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# Annexation or Conquest? The Economics of Empire Building

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## *Abstract*

*This paper develops an economic theory of empire building. This theory addresses the choice among three strategies that empire builders historically have used. We call these strategies Uncoerced Annexation, Coerced Annexation, and Attempted Conquest. The theory yields hypotheses that relate the choice among these strategies to such factors as the economic gains from imperial expansion, the relative effectiveness of imperial armies, the costs of projecting imperial military power, and liquidity constraints on financing imperial armies. This theory also yields hypotheses about the scope of imperial ambitions. The paper uses examples from the history of the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.*

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This paper develops an economic theory of empire building. This theory addresses both the choice among strategies that can be used to expand an empire as well as the scope of imperial ambitions. The paper uses examples from the history of the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.

To focus the analysis, we consider the following story: The Romans are thinking about expanding their empire by incorporating a country now ruled by Barbarians. Historical accounts suggest that the Romans, like other empire builders, can choose among three different strategies.

In an **Uncoerced Annexation** the Romans compensate the Barbarians sufficiently to induce the Barbarians to agree to the annexation of their country by Rome. This strategy requires that incorporating the country into the Roman empire would yield either economic or military advantages and that the Romans share these gains with the Barbarians. The attraction of this strategy to the Romans is that it avoids the expense of sending Legions either to threaten or to attempt to conquer the Barbarians.

In a **Coerced Annexation** the Romans induce the Barbarians to agree to the annexation of their country under the threat that the Romans will attack and try to conquer the country. This strategy requires the Romans to send Legions of sufficient strength to the borders of the Barbarian country that the Barbarians decide to capitulate rather than to resist the Romans. The attraction of this strategy to the Romans is that it avoids having to share the gains from annexation with the Barbarians.

In an **Attempted Conquest** the Romans attack the Barbarian country. This strategy is cheaper than the strategy of Coerced Annexation. It requires sending Legions to the Barbarian country, but fewer Legions than would be necessary to induce the Barbarians to capitulate. But, this strategy is risky in that an Attempted Conquest can fail.

Our theory assumes that the Romans will employ any one of these strategies only if its expected value to the Romans is both positive and at least as large as the expected value to

the Romans of any other strategy. If none of these strategies have a positive expected value, then the Romans will not attempt to incorporate the Barbarian country into the Roman empire.<sup>1</sup>

## 1. Uncoerced Annexation

Let  $\omega$  denote the present value of the expected stream of rents that the Barbarians receive as rulers of the country, and let  $\Omega$  denote the expected gross present value to the Romans of annexing the country, where  $\Omega = k \omega$ ,  $k \geq 0$ . If  $k$  is larger than one, then the Romans expect that they can profitably compensate the Barbarians for agreeing to annexation by offering the Barbarians a stream of payments whose present value is at least as large as  $\omega$ .<sup>2</sup>

The factor  $k$  could be larger than one for a variety of reasons. One set of possibilities is directly economic. For example, Roman rule could bring more efficient public administration. It is also possible that as a part of the Roman empire or as an ally of Rome the country would have expanded trading opportunities.

Another set of possibilities is military and, hence, indirectly economic. For example, by annexing the Barbarian country the Romans could gain either access to resources or geopolitical advantages that would make it easier either to defend the existing borders of their empire or to incorporate other desirable countries into their empire.

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<sup>1</sup>A related paper by Ronald Findlay (1996) develops an economic analysis of the size of empires. Findlay's model abstracts from the distinction between annexation and conquest, which is the main concern of the present paper. Also, Findlay treats territorial expansion as a continuous choice variable, rather than modeling the incorporation of discrete countries together with their populations into an empire, as in the present paper.

<sup>2</sup>The formal arrangements of an Uncoerced Annexation can range from direct Roman rule to an alliance with Rome under Roman leadership. If, as is likely, the Romans and the Barbarians doubt the credibility of each other's commitments, then each party can ensure the performance of the other party by making its own obligations, as well as the prerogatives of the other party, conditional on the other party meeting its obligations. As we will see, Roman history reveals that in fact conditionality was an important component of Uncoerced Annexation.

Let  $R_A$  denote the expected value to the Romans of a strategy of Uncoerced Annexation. Allowing for the possibility that  $k$  is larger than one, we have

$$(1) \quad R_A = \max\{R_a, 0\}, \quad \text{where } R_a \leq (k-1)\omega.$$

In equation (1),  $R_a$  equals the difference between the expected gross present value to the Romans of annexing the country and the present value of the compensation that the Romans pay to the former Barbarian rulers.

## 2. Coerced Annexation or Attempted Conquest

Let  $Q$  denote the probability, as perceived by both the Romans and the Barbarians, that, if the Romans send Legions to the Barbarian country, then they will successfully incorporate the Barbarian country into the Roman empire. Let  $g$  denote the size of the Legions that the Romans send to the country, and let  $h$  denote the amount of resources that the Barbarians allocate to defending their country from the Roman threat. Assume that, as in a standard contest-success function,

$$(2) \quad Q = \begin{cases} \frac{\theta g}{\theta g + h} & \text{for } g > 0, \text{ where } \theta > 0, \\ 0 & \text{for } g = 0. \end{cases}$$

Equation (2), together with the parameter  $\theta$ , maps  $g$  and  $h$  into the perceived probability,  $Q$ . The value of  $\theta$  depends on anticipations or perceptions of the factors that determine the effectiveness of Roman Legions against Barbarian defenses. These factors can include the effectiveness of the military tactics that the adversaries employ, the quality of their training, equipment, and leadership, and the amounts of popular support and support from third parties that they enjoy.

According to equation (2)  $Q$  is positive if and only if  $g$  is positive. Also, if  $g$  is positive, then  $Q$  is an increasing function of  $g$  and a decreasing function of  $h$ . If  $g$  is positive and if  $h$  equals zero, then  $Q$  equals one.

To analyze Roman strategy, we must analyze first how the Barbarians respond to a Roman threat. Let  $B_C$  denote the expected value to the Barbarians of resisting a Roman attempt to conquer their country. If the Romans send Legions to their country, then the Barbarians choose  $h$  to maximize  $B_C$ , where

$$(3) \quad B_C = (1 - Q) \omega - h.$$

Assume that in choosing  $h$  the Barbarians take  $g$  as given. This assumption implies that a Roman decision to allocate resources to sending Legions to the country is irreversible. In effect, this model assumes that the Romans are a Stackelberg leader.<sup>3</sup>

From equations (2) and (3) the solution to the Barbarians' choice problem is

$$(4) \quad h = \max\{\sqrt{\theta g \omega} - \theta g, 0\}.$$

Equation (4) implies that the Barbarians choose a positive value of  $h$  if and only if  $g$  is positive but smaller than  $\omega/\theta$ .<sup>4</sup> Thus, a strategy of Coerced Annexation requires  $g$  to be at least as large as  $\omega/\theta$ , which is the minimum positive value of  $g$  that would induce the Barbarians to choose  $h$  equal to zero and to capitulate. Alternatively, a strategy of Attempted Conquest requires a positive, but smaller, value of  $g$ , in which case the Barbarians resist the Romans by choosing a positive value of  $h$ .

