2 is a stylized approximation of the knowledge held by market participants in the long run.

Second, real world networks are dynamic structures, links are added and removed, and buyers' demand changes over time. Nevertheless, the aggregate attributes of networks (such as the degree distribution) seem to be stable over time. Moreover, the local environments

[37]Non-network examples include models of search with memory constraints (e.g. Dow 1991), or limited attention (e.g. Schwartzstein 2009), as well as models of costly information acquisition (e.g. Verrecchia 1982).

of most individuals changes only infrequently. The study of agents' ability to learn the network structure in a changing environment poses many interesting open questions that are beyond the scope of this paper. For now, we suggest that there are market environments in which incomplete knowledge of the network persists over time even for Bayesian agents. We offer below an example of one such environment. While the description of the environment requires more notation, it relies on simple assumptions: [1] buyers are divided into separate groups (buyers from the same group can be connected to different subsets of sellers); [2] only a small subset of the buyers in each group have demand in a given period; [3] all of the buyers that belong to the same group share information about past transactions; and [4] a seller s is connected to buyers from d_s groups. Under these assumptions, as the market becomes large, it is impossible for buyers and sellers to learn much beyond K_s (K_b). At the same time, the repeated nature of the interactions remains intact.

11.1 A market environment in which incomplete knowledge of the network persists over time even for Bayesian agents

Let buyers live in different locations, in every location $l \in L$ there is a set B^l of buyers. A buyer from location l is connected to d_l sellers. A seller s has connections to buyers from d_s locations. For each location l, let the degree distribution of all of the sellers connected to buyers from l be identical to the degree distribution of sellers in the market, and be i.i.d. across buyers from l and across connections of each buyer $b \in B^l$. Denote by m^u the (fixed) underlying network of locations and sellers which is defined as follow: a seller and a location are connected if the seller is connected to at least one buyer from location l.

In every period, only a subset of buyers have unit demand. We call such buyers active. Let $b^{l,active}$ buyers be active in location l in every period, chosen randomly and i.i.d. across locations and periods with the following restriction: a seller s has a connection to one active buyer from each location from a (fixed) set of d_s locations in every period. Within a period, sellers and buyers that are connected meet in a random order (as described in section 3). After transacting, a buyer learns the true quality of the good, and shares it with all of the other buyers in her location.

Note that the degree distribution of the network between sellers and active buyers, $G = \langle G^S, G^B \rangle$, is constant across periods and is determined by L, $\{d_l\}_{l \in L}$, S, $\{d_s\}_{s \in S}$, and $\{b^{l,active}\}_{l \in L}$.

Holding $\{b^{l,active}\}_{l \in L}$ fixed, as $|B|, |S|, |L|, |B^l| \to \infty$, the network m that is generated in every period has a strong random component. Focusing on large markets with random selection of active sellers and buyers creates an environment in which the network structure changes over time without changes to agents' local environments or to the degree distribution. Consequently, complete knowledge of the network is obsolete, and our analysis holds without any changes for anything between agents who know the full network structure in every period and agents who know only basic information that includes their own degree, the degree of their direct neighbors, and the degree distribution G. Clearly, precise conditions are required to establish that the network m is chosen uniformly at random from all of the networks with $\sum_{l \in L} b^{l,active}$ buyers, $|S|$ sellers, and with degree distribution G. We leave the exact conditions necessary as an open question for future research. However, as our analysis throughout the paper suggest, our results are not sensitive to the small changes in the details of the randomization process behind sellers' beliefs, and much of the proofs can be replicated with alternative randomization schemes for the selection of networks.

12 Appendix B: proofs

Theorem 1 - Proof. We note that if there exists any STNE in network m, there exists also a STNE in network m in which buyers employ grim trigger strategies (after being cheated, buyers do not buy from the cheating seller ever again). Thus, for the reminder of the proof, we focus on the case that buyers and sellers employ grim trigger strategies. Furthermore, since buyers never have incentives to deviate unilaterally when all sellers employ trigger strategies, we are left to prove the conditions for sellers only.