Let  $R_C$  denote the expected value to the Romans of sending Legions to the Barbarian country. Assume for now that the cost of sending Legions of size  $g$  is  $g$ . Accordingly, if

<sup>3</sup>Several recent models of the allocation of resources to conflict have explored the implications of Stackelberg leadership. For example, Dmitriy Gershenson (2002) assumes that a group that wants to change the distribution of economic rents is a Stackelberg leader. His model allows the possibility, which is analogous to coerced annexation, that the leader can induce the follower to acquiesce in the desired change. In contrast, Herschel Grossman and Minseong Kim (1995) associate Stackelberg leadership with the defense of claims to property. Their model allows the possibility that the leader can deter the follower from challenging the status quo.

<sup>4</sup>This analysis abstracts from the possibility that the Romans impose additional penalties on Barbarians who resist the Romans unsuccessfully rather than capitulate.

the Romans send Legions to the country, then they choose  $g$  to maximize  $R_C$ , where

$$(5) \quad R_C = Q \Omega - g.$$

In choosing  $g$  the Romans take into account both the direct effect of  $g$  on  $Q$ , as given by equation (2), as well as the indirect effect on  $g$  on  $Q$  through the effect of  $g$  on  $h$ , as given by equation (4).<sup>5</sup>

From equations (2), (4), and (5), the solution to the Romans' choice problem is

$$(6) \quad g = \min\{k^2\theta\omega/4, \omega/\theta\}.$$

Because  $k^2\theta\omega/4$  is smaller than  $\omega/\theta$  only if the product of  $\theta$  and  $k$  is smaller than two, equation (6) implies that, if the Romans send Legions to the country, then they choose  $g$  large enough to induce the Barbarians to capitulate if and only if the product of  $\theta$  and  $k$  is at least as large as two. If  $\theta k$  is smaller than two, then the Romans choose a smaller value of  $g$  the smaller is  $k$ ,  $\theta$ , or  $\omega$ .

Substituting equations (4) and (6) into equation (2), we find that, if the Romans send Legions to the country, then in equilibrium the probability that they will successfully incorporate the country into the Roman empire is

$$(7) \quad Q = \min\{\theta k/2, 1\}.$$

Equation (7) implies that, if and only if the product of  $\theta$  and  $k$  is as large as two, in which case the Barbarians capitulate, then  $Q$  equals one. But, if  $\theta k$  is smaller than two, in which case the Barbarians resist, then  $Q$  is smaller the smaller is  $\theta k$ . Equation (7) also implies that, although  $Q$  depends on  $k$ , which is the ratio of  $\Omega$  to  $\omega$ ,  $Q$  does not depend on the absolute value of  $\omega$ , because in equilibrium both  $g$  and  $h$  are proportionate to  $\omega$ .

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<sup>5</sup>Another consideration, which would give the Romans an incentive to choose  $g$  to be positive but smaller than  $\omega/\theta$ , could be that an Attempted Conquest, by demonstrating the effectiveness of Roman Legions, would increase the value of  $\theta$  in future confrontations with Barbarians. The present analysis abstracts from this consideration.

Substituting equations (6) and (7) into equation (5), the expected value to the Romans of sending Legions to the country is

$$(8) \quad R_C = \begin{cases} \theta k^2 \omega / 4 & \text{for } \theta k < 2 \\ (\theta k - 1) \omega / \theta & \text{for } \theta k \geq 2. \end{cases}$$

Equation (8) says that  $R_C$  is larger the larger are  $\theta$ ,  $k$  and  $\omega$ .

### 3. Roman Strategy

From equations (1) and (8) we can infer the conditions under which the Romans will use each of the three possible strategies. Figure 1 illustrates these results.

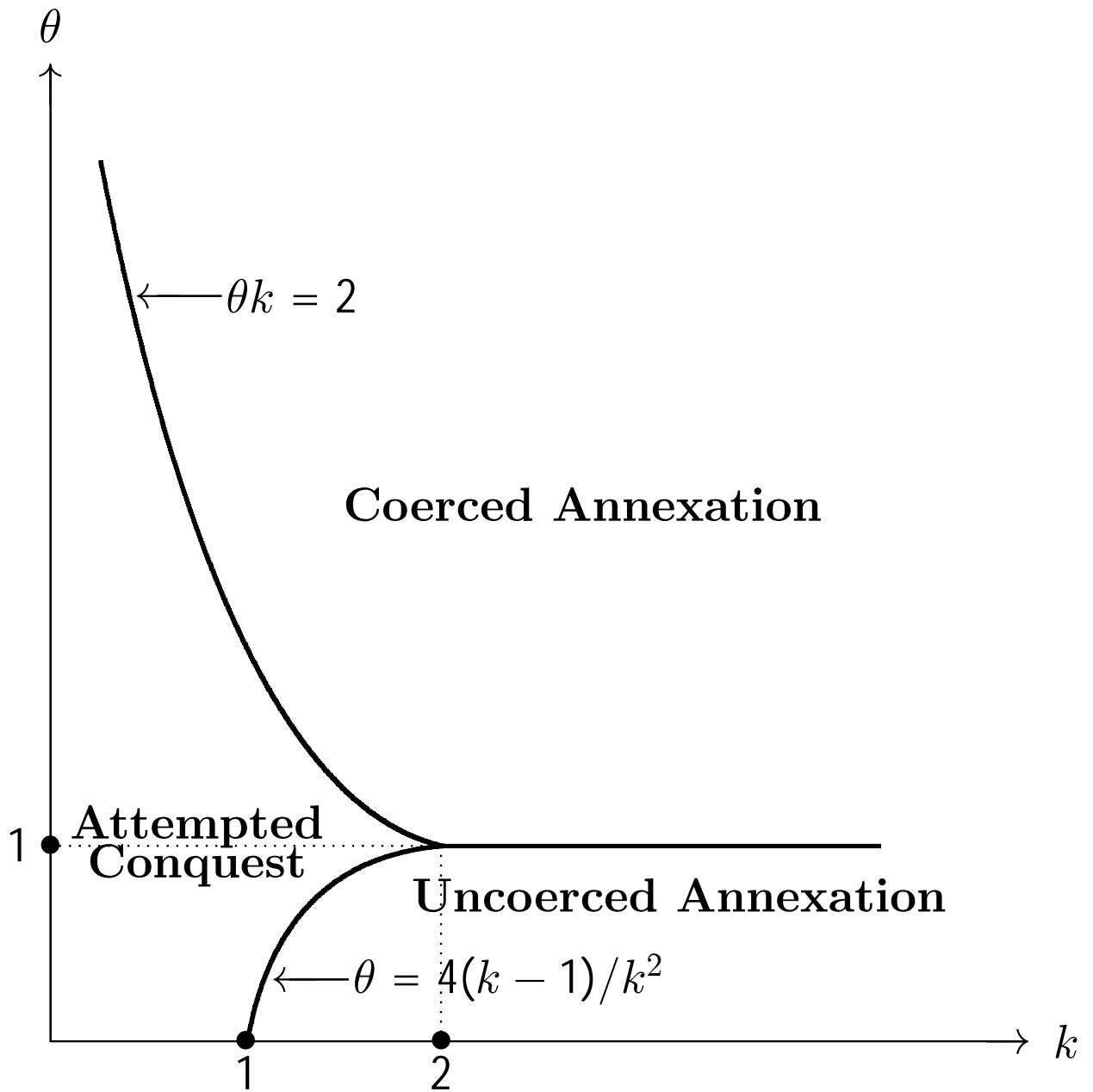
• **Uncoerced Annexation:** The Romans agree to compensate the Barbarians sufficiently to induce the Barbarians to agree to the annexation of their country by the Roman empire only if  $0 < R_A \geq R_C$ . Assuming, for simplicity, that  $R_a$  equals  $(k-1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(9) \quad \begin{array}{l} \text{either } 1 < k < 2 \text{ and } \theta \leq 4(k-1)/k^2 \\ \text{or } k \geq 2 \text{ and } \theta \leq 1. \end{array}$$

[See Appendix A for the derivation of condition (9).] As we see in Figure 1, condition (9) obtains only if  $k$  is larger than one and  $\theta$  is smaller than one. This analysis implies that the Romans use a strategy of Uncoerced Annexation only if annexing the country would yield large economic or military benefits to the Romans and only if the Romans, and the Barbarians, anticipate that Roman Legions are not too effective against the Barbarian defenses.<sup>6</sup>

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<sup>6</sup>The assumption that  $R_a$  equals  $(k-1)\omega$  implies that the Romans agree to make a stream of payments with present value  $\omega$  to the Barbarian rulers of the country and that, consequently, the Romans get all of the expected net gain from incorporating the country into the Roman Empire. If the Romans do not get all of the expected net gain, then, to satisfy the condition  $0 < R_A \geq R_C$ , either  $k$  must be larger or  $\theta$  must be smaller. An interesting extension would be to model the bargaining game between the Romans and the Barbarians.



**Figure 1: Annexation or Conquest?**

$k$ : gain from imperial expansion

$\theta$ : relative effectiveness of imperial armies



• **Coerced Annexation:** The Romans send Legions of sufficient size to induce the Barbarians to capitulate only if  $\theta k \geq 2$  and  $0 < R_C \geq R_A$ . Assuming that  $R_a$  equals  $(k - 1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(10) \quad \theta \geq \max\{1, 2/k\}$$

As we see in Figure 1, condition (10) obtains if  $k$  is larger than two and  $\theta$  is larger than one. If  $k$  is not larger than two,  $\theta$  must be larger the smaller is  $k$ . This analysis implies that, if the Romans expect that the country will have substantial economic or military value as part of their empire, and if the Romans, and the Barbarians, anticipate that Roman Legions are highly effective against the Barbarian defenses, then the Romans use a strategy of Coerced Annexation.

• **Attempted Conquest:** The Romans attack and try to conquer the country only if  $\theta k \leq 2$  and  $0 < R_C \geq R_A$ . Assuming that  $R_a$  equals  $(k - 1)\omega$ , equations (1) and (8) imply that these conditions are satisfied if and only if

$$(11) \quad \max\{4(k - 1)/k^2, 0\} < \theta \leq 2/k.$$

As we see in Figure 1, condition (11) obtains only if neither  $\theta$  nor  $k$  are too large. This analysis implies that the Romans attack and try to conquer a country only if the Romans, and the Barbarians, anticipate that Roman Legions are not too effective against the Barbarian defenses and only if the Romans do not expect that annexing the country would yield economic or military advantages that are too large. To understand this result, observe that equation (6) implies that, with small values of  $\theta$  and  $k$ , although the Romans attack and try to conquer the country, they do not allocate as large an amount of resources to this effort as an alternative strategy of Coerced Annexation would require.

Conditions (9), (10), and (11) show how the Romans' choice of strategy depends on the parameters,  $\theta$  and  $k$ . These conditions also reveal that the Romans' choice of strategy does not depend on the absolute value of  $\omega$ , again because in equilibrium both  $g$  and  $h$  are proportionate to  $\omega$ .

#### 4. The Scope of Imperial Ambitions

In the preceding section, either condition (9), or condition (10), or condition (11) was satisfied. Thus, this analysis implied that the Romans want to incorporate every Barbarian country into the Roman empire. This implication is counter factual.

In order to bound the scope of imperial ambitions, assume that the cost to the Romans of sending Legions of size  $g$  to the Barbarian country is not  $g$ , but  $c$ , where

$$(12) \quad c = \begin{cases} 0 & \text{for } g = 0 \\ f + g & \text{for } g > 0. \end{cases}$$

The parameter  $f$  in equation (12) represents a fixed cost of sending Legions.

The preceding analysis implicitly assumed that  $f$  equals zero. Because the value of  $f$  does not affect the marginal cost of sending Legions to the Barbarian country, equations (6) and (7) obtain whether or not  $f$  is positive. But, if  $f$  is positive, then equation (8) generalizes to

$$(13) \quad R_C = \max\{R_f, 0\},$$

$$\text{where } R_f = \begin{cases} \theta k^2 \omega / 4 - f & \text{for } \theta k < 2 \\ (\theta k - 1) \omega / \theta - f & \text{for } \theta k \geq 2. \end{cases}$$

Equation (13) implies that  $R_C$  is not positive if and only if

$$(14) \quad \begin{array}{l} \text{either } \theta k < 2 \text{ and } f/\omega \geq \theta k^2/4 \\ \text{or } \theta k \geq 2 \text{ and } f/\omega \geq (\theta k - 1)/\theta. \end{array}$$

Condition (14) is satisfied for a combination of a sufficiently large value of the ratio,  $f/\omega$ , and sufficiently small values of the parameters,  $k$  and  $\theta$ . Thus, condition (14) implies that, if the fixed cost of sending Legions is large relative to the wealth of the Barbarian country and if the Romans, and the Barbarians, anticipate that Roman Legions are not too effective against the Barbarian defenses, then the Romans will expect neither Coerced Annexation nor Attempted Conquest to be profitable.