Consider a network m and assume that all buyers and sellers employ grim trigger strategies. Consider a seller s who considers whether to deviate or cooperate with all of the buyers that are connected to her, or to deviate in interactions with a subgroup of the buyers connected to her $\widehat{B}_s \subseteq B_s$, starting with some buyer $b \in \widehat{B}_s$. Let $\overline{u}_s(m)$ be the expected utility of seller s from using her best response given her knowledge and belief as implied by the network m and assumptions 2 and 3. Let $u_s^c(m)$ be the expected utility of seller s from using the strategy 'always cooperate' given her knowledge and belief as implied by the network m and assumptions 2 and 3.

Let seller s meet with buyer b in network m. If seller s deviates in her interaction with buyer b her expected utility is $\Pi^{s,D}+\delta_s\overline{u}_s(m\backslash(s,b))-\delta_s u_s^c(m)$. Thus, the strict best response of seller s is always cooperate with all buyers connected to her if and only if,

$$IC_s(m) \triangleq \min_{b\in B_s}\left\{\delta_s\left(u_s^c(m)-\overline{u}_s(m\backslash(s,b))\right)-\Pi^{s,D}\right\} > 0 \tag{7}$$

Let $\left\{m\left(n_b^i,G\right)|s,d,\{d_j\}_{j=1}^d\right\}_{i=1}^{\infty}$ be an increasing sequence of networks such that seller s belongs to all networks in $m\left(n_b^i,G\right)$ and for any i, the degree of seller s in $m\left(n_b^i,G\right)$ is d and the degrees of all of the buyers that are connected to seller s are captured by $\{d_j\}_{j=1}^d$. Then we note the following observation.

Lemma 2 *(FG) Let G be any finite support degree distribution. Then, for any increasing sequence of networks $\left\{m\left(n_b^i,G\right)|s,d,\left\{\overline{d}_b\right\}_{b\in B_s}\right\}_{i=1}^{\infty}$ and for any l, $\lim_{d\to\infty} IC_s(T^d\left(m\left(n_b^l,G\right)\right))$, and $\lim_{i\to\infty} IC_s\left(\left\{m\left(n_b^i,G\right)|s,d,\left\{\overline{d}_b\right\}_{b\in B_s}\right\}\right)$ both exist, and equal one-another.*

Note that when G is finite there is only a finite combination of d_s and $\{d_b\}_{b\in B_s}$ feasible under G. Therefore, by Lemma 2, there exists $\overline{i}$ such that for any $i>\overline{i}$ a STNE with network $m\left(n_b^i,G\right)$ exists if and only if for every seller s and buyer b that are connected in $m\left(n_b^i,G\right)$,

$$IC_s(T^{\infty}\left(m\left(n_b^i,G\right),s\right) > 0 \tag{8}$$

In the final step of the proof we show that for any network m,

$$sign\left\{\min_{s,b}\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)-\Pi^{s,D}\right\}=sign\left\{\min_{s} IC_s(T^{\infty}\left(m,s\right)\right\} \tag{9}$$

Noting that

$$u_s^c\left(m\right)=\frac{1}{1-\delta_s}\sum_{b\in B_s}\Pr\left(I_m^t(s,b)\right) \tag{10}$$

We can rewrite (1) in the following way

$$FV_{s,b}\left(m\right)=\delta_s\left(u_s^c\left(m\right)-u_s^c\left(m\backslash\left(s,b\right)\right)\right) \tag{11}$$

It follows immediately from (7) and (11) that for any $s\in S$, $\min_{s,b}\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)-\Pi^{s,D}<0$ implies that $\min_s IC_s(T^{\infty}\left(m\left(n_b^i,G\right),s\right)<0$. We are then left to prove that $\min_s IC_s(T^{\infty}\left(m\left(n_b^i,G\right),s\right)<0$ implies that $\min_{s,b}\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)-\Pi^{s,D}<0$.