Recall that equation (1) implies that  $R_A$  is not positive if  $k$  is not larger than one. Thus, if both  $k$  is not larger than one and condition (14) obtains, then neither  $R_A$  nor  $R_C$  is positive. Countries for which both  $k$  is not larger than one and condition (14) obtains are beyond the scope of Roman imperial ambitions. The theory implies that the Romans are not interested in incorporating such countries into the Roman empire either through Uncoerced Annexation, Coerced Annexation, or Attempted Conquest.

Even if  $R_A$  and  $R_C$  are positive, a positive value of  $f$  makes  $R_C$  smaller and, hence, would imply that  $R_A$  is larger than  $R_C$  for a larger range of values of  $k$  and  $\theta$ . In other words, a fixed cost of sending Legions makes Coerced Annexation and Attempted Conquest less profitable and makes Uncoerced Annexation relatively more attractive. [See Appendix B for a derivation of the conditions for  $0 < R_A \geq R_C$  with  $f > 0$ .]

## 5. Liquidity Constraints

The preceding discussion has assumed implicitly that the Romans can mobilize sufficient resources to send to the Barbarian country Legions with a size that equals the value of  $g$  given by equation (6). This assumption abstracts from the possibility that the Romans face a binding liquidity constraint on their ability to mobilize resources.

To relax this abstraction, let  $\bar{g}$  denote the maximum amount of resources that the Romans can mobilize to send Legions to the Barbarian country. If condition (10) obtains, but  $\bar{g}$  is smaller than  $\omega/\theta$ , which is the value of  $g$  that a strategy of Coerced Annexation requires, then  $\bar{g}$  is a binding liquidity constraint. This constraint would prevent the Romans from using a strategy of Coerced Annexation that they otherwise would prefer.<sup>7</sup>

If a binding liquidity constraint precludes a strategy of Coerced Annexation, then the Romans instead would use either a strategy of Attempted Conquest or a strategy of Unco-

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<sup>7</sup>A binding liquidity constraint also could cause the Romans to use a strategy of Uncoerced Annexation instead of a strategy of Attempted Conquest that they otherwise would prefer. This situation could occur only if condition (11) obtains, with  $k$  larger than one, and, from equation (6),  $\bar{g}$  is much smaller than  $k^2\omega\theta/4$ .

erced Annexation. The Romans would use a strategy of Attempted Conquest, instead of a strategy of Coerced Annexation, only if  $\bar{g}$  is large enough, and  $k$  is small enough, that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is equal to or larger than  $R_A$ . Alternatively, the Romans would use a strategy of Uncoerced Annexation, instead of a strategy of Coerced Annexation, only if  $\bar{g}$  is small enough, and  $k$  is large enough, that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is equal to or smaller than  $R_A$ .<sup>8</sup>

## 6. The Theory Applied to Roman History

From the fifth century BCE until the early part of the third century BCE, the incipient Roman empire was often engaged in bloody conflicts with its neighbors, both Latin and Etruscan. Our theory, together with facts, suggests reasons for why in this period the Romans used a strategy of Attempted Conquest. First, because civilization at this early date probably was not much more advanced in Rome than in the Latin and Etruscan states, the value of  $k$  associated with incorporating these states into the Roman empire was probably not much larger than one. This fact precluded a strategy of Uncoerced Annexation. Second, the incipient Roman empire probably faced binding liquidity constraints on its ability to finance its Legions. In those days the Legions were composed mainly of conscripts and other part-time soldiers, whose mobilization was ad hoc. This fact precluded a strategy of Coerced Annexation. In addition, the Romans and their adversaries presumably learned only from experience about the high effectiveness of Roman Legions. This observation suggests that Coerced Annexation initially would not have been a feasible strategy even in the absence of binding liquidity constraints.

During the third century BCE the Romans incorporated the remaining states in the Latin League and Etruria into the Roman empire, and during the second century BCE the Romans added most of the Greek states to their empire. In this period, in contrast to earlier

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<sup>8</sup>Substituting equations (2) and (4) into equation (5), and replacing  $g$  with  $\bar{g}$ , we find that the value of  $R_C$  associated with  $g$  equal to  $\bar{g}$  is  $k\sqrt{\theta\bar{g}\omega} - \bar{g}$ .

times, the Romans used mainly a strategy of Coerced Annexation, as many of the states of Italy and Greece capitulated without the Roman Legions attacking.<sup>9</sup> As Edward Luttwak (1976, page 2) tells us,

Forces visibly ready to fight but held back from battle could serve...to control lands and peoples by intimidation —ideally to the point where... effective domination could be achieved without any use of force at all... indeed [the Romans] conquered the entire Hellenistic world with few battles and much coercive diplomacy.

This change in the Romans' predominant strategy suggests that by the third century BCE not only had experience demonstrated the high relative effectiveness of Roman Legions, but also that the Romans no longer faced binding liquidity constraints. The fact that by the third century BCE the Legions comprised a standing army of professional soldiers, whom the Roman state fed, clothed, and equipped, is consistent with a relaxing of liquidity constraints.

It is noteworthy that at about the same time that the Roman empire was consolidating its control of Italy and expanding into Greece the Romans and the Carthaginians were alternately attempting to conquer parts of each other's empires. Historical accounts of the Punic Wars suggest that neither Rome nor Carthage enjoyed a large enough value of  $\theta$  to warrant a strategy of Coerced Annexation with respect to the other. Also, it seems unlikely that incorporation of any part of either one of the empires into the other one would have involved a large enough value of  $k$  to warrant a strategy of Uncoerced Annexation. Hence, as the theory implies, both Rome and Carthage chose the strategy of Attempted Conquest.