Assume by contradiction that there exists a seller s such that $IC_s(T^{\infty}\left(m\left(n_b^i,G\right),s\right)<0$ and $\min_{b\in B_s}\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)-\Pi^{s,D}\geq 0$. If the optimal strategy of seller s involves a deviation in an interaction with a single buyer b and cooperation with anyone else thereafter then $IC_s(T^{\infty}\left(m\left(n_b^i,G\right),s\right)=\min_{b\in B_s}\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)-\Pi^{s,D}<0$ and we're done. Otherwise, consider a sequence of deviation with all buyers in $\widehat{B}_s\subseteq B_s$, and let $\widehat{b}\in\widehat{B}_s$ be the last buyer such that seller s deviates in an interaction with $\widehat{b}$ and cooperates with anyone else thereafter. Thus,

$$\min_b\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}\left(T^{\infty}\left(m\backslash\left(\widehat{B}_s\backslash\widehat{b}\right),s\right)\right)-\Pi^{s,D}<0.$$

Substituting in (3) yields that for the buyer b that solves the minimization problem, $FV_{s,b}\left(T^{\infty}\left(m\backslash\left(\widehat{B}_s\backslash\widehat{b}\right),s\right)\right)>FV_{s,b}\left(T^{\infty}\left(m,s\right)\right)$ which completes the proof. ■

Lemma 1 - Proof. Consider the following algorithm for matching buyers and sellers in a network $m=\langle S,B,E\rangle$. First, choose an ordering σ of E uniformly at random from all of the $|E|!$ possible orderings of E. Second, repeat the following action iteratively: examine the link (s',b') that was chosen first in the ordering among the links that have not been removed in a previous step. If s' is active, match s' to b' and remove from the ordering all the links (s,b') and (s',b) for all $s\in S_b, b\in B_s$. We note that in any STNE, in any period t, the algorithm above can be coupled with the market activity, such that (s',b') are matched if and only if they trade with each other in period t. Consequently, we can interpret $\Pr\left(I_m^t(s,b)\right)$ as the probability that edge (s,b) is selected by the appropriate randomized matching algorithm.

Following this interpretation, Lemma 1 follows immediately from Proposition 1 of Gamarnik and Goldberg (2010) who study randomized greedy algorithms for matchings in a graph, and the relationship between the local and global properties of the set of matchings of a graph.[38]
■

[38] I thanks David Goldberg for suggesting this proof. An earlier and much longer proof that introduces an algorithm for approximating $\Pr\left(I_m^t(s,b)\right)$ in large networks is available form the author.

Proposition 3 - Proof. We prove first all of the inequality that involve a comparison of $FV^T\left(d^B,d^S\right)$ and $FV^T\left(d^B,d^S+1\right)$. Since $FV^T\left(d^B,d^S\right)=\mu\cdot(\pi-c)\cdot\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\cdot\left[1-\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\right]^{d^S-1}$, Lemma 1 implies that when d^S is large and when d^B and μ are small, an increase in d^S decreases $FV^T\left(d^B,d^S\right)$ by both increasing $\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)$ and the power argument.

We now prove the inequalities that involve a comparison of $FV^T\left(d^B,d^S\right)$ and $FV^T\left(d^B+1,d^S\right)$. To see that when $d^S>1$ there exist small enough $d^B\geq 1$ for which the result for small d^B hold, it is immediate that for any $d^S>1$, $\left(FV^T|d^B=1\right)<\left(FV^T|d^B=2\right)$. The result for small μ follows the same reasoning. Similarly, to see that there exists small enough d^S for which the result for small d^S hold, it is immediate that for $d^S=1$, $\left(FV^T|d^B=1\right)>\left(FV^T|d^B=2\right)$.

For the remainder of the proof, we treat d^B as a continuous variable. We show that $\partial FV_{s,b}/\partial d^B<0$ for large μ and d^B, and that $\partial FV_{s,b}/\partial d^B<0$ for large d^S. First, note that

$$\begin{aligned}\partial FV^T/\partial d^B &= \mu\cdot\left[1-\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\right]^{d^S-2}\cdot\left[\partial\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)/\partial d^B\right]\cdot\\ &\quad\left\{1-\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\cdot d^S\right\}\cdot(\pi-c)\,.\end{aligned}$$

Thus, the sign of $\partial FV^T/\partial d^B$ is determined as the opposite of the sign of $\left\{1-\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\cdot d^S\right\}$ (recall that $\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)$ is decreasing in d^B by Lemma 1). If $\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)$ and d^S are small, $1-\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)\cdot d^S>0$ and $\partial FV^T/\partial d^B<0$, and vice versa. It is only left to note that by Lemma 1, $\Pr\left(I^t_{T^\infty(m,s,b)}(s,b)\right)$ is decreasing in d^B and μ, and increasing in d^S. ■

Theorem 4 - Proof. Part 1a: Consider the case where $d^S=2$ and $d^B=2$ that is captured in Figure 6.

We start by analyzing $FV^{\mathcal{S}}(2,2)$ in Figure 6b. Assume that both b and b' are willing to purchase from s conditional on having demand when they meet. Then, (conditional on having a unit supply) s sells in period t with probability 1. Now assume that b is unwilling to purchase from s and that b' is willing to purchase from s conditional on having demand when they meet. The probability that b' has demand when he meets s is $\left(1-\frac{1}{3}\mu\right)$. To see why, note that b' has demand when meeting s unless the link (s',b') is the first one to be chosen among $\{(s,b'),(s',b'),(s',b)\}$. Therefore

$$FV^{\mathcal{S}}(2,2)=\left[1-\left(1-\frac{1}{3}\mu\right)\right](\pi-c_s)=\frac{1}{3}\mu\cdot(\pi-c_s) \tag{12}$$

We now turn to consider $FV^T(2,2)$ in Figure 6a. Let x be the probability that when s' and b' meet, b' has demand. More generally, for any seller and buyer that are connected, x is the probability that the buyer has demand when they meet. Then

$$FV^T(2,2)=x(1-x)(\pi-c_s)$$

Focusing on the link (s', b') we note that $1 - x$ can be rewritten as the union of the two following mutually exclusive events:

1. The event that [1] s produces in period t, [2] when s and b meet, b does not have demand, and [3] s and b' meet before s' and b' meet.

2. The event that [1] s produces in period t, [2] when s and b meet, b has demand, [3] s and b' meet before s and b meet, and [4] s and b' meet before s' and b' meet.

The probability of the former is $\frac{1}{2}\mu(1-x)$ whereas the probability of the latter is $\mu\left(\frac{1}{3}-\varepsilon\right)x$ for some $\varepsilon > 0$. The addition of ε accounts for the fact that b has demand when he meets s indicates that (s, b) is more likely to have been chosen early. Thus,

$$1 - x = \frac{1}{2}\mu(1-x) + \mu\left(\frac{1}{3}-\varepsilon\right)x$$

and

$$FV^T(2,2) = \frac{6-3\mu}{6-\mu-6\varepsilon\mu}\left(1 - \frac{6-3\mu}{6-\mu-6\varepsilon\mu}\right)(\pi - c_s) \tag{13}$$

We conclude that $FV^T(2,2) < FV^{\mathcal{S}}(2,2)$ if $\frac{6-3\mu}{6-\mu-6\varepsilon\mu}\left(1-\frac{6-3\mu}{6-\mu-6\varepsilon\mu}\right) < \frac{1}{3}\mu$, which holds for every $0 \leq \mu$ and $\varepsilon \leq 1$. To complete the proof of part 1, note that $FV^T(1,2) = FV^{\mathcal{S}}(1,2)$ and $FV^T(1,1) = FV^{\mathcal{S}}(1,1)$ because when $d^B = 1$ the global and segregated networks are identical.

Part 1b: Consider the case where $d^S = 2$ and $d^B \to \infty$. Let the definition of x carry over from the proof of Part 1.

Consider a seller s that is connected to buyers b and b'. In the segregated network, the probability that b does not have demand when meeting s and b' has demand when meeting s is the probability that: [1] s is the first to meet b and not the first to meet b'; or that [2] s is the second to meet b and seller s' who met b before s was the first to meet b'. When $d^B \to \infty$ this can be shown to equal $\left(1 - \frac{1}{\mu\cdot(d^B-1)}\right)\cdot\frac{1}{\mu\cdot(d^B-2)}$.