In later years, as the Roman empire expanded into central and northern Europe, Asia Minor, and the Middle East, the Romans' predominant strategy changed again. Now they

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<sup>9</sup>The Epirians, whom Pyrrhus led to Pyrrhic victory, and the Macedonians were exceptions. The Romans eventually conquered both of these states, but it seems that in confronting the Epirians and the Macedonians the Romans did not have a large enough value of  $\theta$  to use the strategy of Coerced Annexation.

used mainly the strategy of Uncoerced Annexation. Historians refer to the small countries that the Romans peacefully incorporated into their empire as “client states”. Examples include Judea, the countries of Germanic peoples like the Chauci, Cherusi, Quadi, and Marcomanni, and various countries in Asia Minor. Our theory also suggests an explanation for this change in strategy.

In attempting to push out the northern and eastern frontiers of the empire the Romans encountered both relatively small values of  $\theta$  and relatively large values of  $k$ . On these frontiers the Roman Legions were not highly effective against Barbarian defenses, as evidenced by the mixed success of the Legions in their campaigns against the Germanic tribes, the Dacians, and Parthians. But, annexing the client states yielded large military advantages to the Romans. Both Luttwak and Erich Gruen (1984) stress that these military advantages included enhanced ability both to defend the existing borders of their empire and to incorporate other desirable countries into their empire. As Luttwak (pages 19-20, 26, 27) describes these advantages,

Since clients would take care to prevent attacks against provincial territory, their obedience lessened the need to provide local security at the periphery of the empire against low-intensity threats...Against high-intensity threats, such as invasions on a provincial or even a regional scale, client states and client tribes could contribute both their own interposed forces and their capacity to absorb the threat — in other words, they could provide geographic depth...Another obvious contribution of client states and client tribes to Roman security was the supply of local forces to augment Roman armies on campaign. Naturally, these troops would fall into the Roman category of *auxilia*, i.e., cavalry and light infantry, rather than legionary forces of heavy infantry...Auxiliary troops contributed by clients had played an important role in the campaigns of the republic, not least because they could provide military specialties missing from the regular Roman

arsenal, such as archers, and especially mounted archers...The complementarity between *auxilia* and legionary forces was an important feature of the Roman military establishment; moreover, the forces maintained by the client states were substantial.

Also, many of these client states had relatively primitive economies, and incorporating these countries into the Roman empire resulted in large economic gains. With a small value of  $\theta$  and a large value of  $k$ , our theory implies that the Romans would chose a strategy of Uncoerced Annexation.

How did the Romans compensate the rulers of their client states to induce them to agree to being annexed into the Roman empire? As Luttwak (pages 32, 37) tells us, conditionality was important.

Loyal and efficient client rulers were rewarded by personal honors, ordinarily receiving Roman citizenship (which Augustus's highly restrictive citizenship policy made an important privilege)...More tangible rewards were also given, primarily territorial. That model client, Polemo I, king of Pontus, received Lesser Armenia from Anthony, and when Augustus detached that territory from Pontus, Polemo received instead the important (but, as it turned out, ungovernable) Bosporan state. Similarly, when Herod - a very efficient client ruler indeed - was still in Augustus's good graces, he was granted in 24-23 B.C. part of the plateau country of Ituraea (Golan-Hauran), at the expense of another client, Zenodorus, who had failed to control the nomadic raiding of his subjects...The major active instrument of client management among the primitive peoples of continental Europe was a systematic policy of subsidization...By channeling money and favors through chosen client chiefs, the Romans helped the latter gain power over their subjects, while the Romans gained power over them...Speaking of the once formidable Marcomanni and Quadi, Tacitus describes both as ruled by client rulers

maintained in power — and controlled — by a combination of occasional armed assistance and financial support.

Luttwak (page 37) also suggests that to enforce deals with client states after they were annexed the Romans supplemented Uncoerced Annexation with elements of Coerced Annexation.

The control mechanism was complex. It was necessary to manipulate the tribes through their chiefs, while controlling the chiefs by means of personal threats and personal inducements; always there was the latent threat of force against the tribe as a whole.

Dacia, a relatively rich country, was an exception to the use of a strategy of Uncoerced Annexation. But, the case of Dacia also accords with our theory. According to Peter Wilcox and G. A. Embleton (1987, page 25),

Dacian culture at this time was far in advance of that of their fellow European barbarians. It was, in all recognizable aspects, an embryo civilization...Trade was well organized and encouraged; silver and gold work, pottery, iron implements and weapons, of extremely high quality, were produced for home consumption and export to the sophisticated Roman world in the south.

Because Dacia already was economically advanced, the economic gains associated with adding Dacia to the Roman empire apparently were not large enough to allow the Romans profitably to compensate the Dacians for agreeing to annexation. Instead, just as our theory implies, with neither  $\theta$  nor  $k$  being large the Romans attempted to conquer Dacia, and eventually were successful. Note that in the case of Dacia  $k$  was not large because Dacia was already a rich country.<sup>10</sup>

<sup>10</sup>Another exception was Judea, where a strategy of Uncoerced Annexation, supplemented by elements of Coerced Annexation, worked well until the Jewish revolt (66-70CE) forced the Romans to shift to a strategy of Attempted Conquest, which, as in Dacia, was eventually successful.



Our analysis of the scope of imperial ambitions also seems to be consistent with Roman history. Luttwak (page 96) attributes the Roman decision not to incorporate Scotland into the Roman empire to the Scotland's being "inherently difficult to settle, urbanize, and Romanize". In addition, because of its location, annexation of Scotland was not attractive for geopolitical reasons. In the terms of our theory, Scotland apparently differed from the client states of central and northern Europe, Asia Minor, and the Middle East in that, in addition to  $\theta$  being too small to make a strategy of either Coerced Annexation or Attempted Conquest attractive,  $k$  was too small to warrant a strategy of Uncoerced Annexation. Note that in the case of Scotland  $k$  was small because Scotland was hopelessly poor.

## **7. The Theory Applied to the History of The Mongol and Ottoman Empires**

This analysis does not only offer insights into the building of the Roman empire. In his discussion of the expansion of the Mongol empire, E.L. Jones (1998, pages 108-115) suggests that the Mongols used mainly a strategy of Attempted Conquest, which was usually successful. The account provided by Jones also suggests that for the Mongols, with their superior technology of warfare and their effective terror tactics,  $\theta$  was much larger than one. For example, according to Desmond Martin (1971) and Michel Hoang (1990), between 1211 and 1215 an army of little more than 110,000 Mongols defeated an army of 500,000 Chinese and conquered the Chin Empire of Northern China. According to Martin and Leo de Hartog (1989) in 1223 a Mongol army of 20,000 decisively defeated a Russian army of 80,000 in the Battle of the River Khalka. As Martin writes (pages 11-12),

Whenever it is possible to obtain reliable information on the strength of Mongol forces, one finds that often they were heavily outnumbered by their enemies. We shall see that in 1211 Chingis Khan marched against the Chin with little more than 110,000 men — decidedly less than a quarter of the forces of his opponent. During 1219 he mobilized perhaps 150,000 effectives for the war against the Khwarazm Shah. On that occasion, while the army opposed to him was neither

quite so large — approximately 400,000 — or as well organized as that of the Chin, he had to march west nearly one thousand miles from his last home base before reaching the enemy's border. . . . In the troops of Chingis Khan numerical inferiority, both on the battlefield and on campaign, was common.

Also, there seems to be no reason to presume that, even if it was not larger than one, the value of  $k$  associated with expansion of the Mongol empire typically was much smaller than one. Thus, abstracting from liquidity constraints, it would seem puzzling that the Mongols did not mainly use a strategy of Coerced Annexation. But, if for the Mongols  $\bar{g}$  was a binding constraint, then this puzzle is solved.

Indeed there are other good reasons to think that the Mongols were subject to liquidity constraints. Most importantly, the Mongols started with a small population relative to their imperial ambitions. According to Martin and Hoang, the Mongol population at the beginning of the 13th century was about one million people, whereas by 1260 the Mongol empire had a population of about 100 million people. In addition, because the Mongols were originally nomads, they probably did not begin their imperial expansion with large amounts of accumulated capital or other resources. Also, at the beginning their empire building it is likely that the difficulty of collecting taxes from a nomadic population constrained their ability to mobilize resources.

Turning to the Ottoman empire, our theory suggests an explanation for the clear difference in the strategies that the Ottomans used in Anatolia and in the Balkans. The Ottomans seemed to have used a strategy of Uncoerced Annexation to incorporate Anatolia into their empire. Metin Kunt (1995, page 10) writes, “Ottoman historical tradition describes the incorporation of Muslim lands as a peaceful process, in some cases as voluntary submission.” In contrast, to incorporate the Balkans into their empire the Ottomans used a combination of Coerced Annexation, especially in annexing Bosnia in 1453 and Herzegovina in 1454, and Attempted Conquest in other countries, notably Serbia.

Kunt suggests that the Ottoman armies were more effective in the Balkans than in Anatolia because the Black Death affected the Balkans much more than Anatolia. Kunt writes (page 11),

Consideration should also be given to the effects of the mid-fourteenth century Black Death on the demography of Anatolia and the Balkan peninsula. In later Ottoman sources we find no mention of the plague which devastated so much of the Mediterranean and Europe, though event-book ‘calendars’ have scattered references for various years. Constantinople and other Byzantine territories suffered, as did part of Islamic West Asia such as Syria and Egypt, but not, it seems, the *gazi* Türkmen emirates. It may be that the Turkic and Mongolian population of Anatolia and parts of Iran were relatively immune to the plague strain, which seems to have had its origin in the great Eurasian steppe, carried by Genoese ships from Crimea through Constantinople to the Mediterranean and beyond to northern Europe. If it is true the plague had less of an impact on the Oğuz of Anatolia than on the Greeks and other peoples of the Balkans, this would have been another factor in the swift success of Murad Bey’s troops in the Balkans.

As our theory suggests, enjoying a larger  $\theta$  in the Balkans, the Ottomans put aside the strategy of Uncoerced Annexation that they used in Anatolia.

## 8. The Theory Applied to Nazi Germany

Turning to modern times, our theory also can help us to understand the strategies that Nazi Germany used to build its empire. We focus on German territorial ambitions in central and eastern Europe.<sup>11</sup> On our reading of history, the Germans used a strategy of Coerced

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<sup>11</sup>Whether the Nazis had ultimate territorial ambitions elsewhere is not clear. In September 1939 Great Britain and France, being unwilling to acquiesce in further German aggression in the East, declared war on Germany. A.J.P. Taylor (1961, page 70) tells us, “Against all expectations, Hitler found himself at war with the Western Powers before he had conquered the East. Nevertheless, Eastern expansion was the primary

Annexation in taking over Austria and then the Sudetenland and the rest of Czechoslovakia, a successful strategy of Attempted Conquest of Poland, a strategy of Uncoerced Annexation in inducing Hungary and Romania to join the Axis alliance, and finally an unsuccessful strategy of Attempted Conquest of the Soviet Union. Our analysis implies that these differences in Nazi strategy resulted either from differences in  $\theta$  or in  $k$  or from liquidity constraints.

Prior to the German annexation of Austria in March 1938, the existence of a substantial pro-Nazi faction within Austria undermined the ability of the Austrian leaders to resist German aggression without the support of Austria's ostensible foreign friends. As William Shirer (1960, page 323) reminds us,

Throughout 1937, the Austrian Nazis, financed and egged on by Berlin, had stepped up their campaign of terror. Bombings took place nearly every day in some part of the country, and in the mountain provinces massive and often violent Nazi demonstrations weakened the governments position.

But, Great Britain, France, and Italy failed to support Austrian independence and to oppose the *Anschluss*. Shirer (1960, page 327), drawing on Schuschnigg's book, *Austrian Requiem*, and on Schuschnigg's Nuremberg affidavit provides the following paraphrasing of Hitler's boasting to then Austrian chancellor Kurt von Schuschnigg when they met at Berchtesgaden, Hitler's mountain retreat, in February 1938.

Don't think for one moment that anybody on earth is going to thwart my decisions. Italy? I see eye to eye with Mussolini. . . England? England will not move one finger for Austria. . . And France? France could have stopped Germany in the Rhineland and then we would have had to retreat. But now it is too late for France.

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purpose of his policy, if not the only one." But, other historians suggest that the Nazis intended all along to incorporate western Europe into their empire. See, for example, Gerhard Weinberg (1994, page 107).

Shirer (page 330) also tells us vividly how Schuschnigg was then forced into signing an agreement that effectively ended Austria's independence "under the terrible threat of armed attack". British and French timidity, together with the existence of a substantial pro-Nazi faction within Austria, meant that in confronting Austria the Germans enjoyed a large value of  $\theta$ . Hence, as our theory implies, the Germans used a strategy of Coerced Annexation.

At Munich in September 1938 Great Britain and France, in their efforts to appease Hitler and avoid war, effectively handed the Sudetenland to Germany. Manfred Messerschmidt (1990, page 657) summarizes the British and French reluctance to support the Czechs in the months leading to the Munich pact.