In the global network (in the limit when $d^B \to \infty$) a seller has $\mu\cdot\left(d^B-1\right)\cdot\left[(1-x) + x\cdot\left(\frac{1}{2}-\varepsilon\right)\right]$ distinct *competitors* for selling to each of the buyers she is connected to. A competitor is an active seller that is connected to the same buyer and that cannot sell to their other connected buyer. Therefore, $x = \frac{1}{\mu\cdot(d^B-1)\cdot\left[1-\frac{1}{2}x-\varepsilon x\right]}$. Again, $\varepsilon > 0$ because the fact that a competitor's other link was useful, implies that it was chosen early, so the probability that the relevant link was chosen before is less than $\frac{1}{2}$. Therefore,

$$FV^T\left(d^B, 2\right) = \frac{1}{\mu\cdot\left(d^B-1\right)\cdot\left[1-\frac{1}{2}x-\varepsilon x\right]}\left(1 - \frac{1}{\mu\cdot\left(d^B-1\right)\cdot\left[1-\frac{1}{2}x-\varepsilon x\right]}\right)(\pi - c_s).$$

As $\mu \nrightarrow 0$ and $d^B \to \infty$, x is small (and in particular $x < \frac{1}{2}$), so a lower bound on x provides a lower bound on $x(1-x)$ and we can focus on demonstrating that $x(1-x) \geq \left(1 - \frac{1}{\mu\cdot(d^B-1)}\right)\cdot\frac{1}{\mu\cdot(d^B-2)}$ for $0 << \mu < \frac{1}{2}$.

From $x = \frac{1}{\mu \cdot (d^B - 1) \cdot \left[1 - \frac{1}{2}x - \varepsilon x\right]}$ we get that $1 + \varepsilon x^2 \cdot \mu \cdot \left(d^B - 1\right) = x \cdot \mu \cdot \left(d^B - 1\right) - \frac{1}{2}x^2 \cdot \mu \cdot \left(d^B - 1\right)$ and $\varepsilon = 0$ provides a lower bound on x. Denote this lower bound as $\underline{x}$ such that $\frac{1}{\mu \cdot (d^B - 1)} = \underline{x} - \frac{1}{2}\underline{x}^2$ and $\underline{x} = \frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\underline{x}^2 \geq \frac{1}{\mu \cdot (d^B - 1)}$. Consequently, $\underline{x} \geq \frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2$. Plugging $\underline{\underline{x}} = \frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2$ into $x(1 - x)$ yields that $(1 - x)x \geq \left[1 - \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right)\right] \cdot \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right)$ and it is sufficient to show that $\left[1 - \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right)\right] \cdot \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right) \geq \left(1 - \frac{1}{\mu \cdot (d^B - 1)}\right) \cdot \frac{1}{\mu \cdot (d^B - 2)}$ for every $\mu < \frac{1}{2}$.

With some algebra this becomes $\mu + \frac{1}{2} \cdot \frac{(d^B - 2)}{(d^B - 1)} + \frac{1}{\mu} \cdot \frac{(d^B - 2)}{(d^B - 1)^2} + \frac{1}{4\mu^2} \cdot \frac{(d^B - 2)}{(d^B - 1)^3} \leq 1$. Recalling that $\mu > 0$ and $d \to \infty$ this is simplified to $\mu + \frac{1}{2} + 0 + 0 \leq 1$ which hold for every $\mu < \frac{1}{2}$.

Part 2: In a segregated network with more buyers than sellers, a seller is guaranteed to trade with or without her marginal link. Consequently, the value of each link is zero. This is not true for a global network. ■

Theorem 5 - Proof. Part 1-(a): consider a network m_1 in which all agents on the short side of the market have degree one and the maximal degree of any agent in the network is 1 (e.g. if $n_s > n_b$ than all buyers have degree 1, n_b sellers have degree one, and $n_s - n_b$ sellers have degree 0). By definition, for every $(s, b) \in E$, $\frac{\delta_s}{1 - \delta_s} \cdot FV_{s,b}(m_1) = \frac{\delta_s}{1 - \delta_s} \cdot \mu \cdot (\pi - c_s) > \Pi^{s,D}$. Applying Theorem 1 and noting that $E\left[V\left(m_1 | \mu = 1\right)\right] = \min\{n_b, n_s\}$ completes the proof.

Part 1-(b): The network m_1 from Part 1-(a) guarantees that $E\left[V\left(m_1 | n_s < n_b\right)\right] = \mu \cdot n_s$ and that $\frac{\delta_s}{1 - \delta_s} \cdot FV_{s,b}(m_1) > \Pi^{s,D}$ as required.