As the British saw, Hitler was in a position to create *faits accomplis* that could only be reversed by a long war. At the beginning of such a war, British aid must be basically confined to economic pressure. Hence, the British government placed more hopes in an Anglo-French policy of urging Prague to be more conciliatory to its German minority. In other words, London basically thought that an initial German success could not be avoided. At the beginning of May [of 1938] Daladier indicated to Bullit, the American ambassador, what conclusion the French drew from this: they would not be able to fight a war in defence of Czechoslovakia. Their attitude was also affected by fear of German air superiority.

According to Shirer (page 417), reporting what a British official told two Czech representatives at Munich,

If you do not accept, he admonished them, as he prepared to go, you will have to settle your affairs with the Germans absolutely alone. Perhaps the French may tell you this more gently, but you can believe me that they share our views. They are disinterested.

Again, because of British and French timidity, the Germans enjoyed a high value of  $\theta$ , and again, as our analysis implies, the Germans used a strategy of Coerced Annexation.

Following the annexation of the Sudetenland, and after again threatening the Czechs with military action, the Germans occupied the rest of Czechoslovakia in March 1939. Once again, it was clear that Great Britain and France would not intervene in defense of Czechoslovakia. Shirer (page 450) writes, “Neither Great Britain nor France made the slightest move to save it, though at Munich they had solemnly guaranteed Czechoslovakia against aggression.” In his discussion of the British and French “guarantee” of Czech independence, Messerschmidt (page 674) writes,

Chamberlain felt committed to it on moral grounds, but he did not envisage the commitment leading to a confrontation with Germany. The French view was much the same. Bonnet and his political friends wanted an accord with Germany. Halifax considered that they must avoid getting into a position in which Great Britain and France might be called on to act against Germany and Italy.

Furthermore, after the Munich pact, the rest of Czechoslovakia laid almost defenseless to the German army. Shirer (pages 421-422) writes

The final settlement of November 20, 1938, forced Czechoslovakia to cede to Germany 11,000 square miles of territory in which dwelt 2,800,000 Sudeten Germans and 800,000 Czechs. Within this area lay all the vast Czech fortifications which hitherto had formed the most formidable defense line in Europe, with the possible exception of the Maginot Line in France.

When the Germans confronted Czechoslovakia in March 1939, again they had a large value of  $\theta$ , and again they used a strategy of Coerced Annexation.

Now we come to Poland. Several factors suggest that, in confronting Poland, the Germans did not have a value of  $\theta$  as large as they enjoyed in confronting Austria or Czechoslovakia. First, the probability of an Allied intervention was higher than in the case of Austria or Czechoslovakia. After the *Anschluss* and the annexation of Czechoslovakia, public opinion

in Great Britain and France was opposed to any further concessions to Germany. The British and French governments acted accordingly. Shirer (page 454) reports that on March 31st, 1939 Chamberlain said before the Commons:

In the event of any action which clearly threatened Polish independence and which the Polish Government accordingly considered it vital to resist with their national forces, His Majesty's Government would feel themselves bound at once to lend the Polish Government all support in their power. They have given the Polish Government an assurance to this effect. I may add that the French Government have authorized me to make it plain that they stand in the same position in this matter.

Second, the Poles were firmer than the Austrians or the Czechs in resisting German threats. As the Germans pressed claims on the port of Danzig and the Polish corridor, Shirer (page 464) writes,

Warsaw was not so easily intimidated as Vienna or Prague. The next day, March 28, Beck sent for the German ambassador and told him, in answer to Ribbentrop's declaration that a Polish coup against Danzig would signify a *casus belli*, that he in turn was forced to state that any attempt by Germany or the Nazi Danzig Senate to alter the status of the Free City would be regarded by Poland as a *casus belli*.

With  $\theta$  in the case of Poland not being large enough to warrant a strategy of Coerced Annexation, and with  $k$  apparently not being large enough to warrant a strategy of Uncoerced Annexation, the Germans, as our theory predicts, used a strategy of Attempted Conquest.

In contrast to Poland, both Hungary and Romania provide examples of Uncoerced Annexation. The distinguishing feature of Hungary and Romania seems to be that  $k$  was large because annexation, which took the form of Hungary and Romania joining the Axis

alliance under German leadership, offered large military advantages to the Germans. These advantages were both geopolitical and material. The Germans needed manpower in order to confront the Soviet Union, and the military forces of both Hungary and Romania would fight, more or less effectively, along side of the Wehrmacht. In addition, the Germans coveted the Romanian oil fields.<sup>12</sup>

To induce the Hungarians and Romanians to join the Axis alliance, the Germans used policies similar to those that the Romans had used in annexing their client states. As a result of the First World War Hungary had lost territory and population, whereas Romania had gained territory and population. In the interwar period Hungarian policy focused on recovering its lost territory and population, whereas Romania was concerned with maintaining its gains. The Germans offered the Hungarians, as well as the Romanians, a share of the spoils of war. Jürgen Förster (1998, pages 412, 416, and 417) tells us,

After the outbreak of the European war Hungarian policy, while maintaining its independence and avoiding an open rupture with the Western powers, was aimed at achieving its revisionist objectives through close alignment with the Axis...[Later] with her Third Army (approximately 146,000 men) Hungary participated in the military and political smashing of Yugoslavia...Hungary reacquired Bácska, the Mur territory, and the Baranya triangle...In consequence, Hungary had nearly doubled her territory, compared with her 1920 frontiers, both in area and in population...

The Germans also offered the Romanians security against the territorial claims of the Soviet Union, Hungary, and Bulgaria. In describing Romanian policy after the Soviet annexation of Bessarabia, Hertza, and northern Bukowina in 1940, Förster writes (page 393)

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<sup>12</sup>From a military standpoint there was little difference between direct German rule, as in the case of Austria, and joining the Axis alliance under German leadership. Also, when it suited other Nazi objectives, especially pursuit of their genocidal policies, the Germans imposed direct rule on Hungary and Romania.



As no help was to be expected from Great Britain, [King] Carol was feverishly trying to gain German support against the Hungarian and Bulgarian revisionist claims...He saw his country's only salvation in a very close alignment with Germany, with which he was willing to co-operate in all fields...On 1 July 1940 Romania renounced the now worthless Anglo-French guarantee of her frontiers and on 11 July left the League of Nations. Carol, moreover, formed a government of respected pro-German ministers...

Then, on June 22nd, 1941, Germany attacked the Soviet Union, launching, with the active support of their Hungarian and Romanian allies, "Operation Barbarossa". In confronting the Soviet Union the Germans surely enjoyed a high value of  $\theta$ . They had amply demonstrated their military might in easily conquering Poland, France, and the Low Countries. Undoubtedly, the Wehrmacht was qualitatively superior to the Soviet army in terms of both weapons and leadership.

But, the Soviet Union had the ability to mobilize massive armed forces. Moreover, the Nazis were not yet in a position to demand large sacrifices for the war effort. Rolf-Dieter Müller (1998, p. 187-188) writes

Once the euphoria of the early summer of 1940 had evaporated, it became obvious that Germany, while dominating large parts of the European continent, had not in fact achieved any abundance of material assets as a result. The performance of the German war economy was still considerably below the pre-war level... Germany was unable to draw on any additional manpower reserves. Repeated demands from military quarters that greater use should be made of female labour were rejected by the political leadership. Any more intensive exploitation of the manpower potential was prevented not only by ideological and administrative obstacles, but also by a lack of enthusiasm for the war among broad circles of the population.

The high command of the German army calculated that “while the enemy would have approximately 155 divisions, German strength would be about the same”. (Shirer, page 822).

Thus, even if  $\theta$  was large enough to warrant a strategy of Coerced Annexation, liquidity constraints apparently prevented the Germans from using this strategy. Accordingly, as our theory suggests, the Nazi Empire chose a strategy of Attempted Conquest of the Soviet Union. From an ex-ante perspective this choice presumably was sound, albeit risky. Unfortunately for the Germans, a combination of bad luck and unexpected Russian heroism caused Operation Barbarosa to fail.

## 9. Summary

This paper has developed an economic theory of empire building. This theory addresses the choice among three strategies that empire builders historically have used: Uncoerced Annexation, Coerced Annexation, and Attempted Conquest. Annexation, whether uncoerced or coerced, is peaceful, whereas Attempted Conquest involves the application of force. Our theory views these strategies as merely different ways to achieve the goal of building a profitable empire in the most profitable way. In the famous words of the 19th century military strategist Carl von Clausewitz (1976, page 87), “War is merely the continuation of policy by other means.”

Our theory implies that the key factors in choosing among these strategies are the economic gains from imperial expansion, the anticipated relative effectiveness of imperial armies, the costs of projecting imperial military power, and liquidity constraints on financing imperial armies. The theory also addressed the scope of imperial ambitions. The paper used historical examples from the Roman, Mongol, Ottoman, and Nazi German empires to illustrate the applicability of the theory.

We conjecture that the theory also would help us to understand the strategies used by other empire builders as well as the scope of their imperial ambitions. Applying the theory

to the building of overseas empires by European powers from the 16th through the 19th centuries would seem to be an especially interesting extension, but we leave this exercise for another paper.

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## Appendix A: Derivation of Condition (9)

$R_A > 0$  obtains only if  $k > 1$ .

Assuming that  $R_a$  equals  $(k-1)\omega$ ,  $R_A \geq R_C$  obtains if and only if

$$\begin{aligned} &\text{either (A) } \theta k < 2 \text{ and } (k-1)\omega \geq \theta k^2 \omega / 4 \\ &\text{or (B) } \theta k \geq 2 \text{ and } (k-1)\omega \geq (\theta k - 1)\omega / \theta. \end{aligned}$$

Observe that  $(k-1)\omega \geq \theta k^2 \omega / 4$  is equivalent to  $\theta k \leq 4(k-1)/k$ .

Also observe that we have  $4(k-1)/k \gtrless 2$  as  $k \gtrless 2$ .

Thus, condition (A) obtains if and only if

$$\begin{aligned} &\text{either (A.1) } k < 2 \text{ and } \theta \leq 4(k-1)/k^2 \\ &\text{or (A.2) } 2 \leq k < 2/\theta. \end{aligned}$$

Next, observe that  $(k-1)\omega \geq (\theta k - 1)\omega / \theta$  is equivalent to  $\theta \leq 1$ .

Thus, condition (B) obtains if and only if

$$(B.1) \quad 1 \geq \theta \geq 2/k.$$

Furthermore, either condition (A.2) or condition (B.1) obtains if and only if

$$(C) \quad k \geq 2 \text{ and } \theta \leq 1.$$

Thus,  $R_A \geq R_C$  obtains if and only if either condition (A.1) or condition (C) obtains.

**Appendix B: Derivation of Conditions for  $0 < R_A \geq R_C$  with  $f > 0$**

$R_A > 0$  obtains only if  $k > 1$ .

Recall that  $R_C \leq 0$  obtains if condition (14) is satisfied.

Thus, assuming that condition (14) is satisfied,  $0 < R_A \geq R_C$  obtains if and only if  $k > 1$ .

Assuming that condition (14) is not satisfied and that  $R_a$  equals  $(k - 1)\omega$ ,  $R_A \geq R_C$  obtains if and only if

$$\begin{aligned} &\text{either (A) } \theta k < 2 \text{ and } (k - 1)\omega \geq \theta k^2 \omega / 4 - f \\ &\text{or (B) } \theta k \geq 2 \text{ and } (k - 1)\omega \geq (\theta k - 1)\omega / \theta - f. \end{aligned}$$

Observe that  $(k - 1)\omega \geq \theta k^2 \omega / 4 - f$  is equivalent to  $\theta k \leq 4(k - 1 + f/\omega)/k$ .

Also observe that we have  $4(k - 1 + f/\omega)/k \geq 2$  as  $k \geq 2(1 - f/\omega)$ .

Thus, condition (A) obtains if and only if

$$\begin{aligned} &\text{either (A.1) } k < 2(1 - f/\omega) \text{ and } \theta \leq 4(k - 1 + f/\omega)/k^2 \\ &\text{or (A.2) } 2(1 - f/\omega) \leq k < 2/\theta. \end{aligned}$$

Next, observe that  $(k - 1)\omega \geq (\theta k - 1)\omega / \theta - f$  is equivalent to  $\theta \leq 1/(1 - f/\omega)$ .

Thus, condition (B) obtains if and only if

$$(B.1) \quad 1/(1 - f/\omega) \geq \theta \geq 2/k.$$

Furthermore, either condition (A.2) or condition (B.1) obtains if and only if

$$(C) \quad k \geq 2(1 - f/\omega) \text{ and } \theta \leq 1/(1 - f/\omega).$$

Thus, assuming that condition (14) is not satisfied,  $R_A \geq R_C$  obtains if and only if  $k > 1$  and either condition (A.1) or condition (C) obtains. Consequently, assuming that condition (14) is not satisfied, the conditions for  $0 < R_A \geq R_C$  are satisfied if and only if  $k > 1$  and either condition (A.1) or condition (C) obtains.