Part 2-(a): Assume by contradiction that for any $\overline{n}_b$ there exists $n_b > \overline{n}_b$ and $n_s > n_b$ such that $L(\cdot | n_b, n_s, \mu) = 0$. Let n_s^t be the number of sellers that are able to produce in period t. The contradiction assumption implies that there exists Δm such that in every period $\min\{n_b, n_s^t\}$ trades take place and that $\min_{s \in S}\left[\frac{\delta_s}{1 - \delta_s} FV_{s,b}(\Delta m) - \Pi^{s,D}\right] > 0$. In particular, in every period t such that $n_s^t < n_b$ the number of trades need be n_s^t, independent of which are the sellers that produce. The only network m that guarantees that n_s^t take place is the complete network in which for every s, $d_s = n_b$, and for every b, $d_b = n_s$. Consider such a network. When $n_b \to \infty$, the probability that in period t a seller s sells if all of the buyers are willing to buy from her, and does not sell if all but one of the buyers is ready to buy from her is bounded above by

$$\min\left\{\frac{n_b}{\mu \cdot n_s}, 1\right\} - \min\left\{\frac{n_b - 1}{\mu \cdot n_s}, 1\right\}. \tag{14}$$

To see why (14) is an upper bound, recall that in network m all seller are symmetric and note that $\min\left\{\frac{n_b}{\mu \cdot n_s}, 1\right\}$ is the probability that any seller manages to sell in a network m when $\mu \cdot n_s$ produce. We claim that $\min\left\{\frac{n_b - 1}{\mu \cdot n_s}, 1\right\}$ is smaller than the probability that s sells in period t if only $n_b - 1$ of the buyers are willing to buy from her. This is because there is a positive probability that some seller s' sells to b before meeting any other buyer. Conditional on that event, the probability that seller s sells in period t is $\min\left\{\frac{n_b-1}{\mu\cdot n_s-1},1\right\}>\min\left\{\frac{n_b-1}{\mu\cdot n_s},1\right\}$.

To conclude the proof, let $n_s=f(n_b)$. Then, for any function $f:\mathbb{Z}^+\to\mathbb{Z}^+$ such that $f(k)>k$ for all $k\in\mathbb{Z}^+$,

$$\lim_{n_b\to\infty}FV_{s,b}(m)=\lim_{n_b\to\infty}\left\{\frac{\delta_s}{1-\delta_s}\cdot(\pi-c_s)\cdot\left[\min\left\{\frac{n_b}{\mu\cdot n_s},1\right\}-\min\left\{\frac{n_b-1}{\mu\cdot n_s},1\right\}\right]\right\}=0$$

which contradicts the assumption that $\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}\cdot FV_{s,b}(\Delta m)-\Pi^{s,D}\right]>0$.

Part 2-(b): Fix n_b and assume by contradiction that for any $\overline{n}_s$ there exists $n_s>\overline{n}_s$ such that $L(\cdot|n_b,\overline{n}_s,\mu)=0$. Let n_s^t be the number of sellers that are able to produce in period t. The contradiction assumption implies that there exists Δm such that in every period $\min\{n_b,n_s^t\}$ trades take place and that $\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}FV_{s,b}(\Delta m)-\Pi^{s,D}\right]>0$. However, given that $\mu<1$, to satisfy that in every period $\min\{n_b,n_s^t\}$ trades take place, Δm must provide each seller with a positive probability of selling in every period that she produces. Thus,

$$\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}FV_{s,b}(\Delta m)-\Pi^{s,D}\right]<\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}\cdot\frac{n_b}{n_s}\cdot(\pi-c_s)-\Pi^{s,D}\right]<0$$

and for $\frac{n_s}{n_b}>\max_{s\in S}\frac{\delta_s(\pi-c_s)}{(1-\delta_s)\cdot\Pi^{s,D}}$, we have that

$$\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}\cdot\frac{n_b}{n_s}\cdot(\pi-c_s)-\Pi^{s,D}\right]<0.$$

This completes the proof by contradiction to $\min_{s\in S}\left[\frac{\delta_s}{1-\delta_s}FV_{s,b}(\Delta m)-\Pi^{s,D}\right]>0$. ■

13 References

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